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ORIGINAL ARTICLE

Performance Analysis of Intelligent Robust Facility Layout Design

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Abstract Design of a robust production facility layout with minimum handling cost (MHC) presents an appropriate approach to tackle facility layout problems in a dynamic volatile environment, in which product demands randomly change in each planning period. The objective of the design is to find the robust facility layout with minimum total material handling cost over the entire multiperiod planning horizon. This paper proposes a new mathematical model for designing robust machine layout in the stochastic dynamic environment of manufacturing systems using quadratic assignment problem (QAP) formulation. In this investigation, product demands are assumed to be normally distributed random variables with known expected value, variance, and covariance that randomly change from period to period. The proposed model was verified and validated using randomly generated numerical data and benchmark examples. The effect of dependent product demands and varying interest rate on the total cost function of the proposed model has also been investigated. Sensitivity analysis on the proposed model has been performed. Dynamic programming and simulated annealing optimization algorithms were used in solving the modeled example problems.

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1 Introduction

Facility layout problem (FLP) is one of the most critical issues in the design of manufacturing systems because it significantly affects the total manufacturing cost. Material handling cost (MHC) is one of the most appropriate measures to evaluate the efficiency of a facility layout. The MHC forms 20%–50% of the total manufacturing cost and it can be decreased by at least 10%–30% by an efficient layout design [1]. According to the nature of the product demands and time planning horizon, the FLP can be categorized into static (single period) facility layout problem (SFLP), dynamic (multi-period) facility layout problem (DFLP), stochastic static facility layout problem (SSFLP), and stochastic dynamic facility layout problem (SDFLP). In SFLP, the flow of materials is deterministic and constant over the entire time planning horizon. DFLP includes deterministic, constant, and different flow of materials in each period. In SSFLP, the product demands are random variable so that their parameters are fixed throughout the single time period planning horizon. SDFLP is a multi-period layout problem including different stochastic demand scenarios in each period. The objective of SFLP and SSFLP is to design an optimal layout in such a way that the total MHC is minimized. DFLP and SDFLP have the aim of designing an optimum layout for each period of the planning horizon by minimizing the total material handling and rearrangement costs. In fact, considering each period as a stage, the multi-period problem can be considered as a multi-stage dynamic system with optimal

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behavior from stage to stage. Design of dynamic, robust, and static layouts are three different approaches to deal with a multi-period facility layout problem. The methods are described as follows.

Dynamic approach: Using this method, an optimal layout is designed for each period so that the total material handling and rearrangement costs is minimized [2]. In practice, because rearrangement of facilities is a costly process, the dynamic approach is a well-known method for design of the optimal layout in dynamic environments.

Robust approach: In the robust layout design approach, only one robust layout is designed for the entire time planning horizon with different stochastic demand scenarios. Actually, this layout is used for each period and thereby, there is no rearrangement cost in this approach. The robust layout is not necessarily an optimal layout for a particular time period, but it is the best layout over the entire time planning horizon so that the total MHC is minimized. Therefore, the robust approach has the advantage of lack of rearrangement cost and the disadvantage of not having an optimal layout for each period. This method is appropriate for environments where the facility rearrangement cost is high.

Static approach: Each period is considered as a static problem so that it is solved regardless of other periods' data. In fact, using this method, an optimal layout is designed for each period without considering the facility relocating cost and the layout configuration can be easily changed from period to period. The static approach is suitable for cases with low facility rearrangement cost.

Modern manufacturers such as cellular and flexible manufacturing systems (CMS and FMS) rely on the philosophy of group technology (GT) so that a family of parts are produced. Using GT, the parts, which are similar in design and manufacturing process requirements, are grouped into one family to achieve some benefits such as reduction in material handling, set up time, and work-in-process inventories. In these systems, different kinds of material handling devices such as conveyor, automated guided vehicle (AGV), rail guided vehicle (RGV), and rotating robot arm can be used [3]. This paper aims to design of a robust layout of machines placed in a cell or shop floor in a stochastic dynamic environment of manufacturing systems.

Simulated annealing (SA) is an algorithm that is used to simulate the physical annealing process of solids in statistical mechanics starts with a known or randomly generated initial solution and a high initial value of temperature. It is formed by two loops namely, the inner loop to search for a neighboring solution, and the outer loop for decreasing the temperature to reduce the probability of accepting the non-improving neighboring solutions in the inner loop [4].

2 Literature Review

IRAPPA-BASAPPA, et al [5], designed a robust machine layout for the DFLP using the quadratic assignment formulation. MADHUSUDANAN-PILLAI, et al [6], proposed a SA algorithm to solve their robust layout design model. SOOLAKI, et al [7], solved a cell design problem in an uncertain environment of manufacturing systems by developing a robust optimization model. FORGHANI, et al [8], minimised the total inter and intra-cell MHC to design a robust facility layout in cellular manufacturing systems regarding random demands. NEGHABI, et al [9], proposed a new model and adaptive algorithm for designing a robust facility layout assuming unknown facilities' length and width in advance. NEMATIAN [10] designed a robust single row facility layout assuming fuzzy stochastic variables. POURVAZIRI, et al [11], developed a hybrid multi-population genetic algorithm to cope with the dynamic facility layout problem. AZADEH, et al [12], solved a dynamic layout problem having equal-sized facilities using data envelopment analysis and diversification strategy of tabu search algorithm. FAZLELAHI, et al [13], suggested a model to design a robust facility layout in dynamic environment utilizing a permutation-based genetic algorithm. DERAKHSHAN ASL, et al [14], dealt with static and dynamic facility layout problems having unequal-sized facilities by using a modified particle swarm optimization approach. SEE, et al [15], applied an ant colony algorithm for solving facility layout problems modeled as a quadratic assignment problem (QAP). LEE, et al [16], used genetic algorithm to solve a facility layout problem formulated as QAPs.

3 Problem Formulation

3.1 Assumptions

1. Equal-sized machines are assigned to the same number of known locations.
2. The discrete representation of the SDFLP is considered.
3. Demands of products are assumed to be dependent normally distributed random variables with known expected value, variance, and covariance that randomly change from period to period. Some reasons for assuming normal distribution for the demands are as follows: Many real world data naturally follow a normal distribution [17]. Several distributions such as binomial and Poisson distributions can be approximated to a normal distribution under particular conditions. Product demands have also been considered as

normally distributed random variables in layout design problems in a number of previous studies as given in Refs. [18–22].

4. The confidence level, which represents the decision maker's attitude about uncertainty in product demands, is considered.

5. The parts are moved in batches between machines by material handling devices. Table 1 displays some examples of batch production in previous studies.

6. Time value of money is considered.

7. There is no constraint for dimensions and shapes of the shop floor.

8. Machines can be laid out in any configuration such as rectangular configurations as shown in Fig. 1, where L1,…, L12 are the known machine locations.

9. The data on number of machines, number of periods, machine sequence, present value of part movement cost, transfer batch size, distance between machine locations, money interest rate for each period (year), present value of machine rearrangement cost, the expected value, variance, and covariance of part demands in each period are known as inputs of the model.

3.2 Parameters and Indexes

The parameters and indexes used in this model are shown in Table 2.

3.3 Proposed Model

In the robust layout design method, a dynamic (multi-period) layout problem is converted to a static (single period) problem. Therefore, according to the assumptions (1) and (2), the following 0-1 integer programming form of the

Table 1 Examples of batch production in previous studies

Manufacturing system	Reference No.
FMS	[3, 23, 24]
CMS	[5, 6, 25, 26]

L1	L2	L3	L4
L5	L6	L7	L8
L9	L10	L11	L12
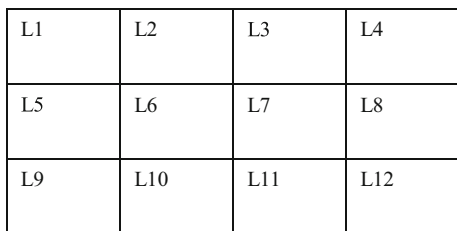
Fig. 1 Rectangular configurations

Table 2 Notations of the proposed model

Notation	Description
K	Number of parts
M	Total number of machines/machine locations
T	Number of periods under consideration
k	Index for parts ($k = 1, 2,…, K$)
t	Index for period ($t = 1, 2,…, T$)
i, j	Index for machines ($i, j = 1, 2,…, M$); $i \neq j$
l, q	Index for machine locations ($l, q = 1, 2,…, M$); $l \neq q$
N_{ki}	Operation number for the operation done on part k by machine i
f_{ijk}	Materials flow for part k between machines i and j
f_{ij}	Materials flow for all parts between machines i and j
D_{tk}	Demand for part k in period t
B_k	Transfer batch size for part k
C_{tk}	The movement cost per batch size per unit distance for part k
C_k	Present value of the movement cost per batch for part k
I_r	Interest rate
d_{lq}	Distance between machine locations l and q
x_{il}	Decision variable of the model
$C(\pi)$	Total MHC for layout π
Z_p	Standard normal Z value for percentile (confidence level) p
$E()$	Expected value of a parameter
$Var()$	Variance of a parameter
$Cov()$	Covariance
$U(\pi_{rm}, p)$	Maximum value (upper bound) of $C(\pi)$ with the confidence level p
OFV_{rm}	Total cost of the robust machine layout

quadratic assignment problem (QAP) formulation suggested by KOOPMANS, et al [27] is used to develop the robust machine layout design model for the SDFLP:

$$Minimize \sum_{i=1}^{M} \sum_{j=1}^{M} \sum_{l=1}^{M} \sum_{q=1}^{M} f_{ij} d_{lq} x_{il} x_{jq} \qquad (1)$$

$$Subject\ to:$$

$$\sum_{i=1}^{M} x_{il}; \ \forall l \qquad (2)$$

$$\sum_{l=1}^{M} x_{il}; \ \forall i \qquad (3)$$

$$x_{il} = \begin{cases} 1 \text{ if facility } i \text{ is assigned to location } l \\ 0 \text{ otherwise} \end{cases} \qquad (4)$$

The objective function Eq. (1) is a quadratic function of the decision variables. In this equation, f_{ij} denotes the flow of materials between facilities i and j. The distance between locations l and q is denoted by d_{lq}. In fact, the objective

function represents the total MHC, which is calculated as the summation of the product of materials flow between facilities and distance between the locations of these facilities. The linear constraints (2) and (3) ensure that each location must contain only one facility and each facility must be assigned to exactly one location respectively. Eq. (4) represents the 0-1 integer decision variables that are the solutions of the problem so that they determine the location of each facility. The robust layout for this multi-period machine layout problem is designed in such a way that the average flow of each part in different periods is considered as the part flow for the entire time planning horizon. The flow of materials for part k between machines i and j in period t (f_{tijk}) can be calculated by using Eq. (5), where the condition $|N_{ki} - N_{kj}| = 1$ refers to two consecutive operations, which are done on part k by machines i and j. The flow of part k between machines i and j over the time planning horizon (f_{ijk}) is given in Eq. (6), which is reformed as Eq. (7) by combining with Eq. (5). According to Eq. (5), in Eq. (7) and the subsequent equations resulting from this equation, the condition $|N_{ki} - N_{kj}| = 1$ should be considered. The total flow between machines i and j resulting from all parts (f_{ij}) is calculated by Eq. (8), which is written as Eq. (9), after combining with Eq. (7). In Eq. (9), D_{tk} is a normally distributed random variable with the expected value $E(D_{tk})$ and variance $Var(D_{tk})$. Therefore, f_{ij} is a normally distributed random variable with the expected value and variance given in Eqs. (10) and (11) respectively.

$$f_{tijk} = \begin{cases} \dfrac{D_{tk}}{B_k} C_{tk} & \text{if} \quad |N_{ki} - N_{kj}| = 1 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

$$f_{ijk} = \frac{\sum_{t=1}^{T} f_{tijk}}{T} \tag{6}$$

$$f_{ijk} = \frac{1}{T} \sum_{t=1}^{T} \frac{C_{tk}}{B_k} D_{tk} \tag{7}$$

$$f_{ij} = \sum_{k=1}^{K} f_{ijk} \tag{8}$$

$$f_{ij} = \sum_{t=1}^{T} \sum_{k=1}^{K} \frac{C_{tk}}{T \cdot B_k} D_{tk} \tag{9}$$

$$E(f_{ij}) = \sum_{t=1}^{T} \sum_{k=1}^{K} \frac{C_{tk}}{T \cdot B_k} E(D_{tk}) \tag{10}$$

$$Var(f_{ij}) = \sum_{t=1}^{T} \left(\sum_{k=1}^{K} \left(\frac{C_{tk}}{T \cdot B_k} \right)^2 Var(D_{tk}) \right. \\ \left. + 2 \sum_{k=1}^{K} \sum_{k'=k+1}^{K} \frac{C_{tk} \cdot C_{tk'}}{T^2 \cdot B_k \cdot B_{k'}} cov(D_{tk}, D_{tk'}) \right) \tag{11}$$

In Eq. (1), considering the flow of parts as a normally distributed random variable causes the total MHC of the robust layout π_{rm} (i.e., $C(\pi_{rm})$) to be a normally distributed random variable as well [28]. The expected value and variance of $C(\pi_{rm})$ are given in Eqs. (12) and (13), respectively. Since we consider time value of money, C_{tk} can be calculated using Eq. (14). For a given robust machine layout π_{rm} if the decision maker considers $U(\pi_{rm}, p)$ as the maximum value (upper bound) of $C(\pi_{rm})$ with the confidence level p, then $U(\pi_{rm}, p)$ given in Eq. (15) can be minimized instead of minimizing $C(\pi_{rm})$ [29–31].

$$E(C(\pi_{rm})) = \sum_{i=1}^{M} \sum_{j=1}^{M} E(f_{ij}) \sum_{l=1}^{M} \sum_{q=1}^{M} d_{lq} x_{il} x_{jq} \tag{12}$$

$$Var(C(\pi_{rm})) = \sum_{i=1}^{M} \sum_{j=1}^{M} Var(f_{ij}) \left(\sum_{l=1}^{M} \sum_{q=1}^{M} d_{lq} x_{il} x_{jq} \right)^2 \tag{13}$$

$$C_{tk} = C_k (1 + I_r)^t \tag{14}$$

$$U(\pi, p) = E(C(\pi)) + Z_p \sqrt{Var(C(\pi))} \tag{15}$$

Using Eqs. (10)-(15) the mathematical model for obtaining the robust machine layout π_{rm} in the SDFLP can be written as the following:

Minimization of

$$OFV_{rm} = \left[\begin{array}{c} \displaystyle\sum_{i=1}^{M} \sum_{j=1}^{M} \sum_{k=1}^{K} \sum_{t=1}^{T} \frac{C_k(1+I_r)^t}{T.B_k} E(D_{tk}) \sum_{l=1}^{M} \sum_{q=1}^{M} d_{lq} x_{il} x_{jq} \\ +Z_p \left(\displaystyle\sum_{i=1}^{M} \left(\sum_{j=1}^{M} \left(\sum_{t=1}^{T} \left(\begin{array}{c} \sum_{k=1}^{K} \left(\frac{C_k(1+I_r)^t}{T.B_k} \right)^2 Var(D_{tk}) + \\ 2 \sum_{k=1}^{K} \sum_{k'=k+1}^{K} \frac{C_k . C_{k'}(1+I_r)^{2t}}{T^2.B_k.B_{k'}} cov(D_{tk}, D_{tk'}) \end{array} \right) \right) \right) \left(\sum_{l=1}^{M} \sum_{q=1}^{M} d_{lq} x_{il} x_{jq} \right)^2 \right)^{1/2} \end{array} \right] \tag{16}$$

Subject to: Eqs. (2)-(4).

Eq. (16) symbolized by OFV_{rm} denotes the objective function (also termed cost function) of the robust machine layout design model. In this model, all parameters, which are defined in Table 2, are known in advance. In fact, this model is deterministic, although part demands, which have the known expected value and variance in each period, are random variables. However, the optimal values of the unknown 0-1 integer decision variables x_{il} are obtained by solving the model. The optimal location of each machine is thus determined and in turn, the robust machine layout π_{rm} is obtained throughout the time planning horizon.

4 Verification and Validation of the Proposed Model

This section aims to verify and validate the proposed model using numerical examples (i.e., to generate a large number of test problems at random), benchmark (data from literature), real case studies, and sensitivity analysis methods. The effect of assuming dependent part demands and time value of money (interest rate) on total cost of the proposed model is also investigated. Rectangular layout configuration is considered by computing rectilinear distance between centers of machines. Dynamic programming (DP) method is used for solving the randomly generated and benchmark problems discussed in sections 4.1 and 4.2 respectively. This is done because DP is an exact method to find the optimal solution of the problems. Therefore, the model validation and analysis carried out in the two aforementioned sections will be more secure than the meta-heuristic approaches such as simulated annealing (SA) algorithm used to solve the problems. It is essential to note that the obtained solution using meta-heuristics is not necessarily an optimal solution, particularly for large-sized problems. However, the real case study is solved using the SA algorithm due to the size of the problem because DP cannot solve it in a reasonable time. This algorithm is coded in MATLAB and run using a PC with an Intel 2.10 GHz CPU and 3 GB RAM.

4.1 Numerical Examples

To validate the proposed model, 104 different-sized randomly generated test problems with $2 < M < 9$ and $1 < T < 7$ are applied to the proposed model. The test problems are also solved in accordance with the static layout design approach. By doing so, for each model, 104 cost function values, which are considered as samples of a population, are obtained.

As we know from statistics, the $100 * (1 - \alpha)\%$ confidence interval for difference between means of two populations is calculated as Eq. (17), where n_1 and n_2 are sample size, $\bar{x}_1$ and $\bar{x}_2$ are sample means, σ_1^2 and σ_2^2 are sample variances, and $z_{\alpha/2}$ is standard normal Z value so that $\Pr(-z_{\alpha/2} \leq Z \leq z_{\alpha/2}) = 1 - \alpha$. The sample mean and sample variance are given in Eqs. (18) and (19) respectively.

$$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2)$$
$$+ z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \tag{17}$$

$$\bar{x} = \frac{\sum\limits_{i=1}^{n} x_i}{n} \tag{18}$$

$$\sigma^2 = \frac{1}{n-1}\sum\limits_{i=1}^{n}(x_i - \bar{x})^2 \tag{19}$$

Using Eq. (17), the 95% confidence intervals for the difference between the populations are calculated as follows: $-14250 < \mu_r - \mu_s < 15840$, where μ_r and μ_s are the mean value of the cost function of the robust model and the static model, respectively.

As we know, the static approach generates an optimal layout for each period. Therefore, it is used as a benchmark to evaluate the performance of the robust layout. The obtained confidence interval is almost a symmetric interval. Thereby, the cost function values of each period for the robust and static layouts are near to each other. To illustrate the conclusion, a numerical example is constructed by using a test problem taken from BALAKRISHNAN, et al [32] in such a way that the flow matrix is considered as the matrix of expectation of flow denoted by E. The matrix of variance of flow denoted by V is computed by $V = E/3$. Additional input data of this problem are as follows: it includes six facilities and five periods. As mentioned, rectangular layout configuration is considered by computing rectilinear distance between centers of machines. This problem is applied to the robust and static models by considering a 0.75 percentile level. Finally, the numerical example is solved using DP algorithm and the results are shown in Table 3.

As mentioned in Section 1, the static and robust approaches are suitable for facility layout design in the cases of low and high rearrangement costs, respectively. It is difficult to relate MHCs with rearrangement costs. However, we assume that the aforementioned conditions for rearrangement costs allowing us to use the static and robust approaches are met in advance. In fact, since neither the static model nor the robust model consider the facility rearrangement cost value, there is no need to mention them

Table 3 Results of the example for robust, and static models

| Model	Period	Optimal layout						Total cost in each period	Total cost in planning horizon
Robust	1	5	6	4	1	2	3	26605	132582
	2	5	6	4	1	2	3	25291	
	3	5	6	4	1	2	3	24541	
	4	5	6	4	1	2	3	28107	
	5	5	6	4	1	2	3	29038	
Static	1	6	2	1	3	4	5	25066	122505
	2	5	6	3	1	2	4	24752	
	3	5	2	3	4	6	1	23883	
	4	4	5	1	2	6	3	25184	
	5	5	4	3	2	6	1	23620	

here. In the static approach, each period is solved separately regardless of data from other periods as well as ignoring the low rearrangement cost. Using this approach an optimal layout, which can be easily changed from period to period, is obtained for each period. On the other hand, using the proposed robust layout design model with the objective function given in Eq. (16), only grants one layout for the entire time planning horizon. As shown in Table 3, each of the five periods has different layouts using the static approach. However, the robust layout [564123] is obtained for the whole time planning horizon by using the robust model.

Thereby, the static layout suffers from the rearrangement cost from period to period, while the robust layout is free of this cost. To evaluate the behavior of the robust layout [564123], which is fixed throughout the planning horizon, we calculate its MHC in each period and in the entire planning horizon. Then, the MHC of the layouts obtained by the static approach is also computed for each period and throughout the planning horizon. As mentioned, the static approach generates an optimal layout for each period, and thereby it can be used as a benchmark for evaluating the performance of the proposed robust layout. The MHC of each period for static and robust approaches is compared with each other in Fig. 2. On comparison, in

each period, the MHC of the robust layout is larger than that of the static layout. Therefore, the robust layout is not necessarily an optimal layout for each period. The results also show that in most periods, the MHC of the robust layout is close to that of the static layout, which is the optimal value. In addition, considering the whole time planning horizon, the difference between the total costs of the robust layout (132582) and the static one (122505) is 7.6%. The MHC results in most periods for random demands, however, are relatively close to the optimal value of the static layout, as shown in Fig. 2. Over the entire planning horizon, the difference between the total costs of the robust layout (132582) and the static one (122505) is 7.6%, making the robust layout reasonably efficient for the whole time planning given the randomly varying demands now.

4.2 Benchmark Study

Since there is no historical data on the expectation and variance of demands in each period, the proposed model is tested in a deterministic environment by comparing it with the previous approaches as benchmark data. To this end, a 50% percentile level ($p = 50\%$) equivalent of $z_p = 0$ is applied to the model. By doing so, the second term of the objective function given in Eq. (16) containing variance of parts demand is ignored. This also implies that the data on the variance of the part demands in each period are not required. Hence, the demand of parts in each period, which is known in the deterministic case, is regarded as the expectation of parts demand in the proposed model. For simplicity and without losing the generality of the proposed model, independent product demands are considered, and thereby there is no correlation between demands. As a result, in Eq. (16), the term covariance is zero. The data set used for testing the proposed model is taken from
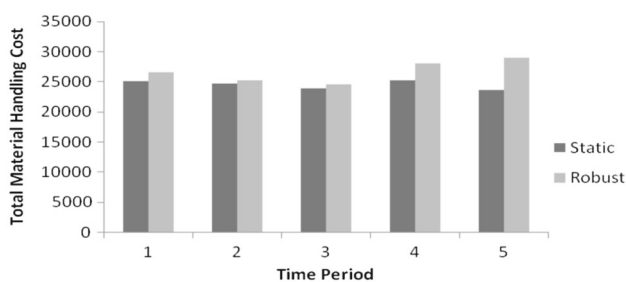
Fig. 2 Total cost of static and robust layouts in each period

Table 4 Data on machine sequence

Part No.	Machine sequence
1	$1 \rightarrow 3 \rightarrow 5 \rightarrow 7 \rightarrow 2 \rightarrow 7 \rightarrow 9$
2	$1 \rightarrow 4 \rightarrow 2 \rightarrow 5 \rightarrow 6 \rightarrow 8 \rightarrow 9$
3	$1 \rightarrow 5 \rightarrow 7 \rightarrow 8 \rightarrow 5 \rightarrow 6 \rightarrow 2 \rightarrow 9$
4	$1 \rightarrow 2 \rightarrow 4 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 2 \rightarrow 3 \rightarrow 9$
5	$1 \rightarrow 7 \rightarrow 6 \rightarrow 4 \rightarrow 2 \rightarrow 8 \rightarrow 3 \rightarrow 5 \rightarrow 6 \rightarrow 9$

Table 5 Part demand in different periods

Part No.	Period				
	1	2	3	4	5
1	10	35	90	40	55
2	30	50	25	65	20
3	45	15	40	70	15
4	70	80	55	90	85
5	85	60	70	20	30

YAMAN, et al [33]. The input data are as follows: The problem includes nine machines and five periods. Data on machine sequence and part demand in different periods are given in Tables 4 and 5, respectively. Rectangular layout configuration is considered so that location grid of 3×3 is used as facilities locations. Part movement cost, and batch size are set to ten and one, respectively.

As mentioned, MADHUSUDANAN-PILLAI, et al [6]. proposed an SA algorithm to solve their robust layout design model, which is termed MP-SA hereafter. Another heuristic method was developed by IRAPPA-BASAPPA, et al [5]. to solve their model, which is denoted by I-M hereafter.

YAMAN, et al's [33] problem is applied to the proposed robust machine layout model. To calculate the MHC of each period, the obtained robust layout is applied to the demands data in different periods of the planning horizon.

Table 6 shows the results of the proposed robust layout design model and the previous methods. On comparison, the proposed model has the best performance with respect to the I-M and MP-SA methods in each period. In addition, the proposed robust model can be applied to both deterministic and stochastic environments, whereas the model developed by IRAPPA-BASAPPA, et al [5] can only be used for deterministic environment. Table 7 shows the total MHC over the entire planning horizon obtained by the proposed robust model and the previous approaches including the MAIN method suggested by CHAN, et al [34], the I-M, and MP-SA methods. Based on the various results, the proposed robust model has the best performance so that it leads to 2.7% improvement in total cost with respect to MP-SA method, which is the best previous method. It is necessary to mention that in this section, to validate the proposed robust model only one test problem taken from the literature is solved. Therefore, unlike the approaches done in section 4.1, there is no need to state the significant level for the 2.7% improvement in total cost. Tables 6 and 7 are examples of using our proposed model in the deterministic case.

Facility layout design application example

According to KRISHNAN, et al [35], in the real world case study, six parts are processed using 21 equal-sized machines $(35' \times 35')$. Part movement cost and machine rearrangement cost are presumed to be \$3.75/foot and zero respectively. An aisle space of 10 feet is assumed to be considered around each machine. Euclidean distance between centers of machines is considered in the case study. The data on yearly part demands, which are given in Table 8, are considered as the expectation of the part demands. Since there is no data on variance of part demands, a 50% percentile p equivalent of $z_p = 0$ is considered. Machine sequence data are given in Table 9. The data of the problem are applied to the proposed robust machine layout design model and is solved using an SA algorithm. SA is an

Table 6 MHC of the robust model and previous approaches

Approach	Period					Planning horizon
	1	2	3	4	5	
I-M	28800	27200	31200	32350	22350	141900
MP- SA	27800	26400	29500	31300	22000	137000
Proposed robust model	26900	26100	28700	29900	21700	133300

Table 7 Total MHC of the robust model and previous approaches

Approach	Chan	I-M	MP-SA	Proposed robust model	Change with respect to MP-SA %
Total cost	145700	141900	137000	133300	−2.7

Table 8 Expectation of part demands for real case study

Year	Part					
	A	B	C	D	E	F
2002	15	8	18	18	16	17
2003	19	25	25	21	27	21
2004	33	37	39	43	39	42
2005	58	64	58	56	58	57
2006	46	46	46	46	46	46

intelligent approach that has been widely used to solve the FLP. It is a simulation of physical annealing process of solids in statistical mechanics, which starts with a known or randomly generated initial solution and a high initial value of temperature. It is formed by two loops, namely, the inner loop to search for a neighboring solution, and the outer loop for decreasing the temperature to reduce the probability of accepting the non-improving neighboring solutions in the inner loop. Table 10 shows the results of our method and that of the best previous one including the cost associated with each period and the total cost over the whole time planning horizon. As shown in Table 10, the obtained robust machine layout leads to 7.35% improvement with respect to the best previous one proposed by KRISH-NAN, et al [35]. Using the same reason as in previous section, there is no need to state the significant level for the 7.35% improvement.

4.3 Sensitivity Analysis

In this section, sensitivity analysis is performed for validating the proposed model and to rank the inputs of the model in terms of the degree of their impact on the output of the model (objective function). Sensitivity of the output of the proposed robust machine layout design model with respect to the input parameters including expectation of materials flow, variance of materials flow, and confidence level is investigated by using one-way analysis of variance (ANOVA). Using this technique, the null and alternative hypotheses are usually tested by using the F-test. The null hypothesis states that the means amongst two or more groups are equal and the alternative one indicates that at least two means are different. In ANOVA, it is assumed that the mean of the model outputs for each group is normally distributed random variable with approximately the same variance. Considering the assumptions, the F-value is statistically important at $p < 5\%$ and the null hypothesis is rejected [36]. In ANOVA, an input and an output of a model are referred to as a "factor" and a "response variable" respectively [37]. The factors are ranked according to the F-values [38]. Inputs with higher F-values are more sensitive factors, which more strongly affect the output of a model. The aim of one-way ANOVA is to realize whether data from several groups have the same mean.

To perform the sensitivity analysis, 100 randomly generated test problems are applied to the proposed model in

Table 9 Machine sequence for real case study

Part	Machine sequence
A	10 → 12 → 13 → 5 → 18 → 14 → 21 → 15 → 1 → 11 → 19 → 20
B	10 → 12 → 5 → 4 → 18 → 14 → 21 → 14 → 18 → 15 → 1 → 11 → 19 → 20
C	10 → 8 → 9 → 10 → 5 → 18
D	3 → 4 → 10 → 8 → 9 → 10 → 4 → 18
E	2 → 10 → 8 → 9 → 10 → 5 → 18
F	15 → 17 → 6 → 2 → 16 → 9 → 10 → 7 → 5 → 18

Table 10 Results of real case study

Year	Approach		
	Krishnan	Robust model & SA algorithm	Percentage Savings (%)
2002	$194,638.76	$165,816	14.80
2003	$211,428.07	$200,324	5.25
2004	$347,675.62	$343,410	1.22
2005	$464,675.32	$480,124	3.20
2006	$560,878.42	$458,821	18.20
Total Cost	$1,779,296.19	$1,648,495	7.35

three different cases, namely, Cases E, V, and P, which investigate the sensitivity of the objective function of the model with respect to expectation of materials flow (matrix E), variance of materials flow (matrix V) and confidence (percentile) level (p) respectively. The input data are as follows: For each test problem, the expectation and variance of part demands (E and V) are randomly generated with uniform distribution so that $E \in (1000, 10000)$ and $V \in (1000, 3000)$. Besides, the number of machines and the number of periods are six and three respectively ($M = 6, T = 3$).

In Case E, matrix E is changed by $E' = E + r * E$, whereas matrix V and confidence level p remain unchanged. Considering nine different values of $r \in A = \{01, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$ leads to generate nine different matrices E. Each test problem is solved for each of the nine different Es so that the optimal value of each objective function corresponds to each E is obtained. Similarly, in Case V, matrix V is changed by $V' = V + r * V$, whereas matrix E and confidence level p remain unchanged. Considering the nine aforementioned values of r leads to generate nine different matrices V. Each test problem is solved for each of the nine different V so that the optimal value of each objective function corresponding to

each V is obtained. Finally, in Case P, the confidence level p is set to each element in A while matrices E and V remain unchanged. Similar to the two former cases, each test problem is solved for each of the nine different p values so that the optimal value of each objective function corresponding to each p is obtained. In fact, each case includes nine groups (populations) and each group contains 100 samples of objective function values. Each case, group, and sample is denoted by k, g, and s, respectively, where (k = E, V, P), ($g = 1, 2, \ldots, 9$), and ($s = 1, 2, \ldots, 100$). Mean of samples within group g in case k is represented by μ_g^k. As mentioned, the condition of having normal distribution for the mean of each group is necessary for ANOVA. To meet this condition, 100 randomly generated test problems are considered. This is due to the central limit theorem (CLT), which states that the average of a sufficiently large number (for example, ≥ 30) of independent random variables follows normal distribution [39].

The aim of this section is to test the hypothesis given in Table 11 using ANOVA. Using MATLAB software, ANOVA is applied to the results of the randomly generated test problems for testing the aforementioned hypothesis. The results of ANOVA are given in Table 12. According to the results including F-values and P-values, the null hypothesis H_0 is rejected. In other words, as expected, different values of input parameters containing expectation of product demands, variance of product demands, and confidence level lead to designing different facility layout. Therefore, the model validation concludes. As mentioned, the input with higher F-value is the more sensitive parameter. According to F-values in Table 12, the expectation of product demand and the variance of product demand are the most and least sensitive parameters, respectively. This is because of $(F_E = 7.98) > (F_P = 3.05) > (F_V = 0.37)$, where F_E, F_V, and F_p are F- values in the Cases E, V, and P respectively. Experimentally, we concluded that changes in the parameters including number of machines, number

Table 11 Hypotheses needed for sensitivity analysis using ANOVA

Case	Case description	Hypothesis
E	$E' = E + r * E$ $V' = V$ $p = 0.75$	$H_0^E : \mu_1^E = \mu_2^E = \cdots = \mu_9^E$ H_1^E : At least two means are different
V	$E' = E$ $V' = V + r * V$ $p = 0.75$	$H_0^V : \mu_1^V = \mu_2^V = \ldots = \mu_9^V$ H_1^v : At least two means are different
P	$E' = E$ $V' = V$ $p = r$	$H_0^P : \mu_1^P = \mu_2^P = \ldots = \mu_9^P$ H_1^P : At least two means are different

Table 12 Results of ANOVA (SS means sums of squares; df means degrees of freedom; MS means mean squares; F = SS/df)

Case	Source	SS	df	MS	F	Prob > F (P -value)
E	Columns	1.026×10^{17}	8	1.28×10^{16}	7.98	2.007×10^{-10}
	Error	1.431×10^{18}	891	1.61×10^{15}		
	Total	1.534×10^{18}	899	–		
V	Columns	2.593×10^{15}	8	3.24×10^{14}	0.37	0.9361
	Error	7.787×10^{17}	891	8.74×10^{14}		
	Total	7.813×10^{17}	899	–		
P	Columns	1.394×10^{16}	8	1.74×10^{15}	3.05	0.0022
	Error	5.089×10^{17}	891	5.71×10^{14}		
	Total	5.229×10^{17}	899	–		

Table 13 Example for analyzing demands correlation and interest rate

Part No.	Variance- covariance matrix			Expectation of part demand		Machine sequence
	1	2	3	Period 1	Period 2	
1	10,000	640	4000	1000	1500	$1 \to 2 \to 3$
2		100	4000	10,000	15,000	$2 \to 3$
3			2500	5,000	7500	$1 \to 2$
Machine relocating cost = 1000				Interest rate = 10%		

Table 14 Total cost for three cases

P	Case		
	i	ii	iii
0.1	3960.7	3841.7	4573.4
0.2	4060.2	3982	4706.3
0.3	4132.5	4084.1	4803
04	4193.5	4170.3	4884.5
0.5	4250	4250	4960
0.6	4308.8	4332.9	5038.5
0.7	4369.8	4419.1	5120.1
0.8	4442.1	4521.1	5216.7
0.9	4541.5	4661.5	5349.6

Fig. 3 Demands correlation and time value of money

of periods, confidence level, and the range in which the expectation and variance of product demands are randomly generated can change the sensitivity ranking of the inputs studied in this section.

5 Demands Correlation and Interest Rate Affects Total Cost

In this section, the effect of assuming dependent part demands and time value of money (interest rate) on total cost of the proposed model is investigated. To this end, a numerical example with input data given in Table 13 is applied to the model. For the known solution, the values of objective function of the proposed model is calculated by considering different p percentile levels in the three following cases: (i) independent demands with no interest rate, (ii) dependent demands with no interest rate, and (iii) independent demands with non-zero interest rate. The results are shown in Table 14.

Using the results, the curve of the total cost with respect to confidence level is plotted in Fig. 3. The figure indicates that non-zero interest rate leads to increased total cost over the range of uncertainty. As shown in the figure, the cost function has the same value for $p = 0.5$ for both independent and dependent demands, because this percentile

level, which is equivalent of $z_p = 0$, leads to ignoring the second term of the objective function of the proposed model given in Eq. (16). According to the equation, the second term of the objective functions is variance of MHC, which is function of demands correlation indicated by covariance. Therefore, by ignoring this term, demands correlation does not affect the total cost. In other words, if the user-defined percentile level is 0.5, independency or dependency of demands does not affect the total cost. It is necessary to state that in Eq. (16), for independent and dependent demands, the term of covariance is zero and non-zero respectively. Besides, the total cost is decreased for $p < 0.5$ (equivalent of $z_p < 0$) and increased for $p > 0.5$ (equivalent of $z_p > 0$) percentile levels by considering dependent demands.

6 Conclusions

In this paper, a novel QAP-based mathematical model was proposed for design of a robust machine layout for the SDFLP. The proposed model was validated by using numerical examples, benchmarks, and sensitivity analysis methods. The effect of assuming dependent part demands and interest rate on total cost of the proposed model was investigated. According to the findings, the following

conclusions were obtained: (i) The robust layout is a good layout for all periods in the planning horizon, although it is not necessarily an optimal layout for each period. (ii) According to the cost values in each period and the total cost value in the planning horizon, the robust layout obtained by solving the proposed model, has the best performance with respect to the previous models in literature. (iii) The proposed robust model can be applied to both deterministic and stochastic environments, whereas the previous models can only be used for deterministic environments. The Raytheon Aircraft Company studied by KRISHNAN, et al [34] was a real world FLP including 21 equal-sized machines that was solve by our proposed model. On the basis of findings, the obtained robust machine layout leads to 7.35% improvement with respect to the best previous one proposed by KRISHNAN, et al [34]. (iv) Sensitivity analysis indicated that different values of input parameters containing expectation of product demands, variance of product demands, and confidence level lead to design of different facility layout. Besides, the expectation of product demand and the variance of product demand are the most sensitive and the least sensitive parameters, respectively. However, changes in the parameters including number of machines, number of periods, confidence level, and the range in which the expectation and variance of product demands are randomly generated can change the sensitivity of the inputs. (v) Considering nonzero interest rate leads to increased total cost over the range of uncertainty. (vi) The total cost is decreased for $p < 0.5$ (equivalent of $z_p < 0$) and increased for $p > 0.5$ (equivalent of $z_p > 0$) by considering dependent demands. Besides, the numerical examples solved in Sections 4.3 and 5 indicate that the proposed model is applicable to stochastic environment.

7 Limitation and Future Works

Since the proposed model was developed based on the QAP formulation, it can be applied to any manufacturing systems, particularly the modern ones such as cellular manufacturing systems (CMS) and flexible manufacturing systems (FMS) with equal-sized facilities. The assumption of equal-sized facilities can be regarded as a restriction or a weakness. However, it can be justified as follows: in practice, there are some manufacturing systems such as the real case mentioned in Section 4.2 and the Vought Aerospace Company in Dallas, Texas [3] with equal-sized facilities.

As another subjective limitation, the same machining time on each machine may also be deduced from the assumption of equal-sized machines, which seem to be identical. Conversely, the equal-sized machines can be

multi-functional machines such as the ones used in FMSs. Considering so, the machining time on each machine is not necessary the same and depends on the process done by machine in accordance with production process. According to GROOVER [3], some real world examples of the FMSs established in the United States of America are as follows: Vought Aerospace in Dallas, Texas; Flexible Fabricating systems for automated sheet-metal processing; IngersollRand Company in Roanoke, Virginia; Assembly FMS installed by Allen-Bradley Company; John Deer American Corporation that manufactures agricultural machines; General Electric Company; and Caterpillar Tractor.

If the production horizon increases, such as 12 months × 5 years, the number of variables in the problem formulation will sharply increase and, as expected, the computational time will increase and/or the SA algorithm may yield a poor solution quality. It is also essential to mention that since the SA algorithm is a randomly local search technique, it can solve the large-sized problems in a reasonable computational time so that a suboptimal solution can be obtained.

In this study, alternate process plans have not been considered. Considering alternate process plans leads to increasing the complexity of the proposed models and in turn to increasing the computational time. Some other constraints and issues such as unequal-sized machines/cells, adding and removing machines in different periods, closeness ratio, aisles, budget constraint for total cost, machine reliability and deadlock problem can be considered in future studies.

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