Chin. J. Mech. Eng. (2017) 30:566–577
DOI 10.1007/s10033-017-0109-1

ORIGINAL ARTICLE

Thermal Error Modeling Method with the Jamming of Temperature-Sensitive Points' Volatility on CNC Machine Tools

Enming MIAO[1] · Yi LIU[1] · Jianguo XU[1] · Hui LIU[1]

Received: 11 May 2016 / Revised: 5 August 2016 / Accepted: 6 September 2016 / Published online: 17 March 2017

Abstract Aiming at the deficiency of the robustness of thermal error compensation models of CNC machine tools, the mechanism of improving the models' robustness is studied by regarding the Leaderway-V450 machining center as the object. Through the analysis of actual spindle air cutting experimental data on Leaderway-V450 machine, it is found that the temperature-sensitive points used for modeling is volatility, and this volatility directly leads to large changes on the collinear degree among modeling independent variables. Thus, the forecasting accuracy of multivariate regression model is severely affected, and the forecasting robustness becomes poor too. To overcome this effect, a modeling method of establishing thermal error models by using single temperature variable under the jamming of temperature-sensitive points' volatility is put forward. According to the actual data of thermal error measured in different seasons, it is proved that the single temperature variable model can reduce the loss of forecasting accuracy resulted from the volatility of temperature-sensitive points, especially for the prediction of cross quarter data, the improvement of forecasting accuracy is about 5 μm or more. The purpose that improving the robustness of the thermal error models is realized, which can provide a reference for selecting the modeling independent variable in the application of thermal error compensation of CNC machine tools.

Keywords CNC machine tool · Thermal error · Temperature-sensitive points · Forecasting robustness · Univariate modeling

Supported by Key Project of National Natural Science Fund ofChina(Grant No. 51490660/51490661), and National Natural ScienceFoundation of China(Grant No. 51175142)

✉ Enming MIAO
miaoem@163.com

[1] School of Instrument Science and Opto-electronics Engineering, Hefei University of Technology, Heifei 230009, China

1 Introduction

With the rapid development of science and technology, higher requirements for machining accuracy and reliability of computer numerical control(CNC) machine tools are put forward. During the operation process of CNC machine tools, the components are uneven heated, which causes the thermal deformation and a change in the relative position between the tool and the workpiece, eventually leading to the machining error of piece parts. According to statistics, thermally induced error can account for about 40%–70% [1, 2]. The thermal error is predicted through the compensation models which are established by temperature data and thermal deformation, then the prediction values are used to compensate thermal deformation in advance by the CNC system software, this is called thermal error compensation technology. And it is an effective and economical method to reduce machining error and improve the accuracy of machine tools [3, 4].

In thermal error compensation technology of CNC machine tool, the key is to improve the forecasting accuracy and robustness of thermal error models [5]. At present, the technologies for improving models' forecasting robustness usually contain two parts. The first part is the physical selection of temperature-sensitive points. Since the temperature field of machine tools has non-linear and time variability, its distribution is extremely complex. In order to obtain the temperature distribution, a lot of temperature sensors are required to lay out, so testing costs and the workload of measurement and calculation are increased, at the same time, the models' accuracy is also affected by multi-collinearity among temperature variables. The least number of temperature sensors which have most important influence on thermal error is selected to obtain the best fitting and forecasting effects, and strong robustness of thermal error model. In other words, by selecting the optimal temperature-sensitive points, it can improve the influence weight of temperature variables on thermal deformation, reducing the collinear error among temperature variables, and finally improving the forecasting robustness of thermal error models [6, 7]. The second part is the application of mathematical modeling algorithms. By establishing the thermal error models between thermal deformation and temperature variables based on the mathematics algorithms, the thermal error of machine can be predicted through the real-time temperature values, then with the communication of CNC system, the real-time compensation of thermal error is realized [8, 9]. Nowadays, the mathematics algorithms used in thermal error modeling include the multiple regression analysis [10], time series [11], support vector machines [12], neural networks [13], etc. According to the advantages of explanatory ability of independent variables on dependent variables using different mathematical modeling algorithms, it can also improve the forecasting robustness of thermal error models.

Furthermore, if the selection results of temperature-sensitive points appear deviations, it will directly affect the improving effects of models' forecasting robustness, and the results may even deviate from the expected design. Therefore, the nature to improve the forecasting robustness of thermal error models is the physical selection of temperature-sensitive points, and so far, many researchers have studied a lot on it. In Canada, ATTIA, et al [14], used finite element method to analyze the overall temperature field of CNC, and divided temperature field into a plurality of regular units, the optimum number and best position of temperature measurement points were determined according to temperature field simulation and correlation choosing. At University of Michigan, LO and NI, et al [15, 16], divided temperature sensors into groups, optimized the distribution of temperature measuring points by correlation grouping, representative searching, group searching and variable searching, finally, 4 temperature sensors were chosen for modeling from 46 sensors. In Korea, LEE, et al [17], regarded the minimum residual mean square as a basis for selecting temperature variables, proposed the method of correlation coefficient combined with linear regression, and the number of temperature measuring points was reduced to 4. At Shanghai Jiao Tong University, YANG, et al [18], put forward the grouping optimization method of temperature variables, divided the temperature variables into groups according to variables' correlation, made the permutation and combination with temperature variables and thermal error, and selected 4 temperature-sensitive points for modeling through comparisons eventually. At Hefei University of Technology, MIAO, et al [19, 20], took the advantage of fuzzy clustering and gray correlation degree algorithms to select temperature-sensitive points, finally, the number of temperature measuring points was reduced to 2.

The above researches are carried out under the condition that the temperature-sensitive points are stable. However, MIAO, et al [21], explored the temperature-sensitive points of CNC machine in different quarters, and found that the temperature-sensitive points have volatility, which leads to the decline of the thermal error models' forecasting robustness while forecasting the cross quarter data. Obviously, the volatility of temperature-sensitive points has an important influence on the forecasting accuracy and robustness of the models, but there is a lack of the relevant research about its influential mechanism.

In view of this, the changing characteristics of temperature-sensitive points of machine tool and its influential mechanism on forecasting accuracy and robustness of thermal error models are studied in this paper. After that, the method of establishing thermal error compensation models of machine tool based on single temperature variable is put forward.

First, according to the thermal data of spindle idling of actual CNC machine, the volatility of temperature-sensitive points is verified through fuzzy clustering combined with grey correlation degree [19, 20]. Then, based on the linear correlation coefficient and variance inflation factor [22] algorithms, the influence of the temperature-sensitive points' volatility on the collinear degree among sensitive variables and the correlation degree between sensitive variables and thermal deformation are analyzed. After that, it is concluded that the collinear degree among sensitive variables is enhanced during the multivariate modeling while the temperature-sensitive points are changing, and it leads to a serious impact on the forecasting accuracy and robustness of multivariate thermal error models. Therefore, the method of establishing thermal error models by using single temperature variable under the jamming of temperature-sensitive points' volatility is put forward. In addition, according to thermal error experiments of multi quarter on Leaderway-V450 machine, the proposed univariate modeling method is verified with accurate tests, and the feasibility of it is also verified.

2 Thermal Error Experiment of CNC Machine Tool

2.1 Experimental Apparatus

The thermal error of machine was measured while taking Leaderway-V450 machine tool as the research object. Since *X*-axis and *Y*-axis of this machine are approximately symmetrical structure, and compared with *Z*-axis, the thermal deformation of *X*-axis and *Y*-axis are smaller. Therefore, in order to reduce the work of experimental and data processing, only the thermal deformation of *Z* direction of machine tool spindle was measured and analyzed.

In the experiment, the positions of temperature sensors were to be placed in the vicinity of main heat source of machine tool in Z direction. Among them, sensors T1–T5 were placed in front of bearing of spindle, sensors T6 and T9 were placed on spindle sleeve, sensor T7 was placed at the bottom of spindle cylinder, sensor T8 was placed on spindle motor, and sensor T10 was placed on machine casing for measuring ambient temperature. The installation locations of temperature sensors and inductance displacement sensor are shown as Fig. 1. Due to machine casing is not shown in Fig. 1, the sensor T10 placed in machine casing is not labeled in Fig. 1.

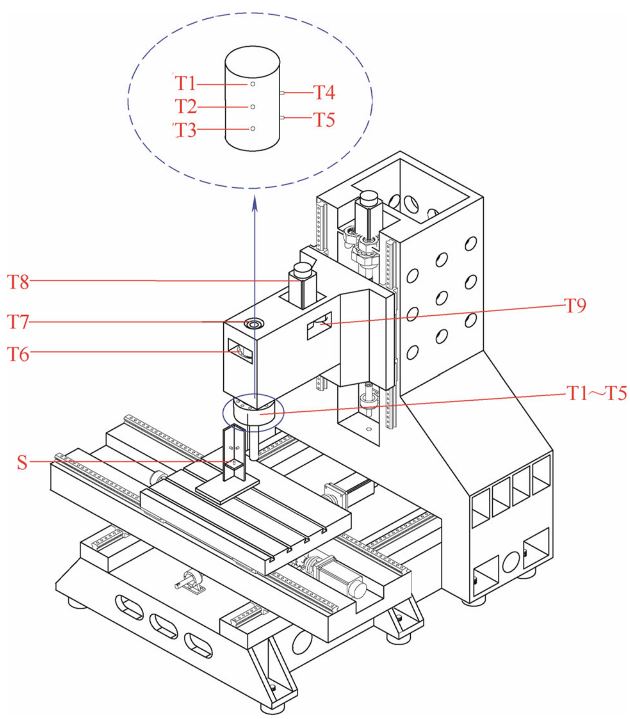

Fig. 1 Installation locations of temperature sensors and displacement sensor

2.2 Experimental Design

In accordance with international standard of *Test code for machine tools—Part 3*: *Determination of thermal effects* [23], four batches of data of CNC machine tool were collected under spindle idling in different seasons. The data of temperature and thermal error were sampled synchronously when measuring, among them, the temperature data was collected through digital sensor DS18B20(measuring accuracy is ±0.2 °C, the highest resolution can reach 0.062 5 °C), and the thermal deformation of *Z* direction was measured by using inductance displacement sensor(measuring accuracy is ±0.5 μm). During the experiment under spindle idling, the spindle was rotated at a constant speed(1000 r/min, 2000 r/min) and the experimental data were collected every three minutes, the duration of every experiment was over four hours. The experiment parameters of batches of K1–K4 are shown as Table 1. Among them, the batches of K1 and K2 were measured in spring, and the batches of K3 and K4 were measured in summer. The thermal deformations of four batches are shown in Fig. 2. Besides, because of the limited space, only the temperature data of K1 are shown as Fig. 3.

Table 1 Experiment parameters of batches of K1–K4

Experiment time	Batches	Spindle speed $S/(\mathrm{r \cdot min^{-1}})$	Ambient temperature T/°C
Spring	K1	2000	10.63–12.00
	K2	2000	10.38–12.38
Summer	K3	1000	28.68–33.75
	K4	2000	31.37–35.06

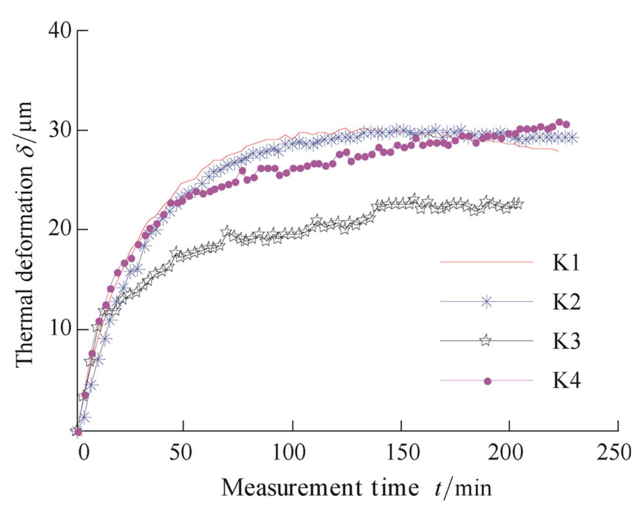

Fig. 2 Thermal deformations of K1–K4

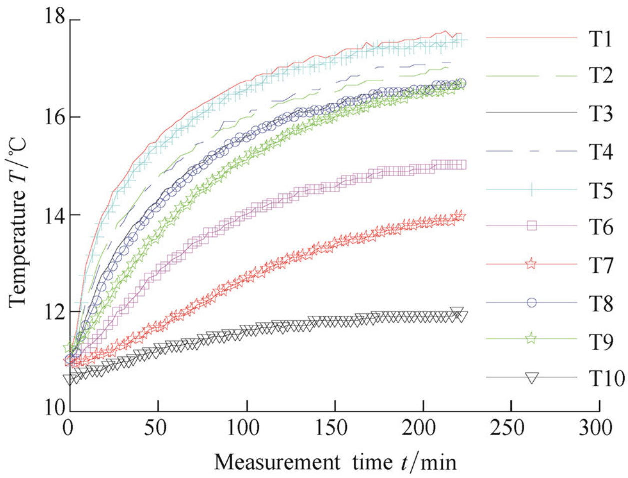

Fig. 3 Temperature data of K1

Table 2 Results of temperature-sensitive points of K1-K4

Batches	Clustering results		Temperature-sensitive points
	Category I	Category II	
K1	T7,T10	others	T1,T7
K2	T7,T10	others	T1,T7
K3	T7,T10	others	T7,T8
K4	T7,T10	others	T7,T8

Note: Based on Refs. [19–21] and engineering experiences, it can meet theaccuracy of thermal compensation models of CNC machine when thetemperature data is classified into two categories. Besides, in this table,"others" refers to all the other sensors except for those sensors in Category I.

3 Volatility of Temperature-Sensitive Points and Its Influential Mechanism on Models

3.1 Volatility of Temperature-Sensitive Points

According to the method of fuzzy clustering combined with gray correlation degree [19, 20], the temperature-sensitive points of K1−K4 are calculated. The principle of this method is: classify all the temperature variables according to their correlation to make the variables from the same category have strong correlation, and the variables from the different categories have weak correlations; then, select one temperature variable which has the maximum correlation with thermal deformation in each category as one of the temperature-sensitive points. The calculated results of temperature-sensitive points of K1–K4 are shown as Table 2.

From Table 2, the selection results of temperature-sensitive points of K1−K4 are not exactly the same, namely, it has a volatility. In addition, the ambient temperature T10 is not included in temperature-sensitive points but instead of T7 which is placed on bottom of spindle cylinder, the reason is that T7 and T10 are in the same category after clustering, there is a high correlation between them, but T7 has a greater influential weight on thermal deformation than T10, hence, T7 is more suitable for modeling.

Table 3 Correlation degree between temperature variables and thermal deformation

Batches	Correlation degree between T1 and thermal deformation C_{d1}	Correlation degree between T8 and thermal deformation C_{d8}
K1	0.428 3	0.415 3
K2	0.434 8	0.420 6
K3	0.403 7	0.421 6
K4	0.410 5	0.427 8

According to Table 2, it is easy to know, the positions of temperature-sensitive points of K1 and K2 are in front of bearing of spindle and at the bottom of spindle cylinder, but the positions of temperature-sensitive points of K3and K4 are on spindle motor and at the bottom of spindle cylinder. The reason for above results is, when the machine stays at a low temperature environment(such as K1 and K2), the correlation between the temperature sensor T8(placed on spindle motor) and thermal deformation is smaller than the correlation degree between the temperature sensor T1(placed in front of bearing of spindle) and thermal deformation. However, when the machine stays at a high temperature environment(such as K3 and K4), the correlation degree of the above two is exchanged, which makes the temperature measuring point T8 become one of the temperature-sensitive points of K3 and K4. The specific correlation degree are shown in Table 3.

3.2 Influential Mechanism of Temperature-Sensitive Points' Volatility on Models' Accuracy

In this section, based on the variance inflation factor and linear correlation coefficient algorithms, the influence of the temperature-sensitive points' volatility on the collinear degree among sensitive variables and the correlation degree between sensitive variables and thermal deformation are analyzed respectively.

3.2.1 Influence of Temperature-Sensitive Points' Volatility on Collinear Degree Among Sensitive Variables

In this paper, the method of variance inflation factor(VIF) [22] is used to calculate the collinear degree among sensitive variables.

For multiple linear regression model, the variance of $\hat{\beta}_j$ can be expressed as

$$VAR\left(\hat{\beta}_j\right) = \frac{\sigma^2}{\sum\left(X_{ij} - \bar{X}\right)^2} \frac{1}{1 - R_j^2} = \frac{\sigma^2}{\sum\left(X_{ij} - \bar{X}\right)^2} VIF_j, \tag{1}$$

$$VIF_j = \frac{1}{1 - R_j^2}, \tag{2}$$

where VIF_j is variance inflation factor, R_j^2 is coefficient of determination about X_j and other independent variables while taking X_j as the dependent variable.

With the increase of collinear degree, the VIF_j values and estimated error are increasing. Thus, VIF_j can be used as an index to judge the collinear degree. Normally, when VIF_j is greater than 10, it can be considered there is a serious collinear degree of variables.

According to Eq. (2), the VIF values of different sensitive variables are calculated and shown in Table 4. From Table 3, the sensitive variables of all batches are (T1,T7) and (T1,T8).

After analyzing the data in Table 4, two conclusions are obtained and shown as follows:

(1) For batches of K1 and K2, the VIF values of (T1,T7) are smaller than 10, it doesn't have a serious collinear degree. But for batches of K3 and K4, the VIF values of (T1,T7) are greater than 10, there is a serious collinear degree. That is to say, for the same location of temperature variables in different batches, the collinear degree of them can change significantly.

(2) (T1,T7) are the temperature-sensitive points of batches of K1 and K2, and they don't have a serious collinearity. However, (T7,T8) are the temperature-sensitive points of K3 and K4, both of the VIF values are greater than 10, so,it is considered that there is a serious collinearity. This can be explained that, when CNC machine stays at a low ambient temperature(such as K1 and K2), its temperature-sensitive points have a lower collinear degree. But when machine stays at a high ambient temperature(such as K3 and K4), the correlation of temperature measurement points is enhanced, which leads to a serious collinear problem among its temperature- sensitive points.

Table 4 VIF values of different sensitive variables

Batches	VIF value of (T1,T7) V_{if}	VIF value of (T7,T8) V_{if}
K1	7.20	62.00
K2	8.57	54.16
K3	41.31	34.89
K4	13.48	19.91

Note: While modeling with the temperature variables of (Ti ,Tj), the VIF of variable Ti is equal to the VIF of Tj , that is to say, VIFi = VIFj , so only one VIF value of each combination is listed in this table.

3.2.2 *Influence of Temperature-Sensitive Points' Volatility on Correlation Degree Between Sensitive Variables and Thermal Deformation*

Linear correlation coefficient(LCC) is one statistical indicator used for reflecting the correlation between variables [22], the calculation formula of LCC is shown as

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}. \tag{3}$$

According to LCC method, the correlation degrees between sensitive variables and thermal deformation in all batches are calculated and shown in Table 5. From Table 3, the sensitive variables of all batches include T1, T7 and T8.

From Table 5, for the same sensitive variable, the LCC values change in a small range, the range of LCC in all batches are 0.089 2, 0.025 0 and 0.089 2. So, the volatility of temperature-sensitive points has little impact on correlation degree between sensitive variables and thermal deformation.

From sections 3.2.1 and 3.2.2, when the temperature-sensitive points changes, the correlation degree between sensitive variables and thermal deformation only changes in a small range, which ensures that the modeling temperature variables have a great influence on thermal deformation. However, the collinear degree among sensitive variables can change significantly, and it seriously affects the forecasting accuracy and robustness of the model, it is also an objective problem of multivariate models. Therefore, in actual thermal error modeling and forecasting of CNC machine, whether the forecasting effects of models is affected by the above problem will be verified in the following sections.

Table 5 LCC values between sensitive variables and thermal deformation

Batches	LCC between T1 and thermal deformation L_{cc1}	LCC between T7 and thermal deformation L_{cc7}	LCC between T8 and thermal deformation L_{cc8}
K1	0.952 5	0.761 9	0.920 7
K2	0.956 6	0.786 9	0.918 3
K3	0.867 4	0.780 7	0.889 3
K4	0.879 8	0.780 3	0.931 3

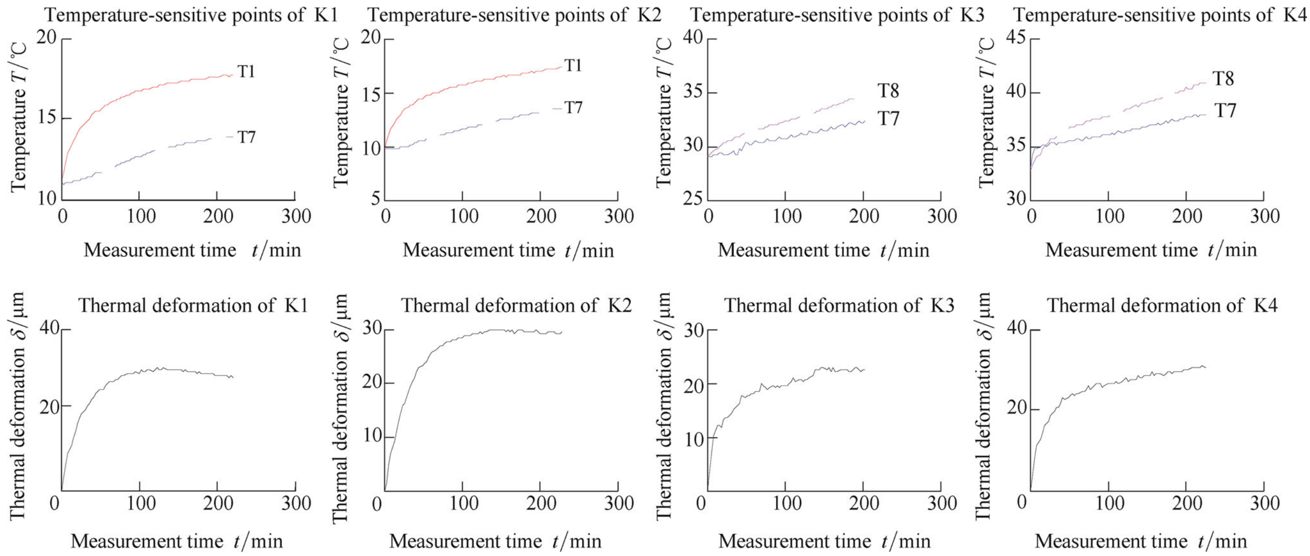

Fig. 4 Data of temperature-sensitive points and thermal deformation of batches of K1–K4

4 Modeling and Accuracy Analysis of Multivariate Thermal Error Models

Taking the temperature-sensitive points of each batch as the modeling variables, and the mathematical models of K1–K4 are established by multiple linear regression(MLR) algorithm which is based on the least square [22]. The results of four models are shown as Eqs. (4)–(7). In addition, the data of temperature-sensitive points and thermal deformation of batches of K1–K4 are shown in Fig. 4.

$$y_1 = -3.2560 + 7.4455\Delta T_1 - 6.0841\Delta T_7, \quad (4)$$

$$y_2 = -5.2452 + 7.4164\Delta T_1 - 5.4829\Delta T_7, \quad (5)$$

$$y_3 = 10.1537 - 3.4197\Delta T_7 + 4.8038\Delta T_8, \quad (6)$$

$$y_4 = 13.2640 - 4.5359\Delta T_7 + 5.2645\Delta T_8, \quad (7)$$

where ΔT_i is incremental temperature value of sensor T_i, y_i is the predictive value of thermal error model.

Use the above MLR models established by the batches of K1–K4 to predict the batches of K1–K4 respectively. Here, the residual standard deviations about predictive value and measured value are used to evaluate the forecasting accuracy of thermal error models, and the smaller the values, the higher the models' accuracy. The calculated results of forecasting accuracy of MLR models are shown in Table 6. Then, according to Table 6, the forecasting accuracy of multivariate models for forecasting different seasonal data are obtained, and the results are shown in Table 7. In Tables 6 and 7, $M_{im}(i = 1, 2, 3, 4)$ is multivariate MLR models established by batch of Ki. Besides, due to the limited space, the forecasting residual value of model M_{1m} is shown as Fig. 5. In Fig. 5, "M_{im}–Ki(i = 1,2,3,4) means the residual of thermal error after forecasting Ki by model M_{im}.

Table 6 Forecasting accuracy of multivariate models μm

Forecasting data	Models			
	M_{1m}	M_{2m}	M_{3m}	M_{4m}
K1	1.40	1.78	12.87	15.32
K2	1.79	1.48	11.32	14.47
K3	16.26	17.42	2.17	3.57
K4	13.64	14.49	3.06	2.01

Table 7 Forecasting accuracy of multivariate models for forecasting different seasonal data

Models	Forecasting data	Forecasting accuracy s/μm
M_{1m}, M_{2m}	K1, K2	1–2
	K3, K4	13–17
M_{3m}, M_{4m}	K1, K2	11–15
	K3, K4	2–4

From Table 7, two conclusions are obtained and shown as follows:

(1) The forecasting standard deviation of K1 and K2 predicted by M_{1m} and M_{2m}, the forecasting standard deviation of K3 and K4 predicted by M_{3m} and M_{4m}, are within 4 μm. However, the forecasting standard deviation of K3 and K4 predicted by M_{1m} and M_{2m}, the forecasting standard deviation of K1 and K2 predicted by M_{3m} and M_{4m}, are greater than 10 μm. It can be known that, due to the large variation of collinear degree among modeling variables caused

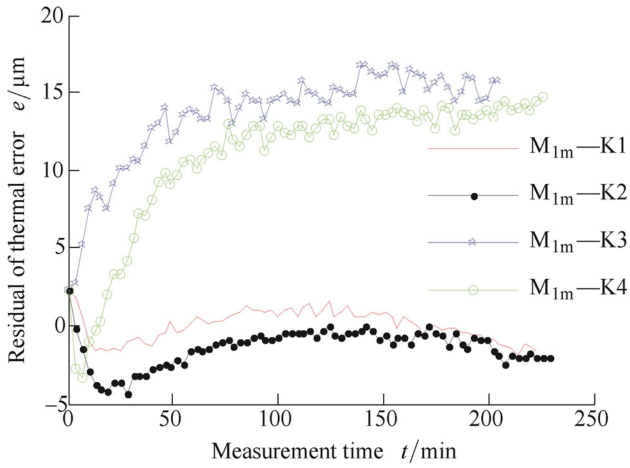

Fig. 5 Forecasting residual value of model M_{1m}

by the temperature-sensitive points' volatility, the forecasting accuracy of multivariate regression models is reduced, so the robustness of models can't be ensured.

(2) The forecasting standard deviation of K1 and K2 predicted by M_{1m} and M_{2m}, is slighter lower than that of K3 and K4 predicted by M_{3m} and M_{4m}, and this difference is only about 1–2 μm. The reason is the temperature-sensitive points of K3 and K4 also have a serious collinear problem, namely, when models M_{3m} and M_{4m} are predicting K3 and K4, there are serious collinear problem in both modeling data and forecasting data. Therefore, when these models are built, the ability of temperature variables for describing the law of thermal deformation is reduced, which leads to a slight decrease of models' forecasting accuracy.

The above analysis shows that, due to the complexity of the structure of CNC machine tool and the time variability and non-linearity of temperature field, the positions of temperature-sensitive points are not identical in different batches, which have a great influence on the collinear degree among temperature-sensitive points, and it is the essential reason that affects the accuracy and robustness of multivariate thermal error compensation models of CNC machine.

Considering there are some LCC values between sensitive variables and thermal deformation which are shown in Table 4 are greater than 0.9, such as the temperature variable T1 in batch of K1, its LCC value reaches to 0.952 5, if the thermal error model is established by T1, it can well describe the trend of thermal error, so the feasibility of the model is worth being discussed. In addition, there is no collinear error when modeling with one independent variable, which avoids the defects of the multivariate models.

5 Modeling and Accuracy Analysis of Univariate Models

5.1 Feasibility Study of Univariate Modeling

In this section, based on the analysis of K1, the feasibility of univariate models is discussed. The key of it is that the correlation among temperature variables and the correlation between temperature variables and thermal deformation can satisfy the accuracy and robustness of thermal error compensation models.

5.1.1 Analysis of Correlation Degree Between Temperature Independent Variables

According to the Eq. (3) of LCC, the correlation matrix of the temperature variables T1–T10 of K1 is calculated. If the correlation among temperature variables are very strong, one temperature variable can be used to collect most of the temperature information of the machine tool.

The results of correlation matrix $\boldsymbol{R}$ of T1–T10 are

$$\boldsymbol{R}=\begin{pmatrix}
1.0000 & 0.9978 & 0.9923 & 0.9950 & 0.9993 & 0.9700 & 0.9896 & 0.9137 & 0.9683 & 0.9623\\
0.9978 & 1.0000 & 0.9980 & 0.9990 & 0.9984 & 0.9829 & 0.9965 & 0.9339 & 0.9814 & 0.9761\\
0.9923 & 0.9980 & 1.0000 & 0.9994 & 0.9939 & 0.9917 & 0.9994 & 0.9500 & 0.9904 & 0.9858\\
0.9950 & 0.9990 & 0.9994 & 1.0000 & 0.9964 & 0.9884 & 0.9985 & 0.9443 & 0.9872 & 0.9824\\
0.9993 & 0.9984 & 0.9939 & 0.9964 & 1.0000 & 0.9733 & 0.9915 & 0.9195 & 0.9718 & 0.9663\\
0.9700 & 0.9829 & 0.9917 & 0.9884 & 0.9733 & 1.0000 & 0.9939 & 0.9782 & 0.9995 & 0.9975\\
0.9896 & 0.9965 & 0.9994 & 0.9985 & 0.9915 & 0.9939 & 1.0000 & 0.9531 & 0.9925 & 0.9883\\
0.9137 & 0.9339 & 0.9500 & 0.9443 & 0.9195 & 0.9782 & 0.9531 & 1.0000 & 0.9817 & 0.9836\\
0.9683 & 0.9814 & 0.9904 & 0.9872 & 0.9718 & 0.9995 & 0.9925 & 0.9817 & 1.0000 & 0.9979\\
0.9623 & 0.9761 & 0.9858 & 0.9821 & 0.9663 & 0.9975 & 0.9883 & 0.9836 & 0.9979 & 1.0000
\end{pmatrix}$$

From the results of correlation matrix $\boldsymbol{R}$, the correlation between T1 and T5 is the highest, and the LCC value reaches to 0.999 3. The correlation between T1 and T8 is the lowest, but the LCC value even reaches to 0.913 7. All the correlation degrees are large from the subjective judgment. Thus, it is very necessary to consider establishing thermal error models with one temperature variable, and as for the effectiveness and reliability of the compensation model, it is also necessary to be verified according to the accuracy of univariate models.

5.1.2 *Analysis of Correlation Degree Between Temperature Independent Variables and Thermal Deformation*

According to the Eq. (3) of LCC, the correlation between temperature variables T1–T10 of K1 and thermal deformation are calculated. If there is a temperature variable which has a strong correlation with thermal deformation, the law of thermal deformation can be well described by this variable.

The calculated results of LCC between temperature variables and thermal deformation are shown in Table 8.

From Table 8, the correlation degree between T1 and thermal deformation is the highest, and the LCC value reaches to 0.952 5. There is a strong correlation between them, so, the temperature variable T1 can reflect the trend of thermal deformation well. From this perspective, we can also consider establishing thermal error compensation models with one temperature variable.

Based on above analysis, a modeling method of establishing thermal error models by using single temperature variable under the jamming of temperature-sensitive points' volatility is put forward.

5.2 Modeling and Accuracy Analysis of Univariate Models

While establishing thermal error modes by one temperature variable, the temperature variable which has the maximum LCC value with thermal deformation should be chosen as modeling independent variable. According to Eq. (3), the modeling independent variables of each batch are calculated and shown in Table 9.

Table 8 LCC between temperature variables T1–T10 and thermal deformation

Temperature variables	LCC values L_{CC}	Temperature variables	LCC values L_{CC}
T1	0.952 5	T6	0.920 7
T2	0.939 8	T7	0.874 8
T3	0.924 3	T8	0.761 9
T4	0.949 4	T9	0.867 7
T5	0.874 8	T10	0.856 9

Table 9 Modeling independent variables of each batch

Batches	K1	K2	K3	K4
Modeling variables	T1	T1	T8	T8

Table 10 Forecasting accuracy of univariate models μm

Forecasting data	Models			
	M1s	M2s	M3s	M4s
K1	2.74	3.35	5.00	4.25
K2	3.81	3.13	4.17	3.81
K3	5.93	6.43	2.22	2.33
K4	7.27	8.30	2.58	2.26

From Table 9, the modeling independent variables of all batches still change, but, according to section 3.2.2, it has been verified that the change of modeling independent variables has little influence on the correlation degree between the modeling variables and thermal deformation. Therefore, all these independent variables listed in Table 9 have strong correlation with thermal deformation, which meets the requirements of the proposed robust modeling method.

According to the results of Table 9, the univariate MLR models of K1–K4 are established and shown as Eqs. (8) – (11):

$$y_1 = 7.038\,4 + 3.392\,3\Delta T_1, \tag{8}$$

$$y_2 = 7.238\,0 + 3.028\,2\Delta T_1, \tag{9}$$

$$y_3 = 9.205\,8 + 2.638\,7\Delta T_8, \tag{10}$$

$$y_4 = 9.871\,3 + 2.915\,9\Delta T_8, \tag{11}$$

where, ΔT_i is incremental temperature value of sensor T_i; y_i is the predictive value of thermal error model.

Use the above MLR models established by the batches of K1–K4 to predict the batches of K1–K4 respectively. In Tables 10, $M_{is}(i = 1, 2, \cdots, 4)$ is univariate MLR models established by Ki. Besides, due to the limited space, the forecasting residual value of model M_{1s} is shown as Fig. 6.

From Table 10, the range of forecasting standard deviation of univariate models is from 2.22 μm to 8.30 μm.

5.3 Accuracy Comparison Between Univariate Models and Multivariate Models

From the forecasting accuracy of univariate models and multivariate models which are shown in Tables 6 and 10, the accuracies of two models are compared, and the results are shown in Table 11. In Table 11, $M_i(i = 1, 2, 3, 4)$.are the univariate models or multivariate models established by Ki.

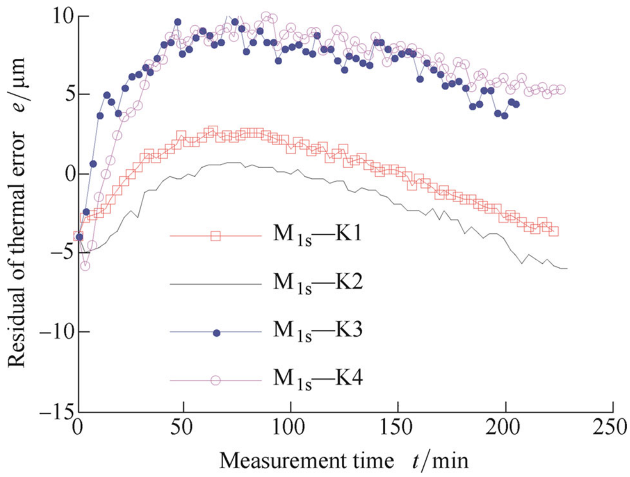

Fig. 6 Forecasting residual value of model M_{1s}

Table 11 Accuracy comparison of univariate models and multivariate models μm

Forecasting data	M1		M2	
	M1 s	M1 m	M2 s	M2 m
K1	2.74	1.40	3.35	1.78
K2	3.81	1.79	3.13	1.48
K3	5.93	16.26	6.43	17.42
K4	7.27	13.64	8.30	14.49
Forecasting data	M3		M4	
	M3 s	M3 m	M4 s	M4 m
K1	5.00	12.87	4.25	15.32
K2	4.17	11.32	3.81	14.47
K3	2.22	2.17	2.33	3.57
K4	2.58	3.06	2.26	2.01

The data in Table 11 are presented by the form of two dimensional histogram, and the accuracy comparison of univariate models and multivariate models established by the batches of K1–K4 are shown as Figs. 7, 8, 9 and 10 respectively.

According to Table 11 and Figs. 7, 8, 9 and 10, the forecasting accuracy comparison between univariate models and multivariate models for forecasting different seasonal data are obtained and shown in Table 12. In Table 12, 'Single-Multi' refers to the difference of forecasting accuracy between univariate models and multivariate models. If its value is greater than 0, it means the forecasting accuracy of univariate model is higher than that of multivariate model.

From Tables 11 and 12, some conclusions can be obtained and shown as follows:

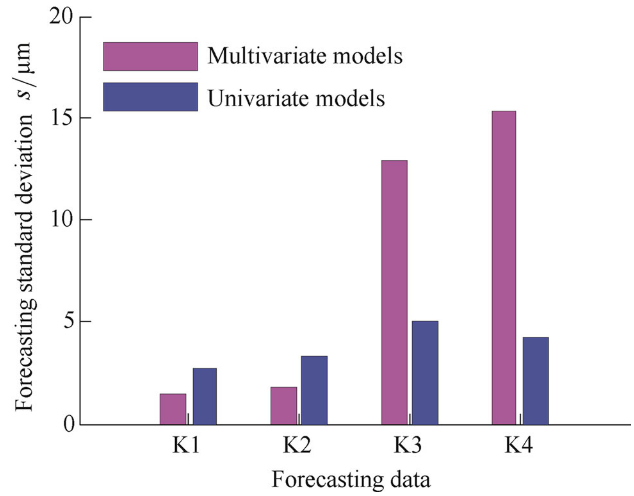

Fig. 7 Accuracy comparison of univariate models and multivariate models established by K1

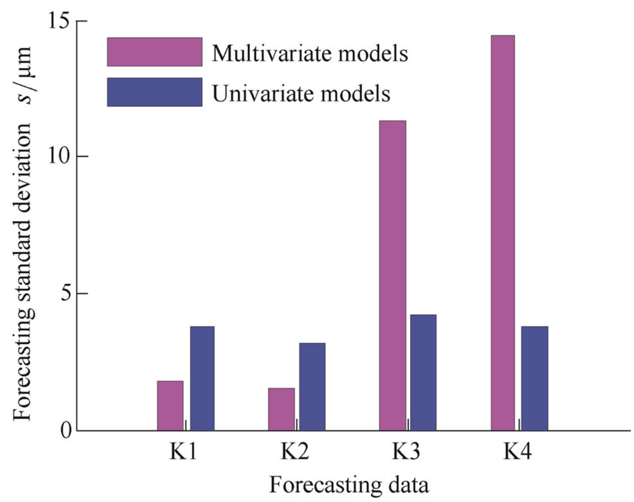

Fig. 8 Accuracy comparison of univariate models and multivariate models established by K2

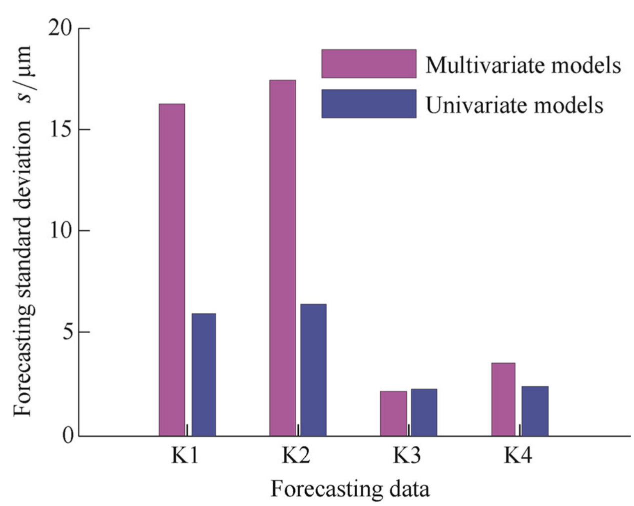

Fig. 9 Accuracy comparison of univariate models and multivariate models established by K3

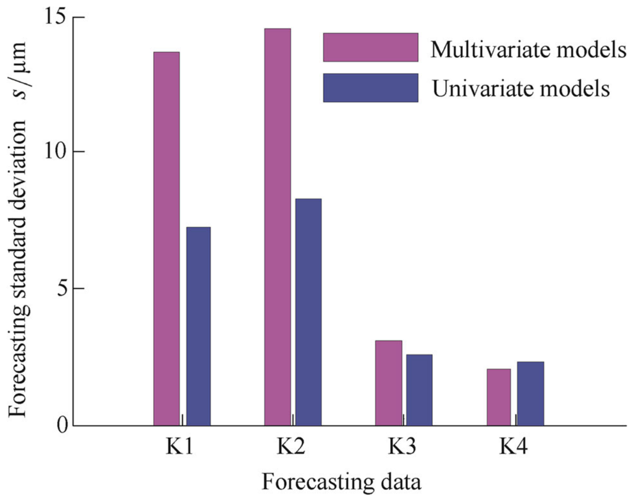

Fig. 10 Accuracy comparison of univariate models and multivariate models established by K4

Table 12 Forecasting accuracy comparison between univariate models and multivariate models for forecasting different seasonal data

Models	Ambient temperature *T*/°C	Forecasting data	Single-Multi *s*/μm
M1, M2	10～13°C	K1, K2	–1～–2
		K3, K4	>10
M3, M4	25～36°C	K1, K2	>7
		K3, K4	–0.5～0.5

(1) The forecasting accuracy of univariate models is less affected by external environment, and the forecasting robustness of univariate models is higher than that of multivariate models. Take the data in Table 11 for example, the range of forecasting accuracy of univariate models established by K1(that is M_{1s}) is from 2.74 μm to 7.27 μm, but the range of forecasting accuracy of multivariate models established by K1(that is M_{1m}) is from 1.40 μm to 16.26 μm, its robustness is significantly lower than that of univariate models. The other models' properties are similar.

(2) From Table 12, while the ambient temperature of modeling data is low, the accuracies of established thermal error models(such as M1 and M2) are expressed as the following: when the ambient temperature of forecasting data is also low(such as K1 and K2), the forecasting accuracy of univariate models is slightly lower than that of multivariate models, but its difference is only about from 1 μm to 2 μm. However, when the ambient temperature of forecasting data is high(such as K3 and K4), due to the great changes of collinear degree among independent variables, the forecasting accuracy of univariate models is much better than that of multivariate models, and the difference of forecasting accuracy between them reaches to 10 μm or more.

(3) From Table 12, while the ambient temperature of modeling data is high, the established thermal error models(such as M3 and M4) are expressed as the following: when the ambient temperature of forecasting data is also high(such as K3 and K4), the accuracy of univariate models is close to that of multivariate models, and the difference is within the range of –0.5–0.5 μm. This is because the correlation of temperature measurement points is enhanced while staying at high temperature, which leads to a serious collinear problem among modeling variables, so the accuracy of multivariate models is suppressed, but since the forecasting data also has a serious collinearity, the accuracy is only slightly decreased. However, when the ambient temperature of forecasting data is low(such as K1 and K2), due to the great changes of collinear degree among variables, the forecasting accuracy of univariate models is much better than that of multivariate models, and the difference is 7 μm or more.

Table 13 Parameters of experimental data

Batches	Spindle speed $S/(\mathrm{r \cdot min^{-1}})$	Ambient temperature *T*/°C	Thermal deformation δ/μm
L1	2000	13.0～16.0	24.5
L2	4000	14.6～19.7	39.5
L3	6000	14.4～19.5	47.8
L4	2000	25.5～27.3	22.5
L5	4000	25.0～29.2	36.5
L6	6000	25.6～29.0	47.3

6 Experimental Verification

To verify the superiority of univariate modeling models in actual thermal error of machine tool, six batches of data were measured under spindle idling on different seasons. Among them, the parameters of batches of L1-L6 are shown in Table 13.

According to above experimental data, the multivariate models and univariate models are established according MLR algorithm, and the batches of L1–L6 are predicted by these models as well. After that, the mean of forecasting

Table 14 Forecasting accuracy comparison between multivariate models and univariate models μm

	Multivariate models	Univariate models
Mn	12.77	7.35
Sd	11.34	5.37

accuracy(*Mn*) and the standard deviation of forecasting accuracy(*Sd*) of each kind of model are obtained and shown in Table 14. In Table 14, *Mn* is the parameter of the average level of forecasting accuracy, and the smaller of *Mn,* the higher of the average models' forecasting accuracy. *Sd* is the parameter of discrete degree of forecasting accuracy, and the small of *Sd*, the stronger of models' robustness.

From Table 14, compared with multivariate models, univariate models improve the models' forecasting accuracy and robustness obviously, the average improvement is more than 5 μm.

7 Conclusions

(1) Due to the complexity of the structure of CNC machine tool and the time variability and non-linearity of temperature field, the positions of temperature-sensitive points are not identical, which have a great influence on the collinear degree among sensitive variables, and it also leads to a greatly reduced in the forecasting accuracy and robustness of multivariate regression model.

(2) A modeling method of establishing thermal error models by using single temperature variable under the jamming of temperature-sensitive points' volatility is put forward. According to the actual data of thermal error measured in different seasons, it is proved that the single temperature variable model can reduce the loss of forecasting accuracy caused by the volatility of temperature-sensitive points, especially for the prediction of cross quarter data, the improvement of forecasting accuracy is about 5 μm or more.

(3) By the use of univariate modeling method, the purpose that improving the forecasting robustness of the thermal error models is realized. It provides a reference for selecting the independent variable in the application of thermal error compensation of CNC machine tools.

(4) The essential reason that affects multivariate models' accuracy and robustness is the great change of collinear degree among modeling independent variables, however, the influencing degree of collinear degree on the accuracy and robustness of multivariate models, still needs to be further studied. In addition, the proposed modeling method is more suitable for the thermal deformation of spindle elongation, but there is a lack of research about the thermal formation of spindle swing and spindle pitch.

References

1. BRYAN J B. International status of thermal error research[J]. *Ann CIRP*, 1990, 39(2): 645–656.
2. ARONSON R B. War against thermal expansion[J]. *Manufacturing Engineering*, 1996, 116(6): 45–50.
3. YANG S, YUAN J X, NI J. Accuracy enhancement of a horizontal machining center by real-time error compensation[J]. *Journal of Manufacturing Systems*, 1996, 15(2): 113–124.
4. YANG J G, YUAN J X, NI J. Thermal error mode analysis and robust modeling for error compensation on a CNC turning center[J]. *International Journal of Machine Tools & Manufacture*, 1999, 39(9): 1367–1381.
5. MIAO E M, GONG Y Y, NIU P C, et al. Robustness of thermal error compensation modeling models of CNC machine tools[J]. *International Journal of Advanced Manufacturing Technology*, 2013, 69(9): 2593–2603.
6. FAN J M, XU L M, ZHAO XM, et al. Sensor placement strategy for thermal error compensation on machine tools[J]. *Chinese Journal of Scientific Instrument*, 2005, 26(8): 83–84. (in Chinese)
7. LIU Q, YAN J, PHAM D T, et al. Identification and optimal selection of temperature-sensitive measuring points of thermal error compensation on a heavy-duty machine tool[J]. *International Journal of Advanced Manufacturing Technology*, 2016, 85(1): 345–353.
8. MIAO E M, GONG Y Y, XU Z S, et al. Comparative analysis of thermal error compensation model robustness of CNC machine tools[J]. *Journal of Mechanical Engineering*, 2015, 51(7): 130–135. (in Chinese)
9. YANG Z, KAN Y, FEI C, et al. Bayesian reliability modeling and assessment solution for NC machine tools under small-sample data[J]. *Chinese Journal of Mechanical Engineering*, 2015, 28(6): 1229–1239.
10. XIANG Z, RUIFEN X U, YAN B U. Optimal design of bicycle frame parameters considering biomechanics[J]. *Chinese Journal of Mechanical Engineering*, 2011, 24(1): 141–145.
11. LI Y X, TONG H C, CAO H T, et al. Application of time series analysis to thermal error modeling on NC machine tools[J]. *Journal of Sichuan University*, 2006, 38(2): 74–78. (in Chinese)
12. ZHANG M J, TANG J, ZHANG X M, et al. Intelligent diagnosis of short hydraulic signal based on improved EEMD and SVM with few low-dimensional training samples[J]. *Chinese Journal of Mechanical Engineering*, 2016, 29(2): 396–405.
13. YANG R, DUAN Z, LU Y, et al. Load reduction test method of similarity theory and BP neural networks of large cranes[J]. *Chinese Journal of Mechanical Engineering*, 2015, 29(1): 1–7.
14. ATTIA M H, FRASER S. A generalized modelling methodology for optimized real-time compensation of thermal deformation of

machine tools and CMM structures[J]. *International Journal of Machine Tools & Manufacture*, 1999, 39(6): 1001–1016.
15. LO C, YUAN J X, NI J. An application of real-time error compensation on a turning center[J]. *International Journal of Machine Tools & Manufacture*, 1995, 35(12): 1669–1682.
16. LO C, YUAN J X, NI J. Optimal temperature variable selection by grouping approach for thermal error modeling and compensation[J]. *International Journal of Machine Tools & Manufacture*, 1999, 39(9): 1383–1396.
17. LEE J, YANG S. Statistical optimization and assessment of a thermal error model for CNC machine tools[J]. *International Journal of Machine Tools & Manufacture*, 2002, 42(1): 147–155.
18. YANG J G, DENG W G, REN Y Q, et al. Grouping optimization modeling by selection of temperature variables for the thermal error compensation on machine tools[J]. *China Mechanical Engineering*, 2004, 15(6): 478–481. (in Chinese)
19. MIAO E M, GONG Y Y, CHENG T J, et al. Application of support vector regression to thermal error modeling of machine tools[J]. *Optics and Precision Engineering*, 2013, 21(4): 980–986. (in Chinese)
20. MIAO E M, NIU P C, FEI Y T, et al. Selecting temperature-sensitive points and modeling thermal errors of machine tools[J]. *Journal of the Chinese Society of Mechanical Engineers*, 2014, 32(6): 559–565.
21. MIAO E M, LIU Y, LIU H, et al. Study on the effects of changes in temperature-sensitive points on thermal error compensation model for CNC machine tool[J]. *International Journal of Machine Tools & Manufacture*, 2015, 97: 50–59.
22. YU X L, REN X S. *Multiple statistical analysis*[M]. Beijing: Statistics Press of China, 1999. (in Chinese)
23. STANDARDS B. IS0 230-3—2007. Test code for machine tools—Part 3: Determination of thermal effects[S]. International Standards Organization, 2007.

Enming MIAO, born in 1971, is currently a PhD supervisor at *School of Instrument Science and Opto-electronics Engineering, Hefei University of Technology*. His research interests include precision machinery engineering, accuracy theory, thermal error compensation of CNC machine tools and the robust structural design of mechinery. Tel: +86-14705511579; E-mail: miaoem@163.com

Yi LIU, born in 1994, is currently a postgraduate at *School of Instrument Science and Opto-electronics Engineering, Hefei University of Technology*. His research interests include accuracy theory and thermal error compensation of CNC machine tools. E-mail:liuyi8045@163.com

Jianguo XU, born in 1991, is currently a postgraduate at *School of Instrument Science and Opto-electronics Engineering, Hefei University of Technology*. His research interests include accuracy theory and thermal error compensation of CNC machine tools. E-mail:xujg110004@163.com

Hui LIU, born in 1992, is currently a postgraduate at *School of Instrument Science and Opto-electronics Engineering, Hefei University of Technology*. His research interests include accuracy theory and thermal error compensation of CNC machine tools. E-mail: IhYQYBHFUT@163.com