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REVIEW

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Design of Accelerated Life Test Plans—Overview and Prospect

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Abstract

Accelerated life test (ALT) is currently the main method of assessing product reliability rapidly, and the design of efficient test plans is a critical step to ensure that ALTs can assess the product reliability accurately, quickly, and economically. With the promotion of the national strategy of civil-military integration, ALT will be widely used in the research and development (R&D) of various types of products, and the ALT plan design theory will face further challenges. To aid engineers in selecting appropriate theories and to stimulate researchers to develop the theories required in engineering, with focus on the demands for theory research that arise from the implementation of ALT, this paper reviews and summarizes the development of ALT plan design theory. The development of the theory and method for planning optimal ALT for location-scale distribution, which is the most applied and mature theory of designing the optimal ALT plan, are described in detail. Taking this as the center of radiation, some problems that ALT now faces, such as the verification of the statistical model, limitation of sample size, solutions of resource limits, optimization of the test arrangement, and management of product complexity, are discussed, and the general ideas and methods of solving these problems are analyzed. Suggestions for selecting appropriate ALT plan design theories are proposed, and the urgent solved theory problems and opinions of their solutions are proposed. Based on the principle of convenience for engineers to select appropriate methods according to the problems found in practice, this paper reviews the development of optimal ALT plan design theory by taking the engineering problems arising from the ALT implementation as the main thread, provides guidelines on selecting appropriate theories for engineers, and proposes opinions about the urgent solved theory problems for researchers.

Keywords: Accelerated life test, Test plan, Optimal design, Reliability assessment

1 Introduction

In reliability engineering, reliability tests [1–13] usually have two main objectives:

(1) Assess the reliability indices of product. For example, assess the reliability level, reliable life, mean time between failures (MTBF), and failure rate.
(2) Improve and perfect products. For example, eliminate product defects and screen unqualified products.

The former is mainly concerned with how to test or estimate the reliability indices of products accurately, quickly, and economically. Determining a suitable statistical theory, method, and technology for the experiment design and data analysis is key for conducting this type of test; this is often called statistics-based reliability testing (SRT). The latter mainly focuses on how the processes of design, material selection, manufacture, assembly, and application affect the storage, performance, and maintenance of the product. The key factor of achieving goals is the profound understanding of the performance evolution law of a particular product throughout its life cycle. This type of test has a higher requirement for engineering experience, and it is often called the engineering-based reliability test (ERT). This classification is only to emphasize the different focus of the two types of tests. In practice, to carry out an SRT correctly, the engineering

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elements, such as usage conditions, failure mode, failure mechanism, test equipment, and cost limits, should be specified; in an ERT, a large amount of data should be collected and analyzed based on statistics.

In practice, ERT and SRT are often used in combination, and play different roles in various stages throughout the life cycle of the product. According to the testing purposes, the reliability test can be further classified into the reliability growth test (RGT), reliability qualification test (RQT), reliability screening test (RST), reliability acceptance test (RAT), and reliability determination test (RDT). According to the relationship of the test stress and the normal work stress, the reliability test can be divided into traditional test and accelerated test (AT). The types of the major reliability tests are shown in Figure 1.

In general, as shown in Figure 1, corresponding to the design, finalization of design, production, delivery, and use phases of the entire product life cycle, the major reliability tests implemented are RGT, RQT, RST, RAT, and RDT, respectively. Among them, RGT and RST belong to ERT; RQT, RAT, and RDT belong to SRT. The main statistical inference method used in RQT and RAT is the hypothesis test, so they are often collectively called the reliability verification test (RVT); the main statistical inference method used in RDT is parameter estimation.

In the initial period of reliability formation, products usually have low reliability and short lifespan, and reliability tests can be carried out by simulating the actual usage conditions. However, with the improvement in product reliability, it gradually becomes difficult to induce product failure effectively using this type of reliability test, and it cannot be used to obtain adequate failure data within an acceptable test time and sample size. To solve this problem, the AT method was developed: the sample was tested within an environment more severe than it would have experienced during normal use. Data was collected at high stress levels and was used to predict the product life at the normal stress level and to improve the product reliability. Among the ATs, ERT mainly include the accelerated RGT [2], highly accelerated life test (HALT) [3], and highly accelerated stress screening test (HASS) [3] (the latter two are often referred to as the reliability enhancement test, RET [4]); SRT mainly includes the accelerated life test (ALT) [5–11] and the accelerated degradation test (ADT) [6–8].

ALT is currently the most widely used method of assessing product reliability rapidly in practice. To ensure that the reliability of products can be assessed accurately, quickly, and economically, the design of an efficient plan is critical before ALT is conducted, and this requires the support of relevant statistical theories. With the promotion of the national strategy of civil-military integration, ALT will be widely used in the research and development (R&D) of various types of products, and the ALT plan design theory will also face more challenges. To aid engineers in selecting appropriate theories and to stimulate researchers to develop the theories required in engineering, following the principle that make engineers be convenient for selecting appropriate theories and methods in accordance with the problems found in practice,

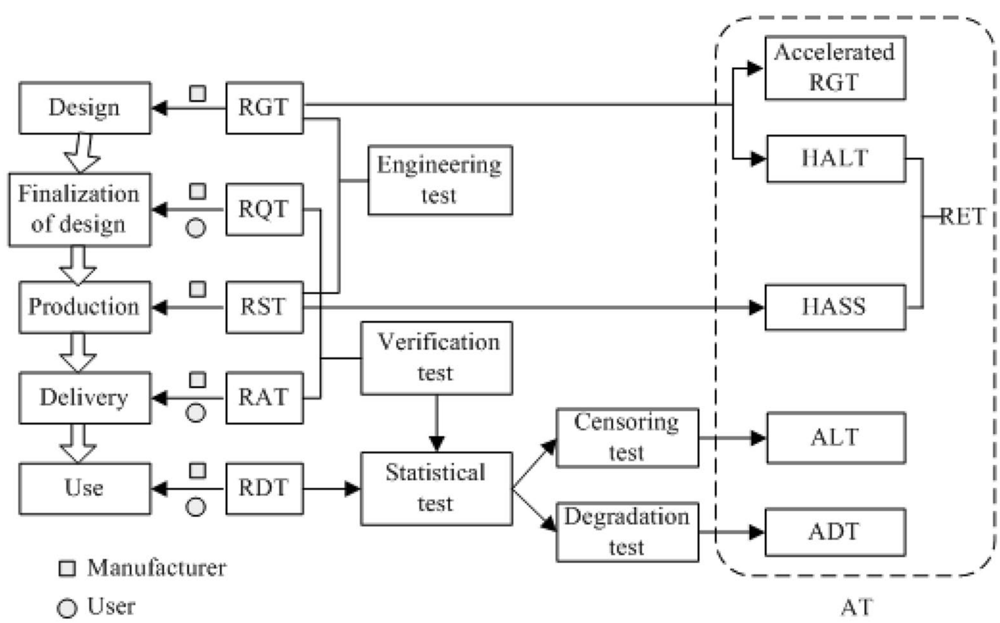

Figure 1 Types, applications and developments of reliability tests

this paper provides a review of the development of ALT plan design theory. Section 2 gives a brief description of the types, statistical essentials and development overview of ALT. Section 3 describes the development of the design theories and methods for planning optimal ALT with location-scale distribution. From the viewpoint of engineering application, except for some most simple product which life follows exponential distribution, one always gives priority to the location-scale distribution to describe the product life distribution, and prepares the test as far as possible to reach the requirements of the test form and sample size according to statistical theories. Then, one has a relatively mature and programmed method for designing test plan and analyzing data. However, there are many problems encountered in practice that cannot be solved by using the method provided in Section 3. Section 4 discusses the current views and possible methods of addressing these problems, and puts forward some opinions on their development trend. Section 5 proposes suggestions of selecting appropriate ALT plan design theories, and gives viewpoints about the urgent solved theory problems and opinions of their solutions.

2 Overview of ALT

2.1 Types of ALT

In an ALT, with the premise that the failure modes and failure mechanisms of product are the same as those under normal stress, the samples are tested at stress levels higher than normal. Then, the product life at the normal stress level can be estimated by extrapolating the life information of samples at high stress levels to the normal level based on the stress-life relationship. The types of ALT can be classified according to four characteristics [6–8, 11–13]:

(1) The mode of stress loading, which includes the main stresses of constant stress, step stress, and progressive stress. The three kinds of loading mode correspond to the constant stress ALT (CSALT), step stress ALT (SSALT) and progressive stress ALT (PSALT), respectively.
(2) The criteria for stopping the test, which includes the main criteria of time-censored (type-I censoring) and failure-censored (type-II censoring).
(3) The strategies of performance inspection for the test unit, which include continuous inspection and periodic inspection. These two strategies generate the life data and group data, respectively.
(4) The number of accelerated stresses, which includes single stress, double stresses and multiple stresses (the number of test stresses greater than or equal to three).

2.2 Statistical Essentials of ALT

The statistical model, statistical analysis method, and test plan design method are the three key elements of ALT. They are closely related to the type of ALT, primarily as follows:

(1) For CSALT, the statistical model includes the life distribution and the stress-life relationship; however, for SSALT and PSALT, the statistical model also includes the equivalent principle of stress level transition;
(2) The different types of stress loading, test stopping criteria and performance inspection methods correspond to different data types and data analysis methods;
(3) The stress-life relationships of single stress, double stress and multiple stress ALT are single, binary, and multivariate functions, respectively. The ALT plan design method is dependent on the stress number.

2.3 Current Situation of ALT

In terms of test purpose, most current ALTs carried out belong to RDT. In terms of test mode, the most widely used test in practice is the CSALT with single stress and time-censoring. With the increasing requirements of product reliability, increasing complexity of operational conditions, and improvement in the technical level of both the instruments and equipment, the multiple stress ALT (MSALT) and PSALT with time-censoring are used in an increasing number of engineering applications.

Corresponding to the classification of statistical thoughts in statistics, the statistical models and methods of ALT can be classified into descriptive statistics, parametric statistics, nonparametric statistics, and Bayesian statistics [9, 10]. In general, the most widely used models and methods almost all belong to the parametric statistics. Among them, the most widely used and advanced theory is the maximum likelihood estimation (MLE) theory for the location-scale distribution and the linear stress-life relationship. With the wide application of ALTs to an increasing number of engineering objects, problems such as difficultly in determining the type of product life distribution and the lack of test data become sharp, and non-parametric and Bayesian methods are gaining attention.

The problem of planning ALTs is the opposite to that of data analysis. Finding the optimal test plan is a common pursuit of engineers and statisticians, and is the most attractive research subject in the application theories of ALT. In principle, a stress loading mode, statistical model, data analysis method, testing condition limitation, and design objective should correspond to a respective problem of designing an optimal ALT plan. Therefore, the test method, statistical models, and data analysis methods of ALT correspond to the following studies of planning optimal ALT covers:

(1) Various test forms, such as CSALT, SSALT, PSALT, type-I censoring ALT, type-II censoring ALT, single stress ALT, double stress ALT, and multiple stress ALT.
(2) Various statistical models, such as descriptive, parametric, nonparametric, and Bayesian statistics models.
(3) Various optimization objectives and constraints such as V-optimization (to obtain the optimal plan by minimizing the asymptotic variance of the MLE of the pth quantile of the product life distribution under normal stress), D-optimization (to obtain the optimal plan by maximizing the determinant of the Fisher information matrix of MLE), single objective, multiple objectives, cost limits, and resource limits.

Among the studies mentioned above, the most widely used and researched ALT plan is the V-optimal continuous-inspection type-I censoring CSALT plan for statistical models with location-scale distribution and linear stress-life relationship.

3 Design Theory and Method of ALT with Location-Scale Distribution

3.1 Development Thread

The study of the statistical theory of ALT began with exponential distribution [5–11]. From an engineering point of view, ALT was applied initially for electronic products, and the exponential distribution was used widely as "standard distribution" in the reliability analysis of electronic products; from a statistical point of view, the exponential distribution led to numerous simple and beautiful analytic conclusions, beloved by theory statisticians. However, the research focus began to change as more engineers and applied statisticians found that it was more appropriate to describe the product life distribution of most products as a function belonging to the location-scale distribution, such as normal distribution, Weibull distribution and log-normal distribution, than exponential distribution.

Nelson et al. [7, 8, 14–19] researched the data analysis method and the optimal plan design method for CSALT with single stress and type-I censoring for lifetimes following the Weibull distribution (or extreme value distribution) and log-normal distribution (or normal distribution). This was a milestone in the development of the statistical theory and methods of ALT. Thus far, the modes of ALT have been extended, the statistical models generalized, and the statistical methods improved; however, the basic ideas and methodological frameworks of Nelson et al. have not yet been exceeded, and the results of their researches are widely used in engineering, and as the important basis and reference for promoting the development of research and comprising the efficiency of the optimal ALT design method. Their studies are briefly described in Section 3.2.

Following Nelson et al. from the aspect of data analysis methods, the dominance of MLE is difficult to shake. Further development of the ALT theory is focused mainly on the optimal design of the test plan, which can be divided into two directions overall:

(1) Following the design ideas and methodology of Nelson et al., expand the optimization model, stress loading modes, and statistical models; and develop the studies of other optimization objectives, constraints, test modes, and statistics models, such as type-II censoring, group data, multiple stresses, SSALT, PSALT, competition failure, and nonlinear stress-life relationship;
(2) Attempt to use design ideas and methodology that are different from those of Nelson et al. to address the problems of unknown model parameters, model deviations and limited sample size during the optimal design, and to explore better methods and plans.

Table 1 summarizes the evolution of the design of the optimal CSALT plan in the first direction. In Table 1, there are some studies beyond the category of the location-scale distribution and parametric statistics, or those involved in the development of the second direction; however, to facilitate the narrative, they are still listed in the table. This paper focuses mainly on the study development of the first direction in the MSALT (Section 3.3) and SSALT (Section 3.4), and the study development of the second direction (Section 3.5).

3.2 Classical Model and Method for Planning Single Stress CSALTs

3.2.1 Statistical Model

(1) The logarithm life of the product follows an extreme value distribution or normal distribution (that is the product life follows the Weibull distribution or lognormal distribution, respectively). The cumulative distribution function (CDF) is $F(y;\mu,\sigma)=\Phi[(y-\mu)/\sigma]$, where μ is the location parameter, σ is the scale parameter, and $\Phi(\bullet)$ is the standard extreme value or the standard normal distribution [14–19];

Table 1 Classical optimal design model of CSALT and extended researches

Essentials of optimization model			Model and method proposed by Nelson et al. [7, 8, 14–19]	Extended researches
Test purpose			To estimate the *p*th quantile $y_p(0)$ of lifetime under normal stress	To estimate multiple quantiles [20–22]; for acceptance of product [23, 24]; to select the optimal product [25]; to estimate the quantile of the different products under normal stress [26]; to estimate the other parameters [27–30]
Test forms	Type of test censoring		Type-I censoring	Failure censored [23, 25, 31, 32]; time failure mixed censored [33]
	Data collection method		Continuous testing	Periodic testing [34–39]
	Strategy for testing the samples		All samples are put in from the start.	Sequential test method [40–43]
	Number of stress levels		Singe stress level	Double or multiple stress levels [44–55]
Statistical models	Failure mode		Single failure mode	Multiple failure modes [28, 29]
	Types of life distribution		Weibull/lognormal distribution	Two parameter exponential distribution [56]; Rayleigh distribution [36]; Burry type XII [38]; nonparametric method [27, 57]
	Parameters relating to stress		Location parameter of location- scale distribution	Location and scale parameter of location-scale distribution [24, 58]
	Form of stress-life relationship		Linear function	Polynomial [26] /nonparametric [27]
	Data form		Life data	Group data [34–39]
Data analysis method			MLE	Unbiased estimation [56]; Bayesian [40, 59, 60]
Optimal design models of test plan	Optimization objective		Minimize the asymptotic variance of estimated $y_p(0)$	Multi-objective [20–22, 26, 27, 61–68]; cost [68]; test time [65, 66]; D-optimal [28, 29, 48, 69]; minimax [70]; robustness [21, 64]; highest stress [71]; others [27, 30]
	Design Variables	Stress level	Minimum stress	Intermediate stress [27, 57, 65–68, 72, 73]
		Allocation of sample size	Proportion in which samples are allocated to the lowest stress level.	Proportion in which samples are allocated to the intermediate stress level [27, 57, 65–68, 72, 73]
		Allocation of test time	Not optimized	Optimize test time allocated to each stress level [65, 66, 68]
	Constraints	Total sample size	Large sample size	Limited sample size [74]
		Total test time	Specify the censoring time allocated to each stress level	Specify the total test time [68]
		Space between stress levels	Unconstrained/Equal difference	Unequal difference [21, 27, 34, 36, 57, 63, 64, 66–68, 70, 72, 73]
		Allocation of sample size	Unconstrained/equal allocation/Specified proportion	Minimize the expected number of failures [27, 39, 57, 66–68, 74]
		Allocation of test time	Equal	Un-equal [65, 66, 68]
		Other	/	Acceleration factor [49]; probability of failure at each stress level [65, 67, 72, 73], etc.

(2) The location parameter μ is a linear function of the standardized stress ξ ($0 \le \xi \le 1$), that is the stress-life relationship is $\mu(\xi) = \gamma_0 + \gamma_1\xi$, where $\gamma_1 < 0$. The Arrhenius model and the inverse power law model, which are the most widely used in engineering, could both be transformed into linear stress-life relationships [7, 8, 14–19];
(3) The scale parameter σ is constant and independent of ξ [14–19];
(4) For each test unit, the failure time is statistically independent [14–19];
(5) The type-I censoring CSALT is considered, and the censoring time at each stress level is τ [14–19].

3.2.2 Statistical Method

The estimation method is MLE. Assume that there are k stress levels, then the sample size on the ith level ξ_i ($i = 1, 2,\ldots, K$) is N_i, and the lifetime of the jth ($j = 1, 2,\ldots, N_i$) samples on ξ_i is (t_{ij}, δ_{ij}) (if the sample fails, then $\delta_{ij} = 1$; if the sample is censored, then $\delta_{ij} = 0$ and $t_{ij} = \tau$). The log likelihood function of the MLE is

$$\ln L = \sum_{i=1}^{K}\sum_{j=1}^{N_i}\left[\delta_{ij}\ln f(t_{ij};\mu(\xi_i),\sigma) + (1-\delta_{ij}) \times \ln(1 - F(\tau;\mu(\xi_i),\sigma))\right],$$

where $f(t_{ij}; \mu(\xi_i), \sigma)$ is the probability density function (pdf) of the extreme value or normal distribution [7, 8, 14, 16].

3.2.3 Method of Designing Optimal ALT Plans

Theoretically, the problem of designing optimal ALT plans could be expressed as [7, 8, 14, 16]: given the prior estimate values $\gamma_{0,e}$, $\gamma_{1,e}$, and σ_e of the model parameters γ_0, γ_1, and σ, respectively, and given the censoring time τ and the failure probability p at the normal stress level, find the number of stress levels K^*, the stress level ξ_i^*, and the sample location ratio p_i^* that minimize the asymptotic variance of the MLE for the pth quantile y_p of product life distribution at the normal stress level.

Nelson and Meeker [14–17] studied the solutions of this problem (called the statistically optimal plan), and drew the following key conclusions:

(1) If the censoring time is not too long, then K^* is always equal to two, and the optimal maximum stress level ξ_H^* is always one;
(2) The optimal minimum stress level ξ_L^* and the sample location ratio p_L^* of ξ_L^* are functions of $a_e = (\ln\tau - \gamma_{0,e})/\sigma$, $b_e = -\gamma_{1,e}/\sigma$ and p, and generally p_L^* is greater than 0.5. This means that allocating more samples on the lowest stress level helps to improve the accuracy of the estimation;
(3) For a given value of a_e, the greater the value of b_e, the smaller the value of the optimal variance factor V_K^*. This means that for the same censoring time, the greater the acceleration factor, the higher the maximum stress level, and thus the higher the estimation accuracy;
(4) For the same value of b_e, the greater the value of a_e, the smaller the V_K^* value, meaning that with the same acceleration factor and highest stress level, the longer the censoring time and the shorter the product life at the normal stress level, the higher the estimation accuracy;
(5) In particular, if the censoring time is too long, the ALT will degenerate into the censored test at the normal stress level, and then $K^* = 1$, $\xi_L^* = 0$, $p_L^* = 1$, and V_K^* approaches a constant.

However, Nelson and Meeker considered that the statistically optimal plan might not have good performance in application because [7, 8, 14–19]:

(1) The optimal plan depends on the values of unknown model parameters γ_0, γ_1, and σ. During the optimization, by substituting γ_0, γ_1, and σ for their prior estimates $\gamma_{0,e}$, $\gamma_{1,e}$, and σ_e, the errors in the prior values may cause the efficiency of the optimal statistical plan to differ substantially from that of actual optimal plan; this may be worse than that of some traditional empirical plans;
(2) If the statistically optimal plan is adopted, the errors in the prior estimates may lead to test failure because the actual test may lack sufficient failure samples at the lowest stress level;
(3) If the true distribution of the product life is not a Weibull distribution or a lognormal distribution, the efficiency of the optimal statistical plan may reduce greatly;
(4) The statistically optimal plan only has two stress levels, and cannot test the correctness of the stress-life relationship;
(5) If the true stress-life relationship deviates from a linear relationship, the performance of the statistically optimal plan may likely reduce greatly;
(6) If the sample size cannot meet the requirements for using an asymptotic theory, the plan performance may not be guaranteed.

To solve these problems, Nelson [14–17] suggested the use of a "compromise plan" with three or four stress levels, to enable the middle stress level to test the stress-life relationship, prevent test failures, and improve the plan robustness to the deviations in the statistical model and model parameters. Furthermore, Meeker [18] proposed several compromise plans, and the criterion that evaluates the ALT plan robustness to deviations in the model parameters and product life distribution. Through computer experiment, over a wide range of values of $\gamma_{0,e}$, $\gamma_{1,e}$, and σ_e, and considering the deviations in the model parameters and life distribution, Meeker studied the actual efficiency of statistically optimal plans and compromise plans. He concluded that the best plan of considering the estimation accuracy and robustness is the optimal compromise plan with three equally spaced test stresses, and the sample location ratio of the middle stress level is 10% or 20% (the ratio at minimum stress level should be determined via optimization; this plan was furtherly simplified by Meeker and Hahn [19] to become the "4:2:1 plan", which has three equally spaced levels and with a sample allocation ratio of 4:2:1 at the lowest, middle, and highest stress levels, respectively).

The optimal compromise plan proposed by Meeker was widely used in engineering and became the "benchmark" for most of the subsequent improved plans, and the method of comparing and selecting the ALT plan with a combined consideration of robustness and estimation accuracy became the basis method followed by almost all studies of designing optimal ALT plans.

3.3 Design of Multiple CSALT Plans

The multiple constant stress ALT (MCSALT) loads two or more accelerated stresses on the product simultaneously. Compared with the single stress ALT, MCSALTs are closer to the real usage conditions for most products and can make products fail faster. With the rapid development of environment simulation technology, the multiple-stress-test-equipment, which can load two or more environment stresses (such as temperature & humidity, temperature & vibration, thermal & vacuum, and temperature & humidity & vibration) on products has been gradually becoming available in the market, and MCSALTs have gradually started to have wide applications in engineering. However, theoretically, when the number of accelerated stresses is greater than one, the stress-life relationship changes into a binary or multivariate function that leads to problems that are different from those of planning single stress ALTs.

Escobar and Meeker [44] carried out the earliest study on the theory and method of planning the optimal MCSALT for location-scale distribution. They used the assumptions mentioned in Section 3.2.1, and generalized the stress-life relationship into a binary linear function $\mu(\xi_1, \xi_2) = \gamma_0 + \gamma_1\xi_1 + \gamma_2\xi_2$ (where $\gamma_i < 0$, $0 \le \xi_i \le 1$, and $i = 1, 2$), and proved the following important conclusions [44]:

(1) The V-optimal MCSALT plan is not unique.

(2) There is a type of V-optimal plan with stress level combinations $\boldsymbol{\xi}_i^* = (\xi_{i1}^*\ \xi_{i2}^*)$ ($i = 1, 2, \ldots, K^*$) distributed on a straight line connecting the normal stress level (0, 0) and the highest stress level (1, 1). Such plans cannot determine all parameters of the stress-life relationship, and are defined as the optimal degenerated plan.

(3) Each optimal degenerated plan corresponds to an infinite number of optimal non-degenerated plans (all parameters of the stress-life relationship can be determined). The stress level combinations of optimal non-degenerated plans distribute on the stress-life relationship contour through the point $\boldsymbol{\xi}_i^* = (\xi_{i1}^*\ \xi_{i2}^*)$, and can be related to the stress level combinations of the optimal degenerated plan by some equations.

Because the optimal plan is not unique, to obtain a determined plan, one should restrict the arrangement mode of the stress level combinations (called test points) in the feasible region of the test (called test region), and restrict the sample location ratio on test points. Escobar and Meeker [44] proposed a method of obtaining the optimal non-degenerated plan (called splitting plan): find the test point $\boldsymbol{\xi}_i^*$ and the sample location ratios p_i^* thereof for the optimal degenerated plan by solving the optimization problem of single stress ALT; then, find the two intersection points $\boldsymbol{\xi}_{i,1}^*$ and $\boldsymbol{\xi}_{i,2}^*$ of the stress-life relationship contour through the point $\boldsymbol{\xi}_i^*$ and the boundary of the test region; and make the sample location ratio of $\boldsymbol{\xi}_{i,1}^*$ and $\boldsymbol{\xi}_{i,2}^*$ inversely proportional to their distance to $\boldsymbol{\xi}_i^*$.

The splitting plan is the V-optimal plan, and is also the D-optimal plan among all V-optimal plans [44]. However, in consideration of the uncertainty of the actual efficiency of the theoretical optimal plan, which is due to the errors of prior estimates on the model parameters, and the need to examine the model and analyze the effect of accelerated stress through ALT, some researchers proposed other arrangement modes of test points. For example, Park, et al. [45], Yang [46, 47] and Guo et al. [48] arranged the test points via orthogonal designs (called an orthogonal plan). Chen et al. [49, 50] arranged the test points based on uniform designs (called a uniform plan). Over a wide value range of model parameters, Gao et al. [51] compared these plans through computer experiment from three aspects, namely the estimation accuracy of the pth quantile, robustness to the deviation of the model parameters, and the estimation accuracy of the model parameters; they concluded that the splitting plan was the best in terms of comprehensive performance.

Another problem in designing the optimal MCSALT plan is that the stresses at the highest level may not be loaded on the product simultaneously, thus resulting in a non-rectangular test region [52]. To solve this problem, Chen et al. [52] demonstrated that the conclusions drawn by Escobar and Meeker remained valid for simply-connected test regions with convex boundaries. In this situation, the test points of the optimal degenerated plan were distributed along the line connecting the normal stress level (0, 0) and the highest stress level (ξ_{H1}, ξ_{H2}), where (ξ_{H1}, ξ_{H2}) was the point at which the value of the stress-life relationship reached the minimum on the boundary of the test region. Based on this, they generalized the splitting plan to simply-connected test regions with convex boundaries. Later, Gao et al. [53] proposed that if the test objective was only to estimate the pth quantile of the product life distribution, it was not necessary to estimate all the parameters in the stress-life relationship. Therefore, the optimal degenerated plan also has practical value, and it can be applicable for both rectangular and non-rectangular test regions. The comparison results from the computer experiment show that for the double-stress test and on the different shapes of the test regions, the optimal degenerated plan has better actual efficiency on average than the corresponding splitting plan, over a wide value range of model parameters and in consideration of the effects of the model parameter error. Furthermore, for a splitting plan for the MCSALT with more than two stresses, the number of test points and the difficulty in finding them increases sharply with the increase in the number of stresses; in addition, the sample allocations at the test points are reduced accordingly, and this increases the risk of test failure. However, the degenerated plans are almost irrelevant to the dimension [54].

Finally, when the number of accelerated stresses is greater than one, the interaction effect between the stresses causes the stress-life relationship to be a nonlinear function. In principle, with a little generalization, the splitting, orthogonal, and uniform plans are all applicable to the nonlinear stress-life relationship [44, 48]. However, Gao et al. [55] found that one could achieve a better plan by using a line segment (chord) to connect the highest and lowest points of the curve or surface that corresponds to the nonlinear stress-life relationship, and by considering the chord as a new stress-life relationship from which to design the test plan and extrapolate the pth quantile. Accordingly, they proposed a "chord method" for planning the V-optimal CSALT with time censoring and continuous inspection. For the problem of planning optimal MCSALT, whether the stress-life relationship is univariate or multivariate, linear or nonlinear, and whether the test region is rectangular or non-rectangular, the method could transform it into the problem of planning a single stress ALT with a linear stress-life relationship.

3.4 Design of SSALT Plans

The step stress test was originally used in ERT to detect the working limits and defects of products. Nelson [7, 75] introduced the hypothesis of cumulative damage, which states that the development of product damage under the same type of stress and failure mechanism was only related to the current state and current stress level, and was independent of the history of stress loading. Based on this hypothesis, Nelson established a rule of equivalent conversion between the life distributions and test times at different stress levels, and proposed the theory and method of applying the step stress test to the SRT. In SSALT, the main method of estimating the model parameters is still the MLE [7, 75]. Bai [76, 77] and Khamis [78, 79] gradually established the theory and method of planning the V-optimal SSALT with type-I censoring. Although it is theoretically possible to find the optimal stress levels and stress switching times on each step of a SSALT with finite multiple steps, the "simple SSALT" that has only two steps was generally used in practice [76]. However, if one needs to check whether the stress-life relationship is linear or not, he should use a three-step SSALT [78].

To assess the product reliability only, it is not necessary to apply incremental stress. The loading order of stress levels can also be optimized to improve the estimation accuracy and reduce the test cost. For the exponential distribution, Miller and Nelson [80] referred to a test with step-down stress (aptly named the step-down test; similarly, the test with gradually increasing stress is called the step-up test, and both the step-up and step-down tests belong to the SSALT), and proved that the statistically optimal CSALT plan, step-down test plan and step-up test plan all have equivalent variance factors. Afterwards, for the Weibull distribution, Khamis [79] compared the variance factors of the optimal CSALT and step-up test plan under the fixed scale parameters and lowest stress level. The results showed that the step-up test was superior. Zhang [81] first studied the step-down test for the Weibull distribution, and pointed out that the effects of the step-down and step-up tests were different owing to the influence of the scale parameters, and the step-down test was superior most of the time in that it need a smaller sample size and shorter test time to reach the same estimation accuracy as the step-up test. Furthermore, Wang et al. [82] demonstrated that if the stress levels and stress switching times were all the same for both the step-down and step-up tests, the step-down test was better than the latter in terms of the estimation accuracy, failure sample size, etc. For the Weibull distribution and lognormal distribution, in the estimation accuracy of pth quantile and the robustness to the deviation of model parameters, Ma and Meeker [83] made comprehensive comparisons on the optimal CSALT, optimal simple step-up and step-down test. They drew the following major conclusions [83]:

(1) The relationship between the estimation accuracy and robustness of the three plans varied with the scale parameters, but without a simple rule that is applicable to all values of the model parameters.
(2) If ranked by the estimation accuracy of the *p*th quantile from high to low, when the scale parameter is less than one, the order is the CSALT, step-up test and step-down test; when the scale parameter is greater than one, the order is the step-down test, step-up test, and CSALT.
(3) If ranked by the robustness from high to low, the order followed is the step-down test, CSALT and step-up test when the scale parameter is less than one, and is the step-down test, step-up test and CSALT when the scale parameter is greater than one.

Although the studies mentioned above are still not sufficient to determine the best ALT mode among the CSALT, step-up test, and step-down test, they prove the following at least: in some cases, in addition to inciting product failure faster than CSALT, the optimal SSALT has a higher estimation accuracy and robustness than the optimal CSALT. This is sufficient to make SSALT a strong competitor to CSALT. Furthermore, one of the major expenses of ALTs in practice is the site cost (calculated according to the number of test equipment and time occupied). The SSALT could reduce test costs: when the total sample size and censoring time are the same, if only one test device is available, then the test time for a SSALT is τ, and 2τ to 4τ for a CSALT; If there is no limit on the number of test devices used simultaneously, then a SSALT only needs one device, but a CSALT needs two to four devices.

3.5 Solutions to Robustness and Limited Sample Size

In the 8th to 14th paragraphs of Section 3.2.3, six doubts about the actual effect of the statistically optimal plan are mentioned, and they can be summarized into the following three aspects of the problem:

(1) Unknown parameters. The 1st and 2nd doubts arose from the query of the actual effect of the statistically optimal plan, considering the errors of the prior estimate value of model parameters;
(2) Model deviations. The 3rd to 5th questions are arose from the concern regarding the inferred errors caused by the assumptions of the statistical model not in line with the actual conditions.
(3) Limited sample size. The asymptotic variance was used as the objective function in designing the optimal plan. If the actual sample size cannot meet the requirements of a large sample size, the efficiency of the optimal plan based on the asymptotic variance is in doubt, which is pointed out in the 6th doubt.

Among these three problems, model deviations and limited sample sizes are very common in many statistical methods, and are not unique to the statistics of the ALT. However, the problem of unknown parameters comes from the "censoring," which leads to the correlation between the optimization objective function and the unknown model parameters; this is unique to the optimal ALT plan design, and increases the difficulty in processing the problems of model deviations and limited sample size. In practice, only after these problems are explained rationally or solved, do the engineers use the optimal plans.

Nelson and Meeker used the "compromise plan" to solve these problems; it is a simple and effective solution. However, there is no precise theory to support whether it was the best way for solving these problems, and therefore some researchers are still trying to find the optimal solution in theory.

Regarding the problem of unknown parameters, Chaloner et al. [20, 61] and Ginebra et al. [70] proposed methods of designing the optimal ALT plan, which use prior distributions and intervals, respectively, to describe the unknown model parameters. Their optimal objectives are to minimize the mathematical expectation of the asymptotic variance of the MLE of the *p*th quantile over the prior distributions, and minimize the maximum value of the asymptotic variance of the MLE of the *p*th quantile over the parameter interval, respectively. The two objectives include the estimation of the parameter range, and the obtained optimal plan was the plan taking into account the parameter estimation errors. In addition, Tang and Liu [40, 41, 59] attempted to consider and control the process of giving the prior estimates in the framework of sequential tests during the plan design. It was perhaps a more complete approach.

For the problem of model deviations, Chaloner et al. [20] and Pascual [21, 62–64] established objective functions containing the effects of model deviation; therefore, they could obtain the optimal plan directly by solving the optimization problems with regard to the model deviations. Specifically, there are two situations:

(1) There are several types of life distributions and stress-life relationships for selection, but the model cannot be determined yet. Therefore, it was necessary to make a test plan with better performance in

all candidate models. Chaloner [20] provided the corresponding optimization model based on the Bayesian method. The objective of the optimization is to minimize the mathematical expectation of asymptotic variances of the p-th quantile corresponding to each possible model; Pascual [62, 63] replaced the asymptotic variance in Chaloner's objective function with an index called asymptotic sample ratio (ASR). The ASR in each possible model is regarded as a component of a vector, and the objective functions are defined by the norms in different forms. In particular, Pascual studied the optimal model under the ∞-norm.

(2) The statistical model was determined during the plan design, but it is expected that the test plan could achieve the best possible estimation accuracy even if the wrong model is selected. For situations where the wrong form of life distribution and stress-life relationship might be chosen, Pascual [21, 64] proposed optimization models that take the asymptotic bias (ABias) and the asymptotic mean squared error (AMSE) as the objective functions respectively.

Refs. [20, 21, 62–64] demonstrate that the ALT plan obtained by the above method is more robust in terms of the model deviation than the compromise plan.

For the problem of limited sample size, Escobar and Meeker [44] used the Monte Carlo (MC) method to simulate the implementation of the optimal plan based on the asymptotic variance, calculate the sample variance of the pth quantile, and investigate the applicability of the theoretical optimal plan by the approximation degree of the sample variance and asymptotic variance. To investigate the approximation degree of the optimal solutions, over the whole feasible area of a one-dimensional optimization problem, Pascual [64] compared the objective function based on the asymptotic variance with the objective function that corresponds to a limited sample size and is calculated by the MC method. The calculation results show that the two objective functions are close to each other in pattern, and without great difference in terms of optimal solutions. Ma and Meeker [74] studied how the sample size and model parameter errors effect the test success rate and estimation accuracy, and introduced a constraint into the optimization model to assure the success rate of ALT with a limited sample size. By combining the optimization based on the asymptotic variance with the graphical method and the stimulation evaluation based on the MC method, they put forward a method of designing the optimal compromise plan with the comprehensive consideration of the effects of limited sample size and unknown parameters. In addition, Meeker [8] suggested that to design the optimal ALT plan based on the objective function obtained by the MC method, rather than the objective function based on the asymptotic variance; Wang [84] conducted a systematic research on this topic, and this type of method is called the "simulation based optimization", from which the optimal plan corresponding to the limited sample size can be obtained.

Overall, under some conditions, these studies indeed produce plans better than the compromise plan. However, these methods have not yet been widely used in engineering, because they are sometimes complicated for engineers, and there is still no sufficient evidence to support their superiority to the compromise plans in all aspects of practice.

4 Additional Problems of ALT

4.1 Statistical Model Test

Whether the statistical model is suitable for engineering practices largely determines the actual effectiveness of the statistical inference. Therefore, it is necessary to test the statistical model before it is used in reliability assessment. The current theories provide methods for testing various common life distributions [5–11]. For the ALT, the verification on the stress-life relationship and the assumptions of cumulative damage (for SSALT and PSALT) is more applicable in engineering. In this respect, although the traditional theories on regression diagnosis are rich [7, 8, 85, 86], few of them are applicable to the censored data and non-normal distribution. In addition to the graphics method in Refs. [7, 8], to meet the need of the ALT for electrical connectors, based on the principle of failure physics, hypothesis testing, and regression theory, Qian et al. [87, 88] and Liu [89] proposed methods of testing the multivariate stress-life relationship and the assumptions of cumulative damage of a time-censored ALT with Weibull distribution, respectively. However, its effectiveness lacks support from precise statistical theory. Recently, Pan et al. [90] proposed a method of designing an optimal ALT plan for the verification of the stress-life relationship in the framework of a generalized linear model.

In general, testing a model needs more sample sizes and stress levels than estimating model parameters. For the ALT, it is difficult to validate the extrapolation effect of the model directly because of the long product life and high reliability. Perhaps, new ideas are required to deal with the uncertainty of the acceleration model. Besides, for some products with complicated structures, their life distribution and stress-life relationship may be difficult to describe in a simple form.

If the model cannot be determined, there are two main processing ideas:

(1) Find a robust plan with good performance in a variety of alternative models [20, 21, 62–64]. These studies were introduced in Section 3.5.
(2) Use the nonparametric methods to reduce the reliance of statistical inference on the specific form of the models. Refs. [91, 92] provide a comprehensive description of the nonparametric models and methods commonly used in the life tests, such as the proportional risks model; Refs. [27, 57] are researches on planning the optimal ALT with nonparametric models. To some extent, the nonparametric method can avoid the problem of the uncertainty of the model, as well as deal with the life distributions and stress-life relationship that are difficult to express with analytic functions; however, they generally require a large sample size [93], which is another main problem faced by ALT.

4.2 Restrictions on Sample Size

To enable the MLE to achieve an acceptable accuracy, the number of failure samples in a test is at least 8 to 10, and typically 30 to 40, or even hundreds. However, the ALT for many products cannot meet this requirement. On one hand, with the improvement in product reliability and lifespan, in some products with long life and high reliability, it is difficult to induce failure within the acceptable test time and sample size (for example, some military components require the 99% reliable life to be up to 24 years). On the other hand, for some very expensive and low-yield parts, equipment, devices, and machines, the number of samples used for the reliability test may be only four to five, one to two, or even zero, and it is difficult to carry out the statistical inference generally. Solving the two aspects of the problem may be beyond the scope of the general ALT and parameter statistics. The statistics for the minimal sample size, Bayesian method, and ADT can reduce the requirements of sample size to a certain extent; thus, this is currently receiving increasing attention [8, 94–101].

4.3 Resource Limits

In practice, the limits on the test conditions, sites, and cost are considered for the implementation of ALTs.

The limits on test conditions mainly refers to the situation in which the performance parameters of the product could only be periodically inspected. Periodic inspection generates group data. In theory, there have special methods of data analysis and optimal plan design for group data [5–11, 34–39, 88]. However, from the application point of view, the method of the data analysis and plan design for life data are relatively simple. Therefore, in practice, the engineers tend to use the method for life data to deal with the group data, by transferring the group data into life data by interpolation methods [102, 103]. However, a systematic study on the accuracy of this method has not yet been reported.

The limits on test sites mainly refer to the restriction in the number of equipment that can be used simultaneously within a certain time. The limits on test costs are mainly due to the sample size, test methods, test time, equipment number and test stress levels. Many studies introduced some constraint on test sites and costs in the optimization model [65, 66, 68]; however, we are not aware of a general model or a systematic study on the characteristics of the solutions of such optimization problems.

4.4 Optimal Mode of Stress Loading

In addition to CSALT, SSALT and PSALT, the researchers proposed other test modes, such as the group ramp test [104, 105], trapezoid test [106, 107], and ramp-constant stress mixing test (RCSMT) [108]. At present, there are still doubts regarding the implementation methods and effects of the SSALT and PSALT, and other methods of stress loading are limited. However, as the performances of SSALT and RCSMT are sometimes superior to that of CSALT [80, 82, 83, 108], at least in theory, it is possible that the modes of stress loading can still be optimized, but no theoretical research on this topic has been produced.

4.5 Optimal Arrangement of Reliability Test

Besides estimating the *p*th quantile, it is often expected in engineering to estimate the model parameters and compare or verify the reliability indexes of the product by the ALT (see Table 1). From a greater perspective, in product reliability engineering, many kinds of ATs are carried out with different objectives and in different stages. The question is how to arrange these tests most efficiently. No quantitative studies on this topic have been reported yet.

4.6 Product Complexity

Thus far, in general, successful ALTs are mainly aimed at the materials, components, and parts under simple stresses (such as constant temperature, voltage, and vibration), with simple failure mechanisms (such as single failure due to oxidation, electrical aging, and wears) and in the single-failure mode. However, these products account for only a small percentage in practice. The complexity in product structure, work stress, failure mechanism, and failure mode increase the challenges of the ALT.

From the aspect of work stress, there are three main problems. (1) The functions of materials and parts in the components and systems are subject to the general environment and other parts in the system. If the ALT on the materials and parts lacks sufficient consideration of these factors, the accuracy and credibility of the test results will be affected. In general, the same material, parts, or components are often used in different environments, and it is impossible to give assessments for all working conditions. The methods of applying the results of ALT to wider environments and assessing the application scope of ALT are the major problems to be solved. Few quantitative research projects are currently addressing this topic. (2) A large number of electromechanical components are loaded with stresses dependent on time, such as alternating current and force. Only a few studies on the ALT under time-dependent stress have been reported [109] (this does not refer to the varying stresses applied in SSALT and PSALT, but to the varying working stresses on the product). (3) A large number of products undergo different stages in their life cycles, including transportation, storage, and application; however, most of the current ALTs are only focused on one stage, and researches on multiple stages are rarely reported.

The aspects of failure mechanism and failure mode are mainly concerned with the theory and method of ALT for products with competitive failures. Related research on this aspect started very early [5–11]. The general strategy is to transform the ALT of competitive failure into ALTs of several single failures under the assumption that each failure mode and mechanism are not related to each other [28, 29], but few cases meet this assumption. From the aspect of engineering application, it is not easy to determine the life distribution and stress-life relationship of a specific product with competitive failure, because once there is more than one failure mode and failure cause, the data collection and physical analysis of failure becomes more difficult. For products with multiple failure mechanisms and modes, or even with relevance to each other (usually referred to some components or systems), it is doubtful that their statistical models can be described with a simple and analytic life distribution and stress-life relationship. Thus, it is more difficult to develop the corresponding method for the ALT.

The structure and components of products mainly involves the ALT on system-level products (including parts, components, machine, and equipment group). For these products, various problems, such as minimal sample size, competitive failure, complex stress and component relevance, often arise simultaneously; and are followed by technical problems. For example, the product size and weight are beyond the range of the test equipment. At present, except for some special systems, there is a lack of effective ideas and methods for dealing with ALTs of system-level products.

5 Conclusions and Outlook

(1) The ALT theory that deals with the location-scale distribution and the linear stress-life relationship is suggested as a first choice in practice, because it is relatively mature and has numerous successful applications in the fields of materials and components.

(2) Regarding the problems of selecting test modes between CSALTs and SSALTs (Sections 3.3 & 3.4), robust plan design (Section 3.5), limited sample size (Sections 3.5 & 4.2), model verification (Section 4.1) and resource limits (Section 4.3), current researches can provide heuristic guidance and case reference for engineering practice. However, the relevant theory still needs further improvement. Among these problems, the weakest research in present studies is the model verification problem. There is an urgent need to establish statistical theories and methods that can deal with the test of stress-life relationship with censored data and non-normal distribution, research the relationship between the ALT used for testing models and that used for estimating model parameters, and study the method of designing optimal plan for testing model.

(3) For ALTs with phased-missions, complex stresses, and competitive failures or for system-level products (Section 4.6), the statistical models, modeling methods and test methods still fall short in terms of sufficient cases and theoretical support. This requires more collaboration among engineers and statisticians to promote its development. These problems could possibly be solved in three stages. Firstly, the AT theories and methods that deal with the phased-mission, time varying stress, and competitive failures should be established gradually by conducting research on the AT estimation-technology of the operational reliability of the components and material with single failure mode and failure mechanism in practice. Then, the AT methods that aim at estimating the reliability of the components and material with competitive failures could be further studied and determined. Finally, the AT methods applied to a system level product can be studied. These three stages have increasing difficulty, and are extremely challenging in terms of estimation theories, methods, and technologies. It is possible that the framework of ALT alone cannot solve these problems completely; ALTs need to be combined with the ADT, dynamic model, and simulation technology based failure physics, and so on.

Authors' contributions

WC put forward the framework and content of the paper, completed sections 1, 2 and 5, and did the global modification and adjustment of the

manuscript; LG drafted the paper, and completed sections 3.1,3.2, 3.3 and 3.5; JP gave suggestions about the framework and content, and completed sections 3.4, 4.5and 4.6; PQ completed sections 4.1, 4.2 and 4.3; QH completed section 4.4, and checked the English draft. All authors read and approved the final manuscript.

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