Chinese Journal of Mechanical Engineering

ORIGINAL ARTICLE

Open Access

Condition-based Maintenance Optimization for Gamma Deteriorating Systems under Performance-based Contracting

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Abstract

With the further development of service-oriented, performance-based contracting (PBC) has been widely adopted in industry and manufacturing. However, maintenance optimization problems under PBC have not received enough attention. To further extend the scope of PBC's application in the field of maintenance optimization, we investigate the condition-based maintenance (CBM) optimization for gamma deteriorating systems under PBC. Considering the repairable single-component system subject to the gamma degradation process, this paper proposes a CBM optimization model to maximize the profit and improve system performance at a relatively low cost under PBC. In the proposed CBM model, the first inspection interval has been considered in order to reduce the inspection frequency and the cost rate. Then, a particle swarm algorithm (PSO) and related solution procedure are presented to solve the multiple decision variables in our proposed model. In the end, a numerical example is provided so as to demonstrate the superiority of the presented model. By comparing the proposed policy with the conventional ones, the superiority of our proposed policy is proved, which can bring more profits to providers and improve performance. Sensitivity analysis is conducted in order to research the effect of corrective maintenance cost and time required for corrective maintenance on optimization policy. A comparative study is given to illustrate the necessity of distinguishing the first inspection interval or not.

Keywords Performance-based contracting, Condition-based maintenance, Gamma process, Profit maximization, Inspection interval

1 Introduction

Service-oriented is an innovation that organization's central work shifts from selling labor and materials to selling compositive products and services [1, 2]. With the continuous development of service-oriented, the role of operation and maintenance (O&M) has become more and more significant. For instance, in the U.S. defense budget for the fiscal year 2020, O&M costs were $292.7 billion, accounting for 41% of the total defense budget. Compared with the fiscal year 2019, O&M costs increased by $9.2 billion [3].

Conventionally, O&M is implemented under material-based contracts (MBC) in which clients pay support providers according to the materials and labor consumed each time [4]. Nevertheless, with the implementation of MBC, the service providers will expose the problem of insufficient innovation. This is because a majority part of the provider's income is from the consumables and services they sold. With the product ages, providers might profit from these individual products. In contrast, clients need to pay more support and maintenance costs [5–7].

In order to well address this challenge, a new kind of support contract, namely "Performance-based contracting" (PBC), has emerged. Under PBC, clients pay for

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the outcomes delivered by providers instead of the individual materials or services [8, 9]. Likewise, providers' compensation has been tied to how much they successfully achieved the required outcomes. If providers do not act and allow the product to obsolete, the consequences will be borne by themselves. PBC promotes providers to innovate in the form of materials, technologies, processes, and policies.

The concept of PBC came from Performance-based Logistics (PBL), which was first proposed in the U.S. military. As an outcome-based support policy, PBL projects and delivers an integrated, affordable performance solution so as to optimize the weapon system's performance and readiness. PBL concentrates more on how much the support solution satisfies the war fighter's requirements, usually adopted availability instead of the consumption of resources to express [10]. In short, purchasing performance outcomes instead of individual materials or services is the essence of PBL. In the following discussion, PBL has the same meaning as PBC.

In recent years, PBC has been widely applied in various fields, therefore the amount of academic research related to PBC is also gradually increasing. Nevertheless, the main research attention is put on contracts [11, 12], risks [13–16], incentives [17], and performance metrics [18]. Tan [11] proposed a new analytical model, which helps to determine the parameters and analyze contracts. Shang et al. [16] improved that energy performance contracting is an effective way to achieve the goals of energy saving and emission reduction. Selviaridis et al. [17] presented a cross-case study. They found that the incentives can be framed through using a promotion, prevention, and hybrid, respectively. Akkermans et al. [18] provided a new approach to buyer-supplier contracting. According to our observation, mathematical modeling, and optimization approaches under PBC are rarely addressed.

Nowadays, maintenance has been extensively recognized as an essential part of asset management and a necessary business function. More and more manufacturers start realizing that the efficiency and reliability of products can be improved more effectively by developing a maintenance plan [19]. Therefore, there is more and more preventive maintenance (PM) is implemented. Conventionally, PM is implemented in the form of unit replacement or overhaul based on runtime, namely Time-based Maintenance (TBM) [20]. TBM is a maintenance policy that implements maintenance activities at regular intervals based on historical maintenance data. However, CBM is a maintenance policy that puts more emphasis on planning maintenance activities through data collected by sensors [21]. With the use of emerging technologies such as wireless telecommunication and various sensors, CBM has developed rapidly and CBM modeling attracts increasing attention [22]. Alaswad and Xiang [21] conducted a comprehensive and systematic survey of CBM studies. They mainly reviewed the inspection interval, optimization objective, degree of maintenance, and solution method of the CBM. With respect to systems subject to continuous deterioration, we generally adopted stochastic deterioration models, like the Wiener process and Gamma process. When degradation is in the form of cumulative damage, the Gamma process is more appropriate, which has been extensively studied in CBM models [23]. However, to our best knowledge, the work related to CBM maintenance optimization for gamma deteriorating systems under PBC is rarely reported.

In order to fill the gap mentioned above, the CBM optimization for gamma deteriorating systems under PBC is investigated in this paper, which can be viewed as a combination of PBC and CBM and is still very limited. The main target of this study is to find the optimal decision variables in order to maximize profit and improve system performance in a relatively low-cost way. Compared with the existing studies, a stepwise linear revenue function is adopted to correlate the availability with the support provider's profit. For demonstrating the superiority of PBC, we compare the presented policy with traditional policy (i.e., cost minimization). Then, a sensitivity analysis of corrective maintenance cost and time required for corrective maintenance is conducted. Finally, we conduct a comparative study of considering first inspection interval or not, which is involved in our proposed model and rarely considered in previous works.

The remainder of this study is organized as follows. Section 2 conducts a literature review based on the mathematical modeling and optimization approaches under PBC. Section 3 presents the degradation process with random effects, problem description, and model assumptions. Section 4 develops a CBM optimization model for gamma deteriorating systems under PBC. In Sect. 5, the solution algorithm and related procedure are presented. A numerical example is provided in Sect. 6, which validates the superiority of the proposed model. Section 7 presents the conclusion and several research directions.

2 Literature Review

In the past ten years, the research concentrates on mathematical modeling and optimization approaches under PBC have been gradually increasing. Before starting further research, we review the papers related to mathematical modeling and optimization approaches of maintenance support under PBC.

In Table 1, we reviewed 18 pieces of literature related to mathematical modeling and optimization approaches of maintenance support under PBC. These papers are ranked according to the publishing time. They were mainly summarized in application domains, solution algorithm, first inspection interval considered or not, and characteristic feature.

Concerning the application domains, the reviewed papers are mainly divided into spares inventory and maintenance optimization. From Table 1, it can be observed that 12 of the reviewed papers [24–35] concentrated on the spares inventory. As the common inventory stock items, spare parts are necessary for maintenance equipment. Generally, spare parts will cost a large part of the product life cycle cost (LCC) [36]. For this reason, PBC is applied in spares inventory, and correspondent optimization models are developed in order to obtain the optimal inventory. Instead, only 6 of the reviewed papers [37–42] studied maintenance optimization under PBC.

With respect to the solution algorithms, one-third of the reviewed papers did not mention the solution algorithm. 11 of the reviewed papers adopted the traditional solution algorithms, such as simulation, allocation algorithm, gradient descent, selection algorithm, discrete algorithm, and coordinate search algorithm. Only 2 reviewed papers applied heuristic intelligent algorithms,

Table 1 Overview of application domains, solution algorithm, first inspection interval considered or not, and characteristic feature by the central reviewed literature

Ref.	Application domain	Solution algorithm	Consider first inspection interval?	Characteristic feature
[24] Kim et al. (2007)	Spares inventory	–	–	Principal-agent model
[25] Nowicki et al. (2008)	Spares inventory	Allocation algorithm	–	METRIC
[26] Mirzahosseinian and Piplani (2011)	Spares inventory	–	–	METRIC; Queueing theory; Markov chain
[27] Jin and Tian (2012)	Spares inventory; Reliability	Simulation	–	Reliability design; Spare parts logistic
[28] Mirzahosseinian and Piplani (2013)	Spares inventory; Reliability	–	–	METRIC; Reliability design; Spare parts logistic
[29] Zhang et al. (2014)	Spares inventory; Reliability	Simulation	–	Reliability design; Spare parts logistic
[30] Jin et al. (2015)	Spares inventory; Reliability	Gradient Descent	–	METRIC; Game theory
[31] Mirzahosseinian et al. (2016)	Spares inventory; Reliability	–	–	METRIC; Reliability design; Spare parts logistic
[32] Riccardo et al. (2016)	Spares inventory	Quick Request selection algorithm; Local Department Kit allocation algorithm	–	METRIC
[37] Qiu et al. (2017)	Maintenance optimization	–	No	Mathematical statistical theory; Virtual age model
[38] Xiang et al. (2017)	Maintenance optimization	Discrete algorithm	No	Stochastic deterioration model
[33] Hur et al. (2018)	Spares inventory	Runge-Kutta methods; Discrete event simulation	–	Markov chain
[34] Patra et al. (2019)	Spares inventory; Reliability	–	–	Principal-agent model; Time-series model
[39] Wang et al. (2019)	Maintenance optimization	Simulation	No	Mathematical statistical theory
[40] Yang et al. (2019)	Maintenance optimization	ABC algorithm	No	Delay-time-based maintenance model
[41] Li et al. (2020)	Maintenance optimization	Coordinate search algorithm	No	Mathematical statistical theory
[42] Wang et al. (2020)	Maintenance optimization	PSO algorithm	No	Degradation-threshold-shock model
[35] Hosseinifard et al. (2021)	Spares inventory	Simulation	–	Service-level agreement
This paper	Maintenance optimization	PSO algorithm	Yes	Stochastic deterioration model

they are artificial bee colony (ABC) algorithm [40] and particle swarm optimization (PSO) algorithm [42], respectively. For low-dimensional problems, the traditional solution algorithm is relatively simple and effective. However, for high-dimensional problems, such as maintenance optimization problems with multiple decision variables, adopting a traditional solution algorithm will take large numbers of time and the accuracy of the solution is relatively low. In contrast, the heuristic intelligent algorithms have high operating efficiency and are less affected by the dimensionality of the problem [43, 44]. Therefore, the application of a heuristic intelligent algorithm is needed for complicated maintenance optimization problems under PBC.

Regarding the first inspection interval, to our best knowledge, there has never been researching considering the first inspection interval in maintenance optimization under PBC. The first inspection interval was firstly proposed by Jia and Christer [45] for modeling functional checking models. They have verified the effectiveness of distinguishing the first inspection interval. The fly in the ointment is that they did not consider the downtime in their numerical example.

In regard to the characteristic feature in maintenance optimization, it can be known from Table 1 that PBC has applied mathematical statistical theory [37, 39, 41], stochastic deterioration model [38], delay-time-based maintenance model [40], degradation-threshold-shock model [42], and so on. At present, the development of stochastic deterioration is the main research aspect of CBM, whether in research organizations or industrial applications [21]. Nevertheless, there is only one paper [38] that studies the application of stochastic deterioration models under PBC. So, the research on stochastic deterioration models under PBC needs to be further strengthened and developed.

Based on the above summarizes, it can be concluded that:

(1) Only a few studies applied PBC in maintenance optimization. Therefore, there is still an imperative need to study maintenance optimization under PBC.
(2) The application of emerging heuristic intelligent algorithms in maintenance optimization under PBC is relatively rare. With the in-depth study of maintenance optimization problems under PBC, traditional algorithms cannot meet the requirements of efficiency and accuracy.
(3) As an effective approach, the first inspection interval has not been considered in the maintenance optimization under PBC so far.
(4) As an important part of CBM, current research on the stochastic deterioration model in the maintenance optimization under PBC is far from enough.

Consequently, this paper attempts to investigate the CBM optimization for gamma deteriorating systems under PBC. Meanwhile, we not only adopt the PSO algorithm but also consider the first inspection interval.

3 Problem Description and Assumptions

3.1 Degradation Process with Random Effects

Generally, degradation is a physical or chemical process in which the internal materials of the system suffer a gradual change under external stresses. For a system, the process of degradation is equivalent to damage. If the accumulated damage beyond the failure threshold over time, the system will eventually fail [23]. Figure 1 shows illustrate the process of degradation failure. Where $X(t)$ represents the performance degradation at time t, L_f denotes the threshold of the failure, and t_f represents the time when $X(t)$ arrives L_f.

The system degradation process is usually stochastic so that it is an effective approach to describe the uncertainty in the system degradation process using a random process. To facilitate mathematical processing, the commonly used stochastic degradation models are mostly homogeneous and stable independent incremental processes, like the Wiener process and Gamma process. For systems where the degradation increases (or decreases) over time (non-monotonic), the Wiener process is more appropriate. However, for systems where the degradation monotonically increasing (or decreasing), the Gamma process is more suitable [21]. Gamma processes

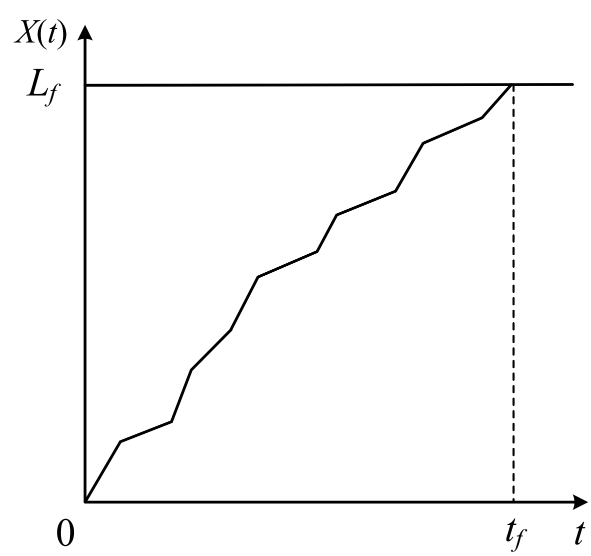

Figure 1 Process of degradation failure

are well suited for modeling the temporal variability of deterioration, they have proven to be useful in determining optimal inspection and maintenance decisions [46]. Therefore, we investigate a system in which the degradation process follows the gamma process.

The gamma process is defined in mathematical terms as follows.

Let X denotes the random variables which have a gamma distribution, where the shape parameter α>0 and the scale parameter $\beta > 0$. The probability density function of X is as follows:

$$Ga(x;\alpha,\beta)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}I_{(0,\infty)}(x), \tag{1}$$

where $\Gamma(\alpha)=\int_0^{\infty}u^{\alpha-1}e^{-u}\mathrm{d}u$, and $I_A(x)=1$ for $x\in A$ and $I_A(x)=0$ for $x\notin A$.

Moreover, if $\alpha(t)$ is a non-decreasing, right-continuous, real-valued function for $t\geq0$, with $\alpha(0)\equiv0$, the gamma process can be described as a continuous stochastic process $\{X(t), t\geq0\}$ with shape function $\alpha(t)$>0 and scale parameter β>0. The process has the following properties:

(1) $X(0)=0$;

(2) $X(t+\Delta t)-X(t)\sim Ga(t)$; $\alpha(\Delta t)$, $\beta)$ for all Δt>0 and $t\geq0$;

(3) $X(t)$ has independent increments.

If shape parameter $\alpha(t)$ is a linear function of time t, the random process will be a stationary gamma process. In this paper, we assume $\alpha(t)=\alpha t$, then the probability density function can be expressed as follows:

$$f(x;\alpha t,\beta)=\frac{\beta^{\alpha t}}{\Gamma(\alpha t)}x^{\alpha t-1}e^{-\beta x}, \tag{2}$$

and the distribution function is as follows:

$$F(x;\alpha t,\beta)=\frac{\beta^{\alpha t}}{\Gamma(\alpha t)}\int_0^x u^{\alpha t-1}e^{-\beta u}\mathrm{d}u. \tag{3}$$

3.2 Problem Description

CBM is one of the most effective maintenance strategies to deal with degradation failure [47]. It refers to a maintenance strategy that makes a maintenance plan by collecting and estimating the real-time conditions of the system. At present, CBM is mostly based on periodic inspections, which is shown in Figure 2. Where T_1 and T denote inspection intervals, L_p denotes preventive maintenance threshold, T_i represents the inspection time, T_p represents the preventive maintenance time, T_f represents the corrective maintenance time.

The design of this type of strategy has primarily concentrated on preventive maintenance threshold and inspection interval. Different from traditional inspection models, we consider the first inspection interval, T_1, because the probability of the cumulative deterioration level above the preventive maintenance threshold is relatively small in the early period. Thus, it is necessary to set the first inspection interval to reduce the inspection frequency and the cost.

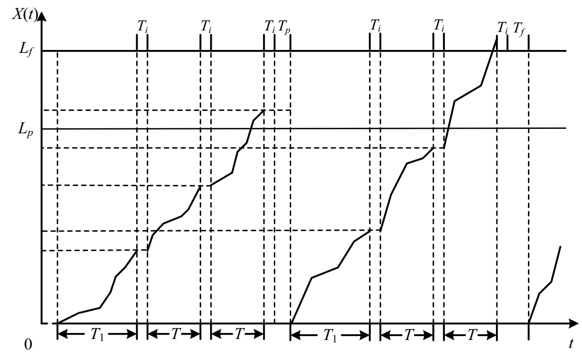

Figure 2 Periodic inspection policy of degradation process

Generally, the cost is the primary consideration when setting the inspection interval and preventive maintenance threshold. The existing maintenance optimization models often adopt the optimization standard that minimizing system maintenance cost instead of performance. However, in some cases when the cost is minimal, the performance of the system may be very low, which can not be acceptable in practice. In this study, we deal with an optimization model of CBM for gamma deteriorating systems under PBC. The difference from the traditional maintenance optimization models is that PBC motivates support providers to implement efficient maintenance strategies which encouraging profits and improving the performance of the system in a relatively low-cost way [27]. Therefore, how to find the optimal decision variables (i.e., first inspection interval, repeat inspection interval, and preventive maintenance threshold) in order to accomplish the target of PBC is what we are committed to studying.

3.3 Modeling Assumptions

The basic modeling assumptions are listed as follows:

(1) A single-component system is a system that treats a component as a system for research. Since maintenance policies for single-component systems are more established and are the basis for maintenance policies of multi-component systems, this paper investigates a repairable single-component system subject to the gamma degradation process. Let $X(t)$ represent the degradation of time t and satisfy the following conditions: (a) when t=0, $X(0)$=0, the system is viewed as an operative condition; (b) the increments of degradation are non-negative and independent.

(2) The inspection of the system is a discrete inspection, and the status of the system can only be detected in an inspection way. All inspections are viewed as perfect. The first inspection interval is T_1 and the repeated inspection interval is T.
(3) If $X(t)<L_p$, the system continues to work until the next inspection.
(4) If $L_p<X(t)<L_f$, the system will be preventively repaired. We assume that the preventive repair is imperfect, namely the system will return to operative condition with probability p. In contrast, the system will stay the same state with the probability $q=1-p$.
(5) If $L_f<X(t)$, the system will break down and corrective maintenance will be performed. Corrective maintenance is perfect, the system can be repaired as new after corrective maintenance. Furthermore, the failure is not obvious which can only be detected in an inspection way, therefore the system will keep running when a failure happens until the inspection.

4 Maintenance Optimization Modeling under Performance-based Contracting

4.1 Optimization Model of Performance-based Contracting

In this section, the expected profit rate under PBC is derived and calculated. As a representative profit-centered strategy, PBC considers both the performance and costs of operation. In this paper, the expected average availability is adopted to measure the performance, which is because higher system stability is required in the defense and industrial field. And then, the expected cost rate per unit time is adopted to measure the costs of operation.

Firstly, the revenue function is used to help to connect the profit and availability. As it is easy to implement and can clearly express the relationship between the profit and availability, the stepwise linear revenue function is adopted in this study to assist in maintenance decision making under PBC, the schematic diagram of stepwise linear revenue is shown in Figure 3, which can be expressed as Eq. (4):

$$ER = \begin{cases} 0, & A < A_{\min}, \\ \theta + \pi(A - A_{\min}), & A \geq A_{\min}, \end{cases} \tag{4}$$

where θ represents the fixed revenue, $\pi(A-A_{\min})$ denotes the incentive based on performance. If $A<A_{\min}$, providers could not obtain any revenue. If $A\geq A_{\min}$, providers can get fixed revenue and incentives.

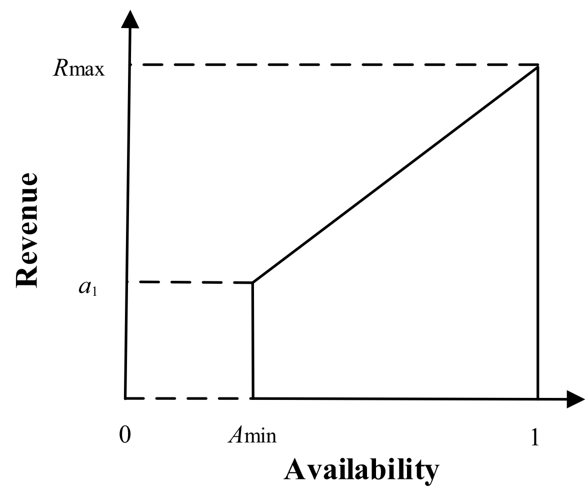

Figure 3 Stepwise linear revenue function

Let EP denote the expected profit rate per unit time. According to the expected cost and the expected revenue, the maintenance optimization model to maximize the expected profit can be expressed as Eq. (5):

$$\begin{aligned} &\max EP(T_1, T, L_p) = ER(T_1, T, L_p) - EC(T_1, T, L_p), \\ &\text{s.t.,} \begin{cases} 0 < T < T_1 < T_{\max}, \\ 0 < L_p < L_f, \end{cases} \end{aligned} \tag{5}$$

where $T_{\max}$ represents the potential time constraint for inspection interval.

In order to reflect the superiority of the proposed model, a benchmark model to minimize cost is proposed as follows:

$$\begin{aligned} &\min EC(T_1, T, L_p), \\ &\text{s.t.,} \begin{cases} 0 < T < T_1 < T_{\max}, \\ 0 < L_p < L_f, \end{cases} \end{aligned} \tag{6}$$

4.2 Calculation of Renewal Probabilities

Before calculating the A and EC in Eq. (4) and Eq. (5), the probabilities of preventive renewal and corrective renewal should be calculated in advance.

In this study, we have a hypothesis that the system is renewed when preventive maintenance is performed perfectly or the cumulative degradation beyond the failure threshold at the inspection. It is noticed that the first inspection interval, T_1, is greater than subsequent inspection intervals, T.

4.2.1 Probability of Preventive Renewal

Let p_λ (λ=1,2,3,...) represents the probability of preventive maintenance is perfectly implemented at the λth inspection. At the first inspection, if L_p<$X(t)$< L_f, PM will be implemented perfectly with probability p. The corresponding probability can be expressed as Eq. (7):

$$p_1 = \Pr\{L_p < X(T_1) < L_f\} \times p. \tag{7}$$

There are two cases where PM is perfectly performed at the second inspection (λ=2). The first situation is that PM is not required at the first inspection but is required and performed perfectly at the second inspection. The second situation is that PM is failed to implement at the first inspection and the cumulative degradation did not beyond the failure threshold during the second inspection interval, at the same time, PM is perfectly implemented at the second inspection. Therefore, the probability of these cases can be expressed as Eq. (8):

$$\begin{aligned} p_2 = & \Pr\{X(T_1) < L_p \& L_p < X(T_1+T) < L_f\} \times p \\ & + \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1+T) < L_f\} \\ & \times (1-p)p. \end{aligned} \tag{8}$$

It is similar to the situation above, there exist three cases where PM is performed perfectly at the third inspection (λ=3). The first situation is that PM is not required during the first two inspections but is required and performed perfectly at the third inspection (shown in Figure 4(a)). The next situation is that PM is not required at the first inspection and fails to implement at the second inspection, meanwhile, it is performed perfectly at the third inspection (shown in Figure 4(b)). The third situation is that PM is required during the first two inspections, but they all fail to implement perfectly, it is performed perfectly at the third inspection (shown in Figure 4(c)). Combining the above three situations, the probability of these cases can be expressed as Eq. (9):

$$\begin{aligned} p_3 = & \Pr\{X(T_1+T) < L_p \& L_p < X(T_1+2T) < L_f\} \\ & \times p + \Pr\{X(T_1) < L_p \& L_p < X(T_1+T) < L_f \& L_p \\ & < X(T_1+2T) < L_f\} \times (1-p)p + \Pr\{L_p < X(T_1) \\ & < L_f \& L_p < X(T_1+2T) < L_f\} \times (1-p)^2 p. \end{aligned} \tag{9}$$

According to the above deduction, the general expression of the probability which performing PM perfectly at the λth inspection can be expressed as Eq. (10):

$$p_\lambda = \begin{cases} \Pr\{L_p < X(T_1) < L_f\} \times p, & \lambda = 1, \\ \Pr\{X(T_1) < L_p \& L_p < X(T_1+T) < L_f\} \times p + \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1+T) < L_f\} \times (1-p)p, & \lambda = 2, \\ \Pr\{X(T_1+(\lambda-2)T) < L_p \& L_p < X(T_1+(\lambda-1)T) < L_f\} \times p + \sum\limits_{n=2}^{\lambda-1} \Pr\{X(T_1+(n-2)T) < L_p\ L_p & \\ < X(T_1+(n-1)T) < L_f \& L_p < X(T_1+(\lambda-1)T) < L_f\} \times (1-p)^{\lambda-n} p + \Pr\{L_p < X(T_1) < L_f \& L_p & \\ < X(T_1+(\lambda-1)T) < L_f\} \times (1-p)^{\lambda-1} p, & \lambda > 2, \end{cases} \tag{10}$$

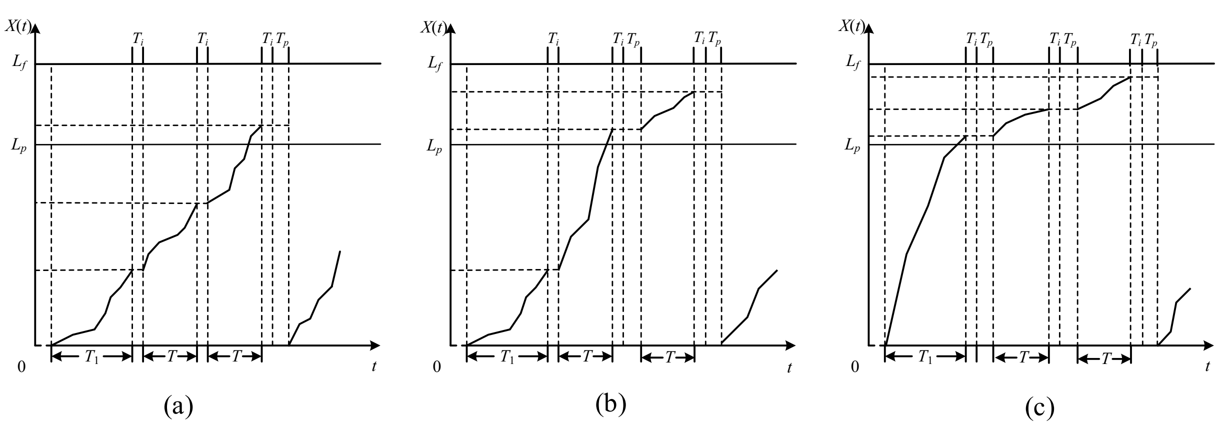

Figure 4 Perfect PM at the 3rd inspection

where

$$\Pr\{X(T_1+(\lambda-2)T)<L_p\&L_p<X(T_1+(\lambda-1)T)<L_f\} = \int_0^{L_p} f(u,\alpha(T_1+(\lambda-2)T),\beta)\int_{L_p-u}^{L_f-u} f(v,T,\beta)\mathrm{d}v\mathrm{d}u, \tag{11}$$

$$\begin{aligned}&\Pr\{X(T_1+(n-2)T)<L_p\&L_p<X(T_1+(n-1)T)<L_f\&L_p<X(T_1+(\lambda-1)T)<L_f\}\\ &= \int_0^{L_p} f(u,\alpha,T_1+(n-2)T),\beta)\int_{L_p-u}^{L_f-u} f(v,\alpha T,\beta)\\ &\quad\times F(L_f-u-v,\alpha(\lambda-n)T,\beta)\mathrm{d}v\mathrm{d}u.\end{aligned} \tag{12}$$

4.2.2 Probability of Corrective Renewal

Then, let q_λ (λ=1,2,3,...) denotes the probability of corrective maintenance (CM) is performed at the λth inspection. The probability of CM is implemented at the first inspection is

$$q_1=\Pr\{X(T_1)>L_f\}. \tag{13}$$

There are also two cases where CM is implemented at the second inspection (λ=2). The first situation is that $X(t)<L_p$ at the first inspection, but $L_f<X(t)$ between the first and the second inspection. The second situation is that PM is failed to implement at the first inspection and the cumulative degradation beyond the failure threshold at the second inspection. Thus, the probability of these cases can be expressed as Eq. (14):

$$q_2=\Pr\{X(T_1)<L_p\&X(T_1+T)>L_f\}+\Pr\{L_p<X(T_1)<L_f\&X(T_1+T)>L_f\}\times(1-p). \tag{14}$$

Similar to the above situation, there are three cases where CM is performed at the third inspection (λ=3). The first situation is that $X(t)<L_p$ during the first two inspections, and $L_f<X(t)$ between the second and the third inspection. The second situation is that PM is required at the second inspection, but the PM fails to implement, and $L_f<X(t)$ between the second and the third inspection. The third situation is that PM is required and performed at both the first two inspections, but these two inspections are both failed to implement, meanwhile, $L_f<X(t)$ between the second and the third inspections. Therefore, the probability of these cases can be expressed as Eq. (15):

$$\begin{aligned}q_3=&\Pr\{X(T_1+T)<L_p\&X(T_1+2T)>L_f\}\\ &+\Pr\{X(T_1)<L_p\&L_p<X(T_1+T)<L_f\\ &\&X(T_1+2T)>L_f\}\times(1-p)+\Pr\{L_p<X(T_1)\\ &<L_f\&L_p<X(T_1+T)<L_f\&X(T_1+2T)\\ &>L_f\}\times(1-p)^2.\end{aligned} \tag{15}$$

Similarly, there are four cases where CM is performed at the fourth inspection (λ=4). The first situation is that PM is not required before the fourth inspection and the cumulative degradation beyond the failure threshold between the third and the fourth inspections (shown in Figure 5(a)). The second situation is that the first PM is required at the third inspection, but the PM is not performed perfectly, and $L_f<X(t)$ between the third and the fourth inspections (shown in Figure 5(b)). The third situation is that the first PM is required at the second inspection, however, both the second and third inspections are not performed perfectly, and $L_f<X(t)$ between the third and the fourth inspections (shown in Figure 5(c)). The last situation is that the PM fails to implement during the first three inspections, and $L_f<X(t)$ between the third and the fourth inspection (shown in Figure 5(d)). The probability of these cases can be expressed as Eq. (16):

$$\begin{aligned}q_4=&\Pr\{X(T_1+2T)<L_p\&X(T_1+3T)>L_f\}\\ &+\Pr\{X(T_1+T)<L_p\&L_p<X(T_1+2T)\\ &<L_f\&X(T_1+3T)>L_f\}\\ &\times(1-p)+\Pr\{X(T_1)<L_p\&L_p<X(T_1+T)\\ &<L_f\&L_p<X(T_1+2T)\\ &<L_f\&X(T_1+3T)>L_f\}\times(1-p)^2+\Pr\{L_p\\ &<X(T_1)<L_f\&L_p<X(T_1+2T)\\ &<L_f\&X(T_1+3T)>L_f\}\times(1-p)^3.\end{aligned} \tag{16}$$

According to the above deduction, the general expression of the probability which performing CM at the λth inspection can be expressed as Eq. (17):

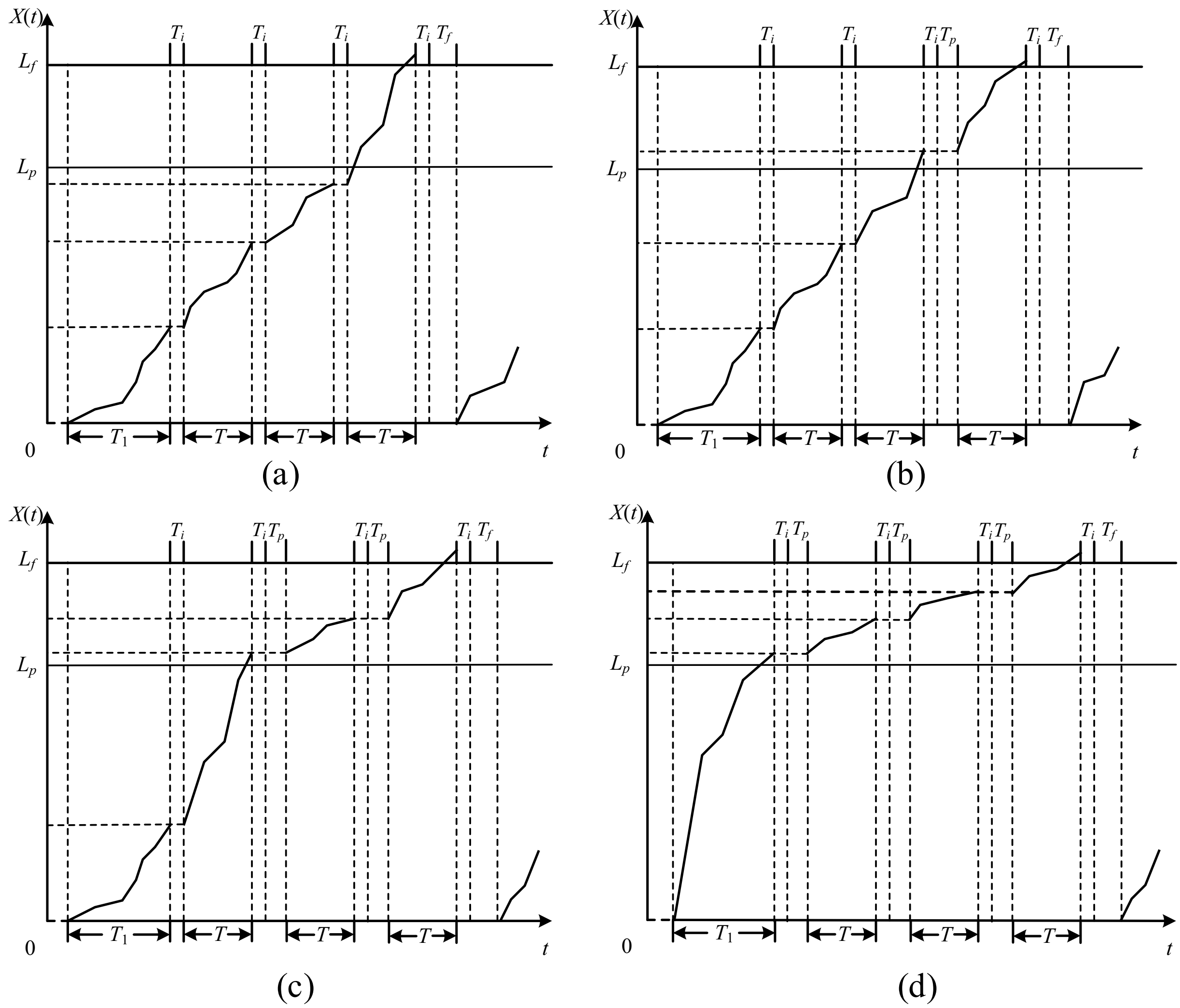

Figure 5 CM at the 4th inspection

$$
q_\lambda = \begin{cases}
\Pr\{X(T_1) > L_f\}, & \lambda = 1,\\
\Pr\{X(T_1) < L_p \& X(T_1+T) > L_f\} + \Pr\{L_p < X(T_1) < L_f \& X(T_1+T) > L_f\} \times (1-p), & \lambda = 2,\\
\Pr\{X(T_1+T) < L_p \& X(T_1+2T) > L_f\} + \Pr\{X(T_1) < L_p \& L_p < X(T_1+T) < L_f \& X(T_1+2T) > L_f\} \times (1-p) & \\
+ \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1+T) < L_f \& X(T_1+2T) > L_f\} \times (1-p)^2, & \lambda = 3,\\
\Pr\{X(T_1+(\lambda-2)T) < L_p \& X(T_1+(\lambda-1)T) > L_f\} + \Pr\{X(T_1+(\lambda-3)T) < L_p \& L_p < X(T_1+(\lambda-2)T) < L_f & \\
\& X(T_1+(\lambda-1)T) > L_f\} \times (1-p) + \sum_{n=2}^{\lambda-2} \Pr\{X(T_1+(n-2)T) < L_p \& L_p < X(T_1+(n-1)T) < L_f \& L_p < X(T_1 & \\
+(\lambda-2)T) < L_f \& X(T_1+(\lambda-1)T) > L_f\} \times (1-p)^{\lambda-n} + \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1+(\lambda-2)T) < L_f & \\
\& X(T_1+(\lambda-1)T) > L_f\} \times (1-p)^{\lambda-1}, & \lambda > 3,
\end{cases}
\tag{17}
$$

where

$$\Pr\{X(T_1+(\lambda-2)T) < L_p \& X(T_1+(\lambda-1)T) > L_f\} = \int_0^{L_p} f(u,\alpha(T_1+(\lambda-2)T),\beta)[1-F(L_f-u,\alpha T,\beta)]\mathrm{d}u, \tag{18}$$

$$\begin{aligned}&\Pr\{X(T_1+(\lambda-3)T) < L_p \& L_p < X(T_1+(\lambda-2)T) \\ &< L_f \& X(T_1+(\lambda-1)T) > L_f\} \\ &= \int_0^{L_p} f(u,\alpha(T_1+(\lambda-3)T),\beta)\int_{L_p-u}^{L_f-u} f(v,\alpha T,\beta)[1 \\ &\quad - F(L_f-u-v,\alpha T,\beta)]\mathrm{d}v\mathrm{d}u,\end{aligned} \tag{19}$$

$$\begin{aligned}&\Pr\{X(T_1+(n-2)T) < L_p \& L_p < X(T_1+(n-1)T) \\ &\quad < L_f \& L_p < X(T_1+(\lambda-2)T) < L_f \& X(T_1+(\lambda \\ &\quad -1)T) > L_f\} \\ &= \int_0^{L_p} f(u,\alpha(T_1+(n-2)),\beta)\int_{L_p-u}^{L_f-u} f(v,\alpha T,\beta) \\ &\quad \times \int_0^{L_f-u-v} f(w,\alpha((\lambda-n-1)T),\beta)[1-F(L_f-u \\ &\quad -v-w,\alpha T,\beta)]\mathrm{d}w\mathrm{d}v\mathrm{d}u.\end{aligned} \tag{20}$$

4.3 Calculation of Average Availability

In the field of maintenance support, availability is the most used metric to measure performance. In our proposed model, availability is adopted to measure the practical performance outcomes. Meanwhile, it is assumed that the incentive is directly related to availability. The average availability is given by

$$A = \frac{T_{\text{up}}}{T_{\text{up}} + T_{\text{down}}}, \tag{21}$$

where T_{up} denotes the expected uptime per cycle, T_{down} denotes the expected downtime per cycle.

4.3.1 Derivation of Uptime

The expected uptime per cycle is expressed as:

$$E(T_{\text{up}}) = \sum_{\lambda=1}^{\infty}(T_1+(\lambda-1)T)\times p_\lambda + \sum_{\lambda=1}^{\infty}(T_1+(\lambda-1)T)\times q_\lambda. \tag{22}$$

4.3.2 Derivation of Downtime

The total expected downtime can be expressed as follows:

$$\begin{aligned}E(T_{\text{down}}) =& E(\text{inspection time}) + E(\text{time required for PM}) \\ &+ E(\text{time required for CM}).\end{aligned} \tag{23}$$

Let $E_{1,\lambda}(T_{\text{down}})$ represent the expected downtime when PM is implemented perfectly at the λth inspection, $E_{2,\lambda}(T_{\text{down}})$ represent the expected downtime when CM is implemented at the λth inspection. Then, the total expected downtime can be expressed as follows:

$$E(T_{\text{down}}) = \sum_{i=1}^{2}\sum_{\lambda=1}^{\infty}E_{i,\lambda}(T_{\text{down}}). \tag{24}$$

The expected downtime when PM is performed perfectly at the λth inspection can be expressed as Eq. (25):

$$E_{1,\lambda}(T_{\text{down}}) = \begin{cases}(T_i+T_p)\times\Pr\{L_p < X(T_1) < L_f\}\times p, & \lambda=1, \\ (2T_i+T_p)\times\Pr\{X(T_1) < L_p \& L_p < X(T_1+T) < L_f\}\times p + (2T_i+2T_p)\times\Pr\{L_p < X(T_1) < L_f l \\ \& L_p < X(T_1+T) < L_f\}\times(1-p)p, & \lambda=2, \\ (\lambda T_i+T_p)\times\Pr\{X(T_1+(\lambda-2)T) < L_p \& L_p < X(T_1+(\lambda-1)T) < L_f\}\times p + \sum_{n=2}^{\lambda-1}(\lambda T_i+(\lambda l \\ -n+1)T_p)\times\Pr\{X(T_1+(n-2)T) < L_p \& L_p < X(T_1+(n-1)T) < L_f \& L_p < X(T_1+(\lambda l \\ -1)T) < L_f\}\times(1-p)^{\lambda-n}p + (\lambda T_i+\lambda T_p)\times\Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1+(\lambda-1)T)l \\ < L_f\}\times(1-p)^{\lambda-1}p, & \lambda>2,\end{cases} \tag{25}$$

The expected downtime when CM is implemented at the λth inspection can be expressed as Eq. (26):

$$E_{2,\lambda}(T_{\text{down}}) = \begin{cases} (T_i + T_f) \times \Pr\{X(T_1) > L_f\}, & \lambda = 1, \\ (2T_i + T_f) \times \Pr\{X(T_1) < L_p \& X(T_1 + T) > L_f\} + (2T_i + T_p + T_f) \times \Pr\{L_p < X(T_1) < L_f \& X(T_1 + T) \\ > L_f\} \times (1 - p), & \lambda = 2, \\ (3T_i + T_f) \times \Pr\{X(T_1 + T) < L_p \& X(T_1 + 2T) > L_f\} + (3T_i + T_p + T_f) \times \Pr\{X(T_1) < L_p \& L_p < X(T_1 \\ + T) < L_f \& X(T_1 + 2T) > L_f\} \times (1 - p) + (3T_i + 2T_p + T_f) \times \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1 + T) \\ < L_f \& X(T_1 + 2T) > L_f\} \times (1 - p)^2, & \lambda = 3, \\ (\lambda T_i + T_f) \times \Pr\{X(T_1 + (\lambda - 2)T) < L_p \& X(T_1 + (\lambda - 1)T) > L_f\} + (\lambda T_i + T_p + T_f) \times \Pr\{X(T_1 + (\lambda \\ -3)T) < L_p \& L_p < X(T_1 + (\lambda - 2)T) < L_f \& X(T_1 + (\lambda - 1)T) > L_f\} \times (1 - p) + \sum_{n=2}^{\lambda-2} (\lambda T_i + (\lambda - n)T_p \\ + T_f) \times \Pr\{X(T_1 + (n - 2)T) < L_p \& L_p < X(T_1 + (n - 1)T) < L_f \& L_p < X(T_1 + (\lambda - 2)T) < L_f \\ \& X(T_1 + (\lambda - 1)T) > L_f\} \times (1 - p)^{\lambda - n} + (\lambda T_i + (\lambda - 1)T_p + T_f) \times \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1 \\ + (\lambda - 2)T) < L_f \& X(T_1 + (\lambda - 1)T) > L_f\} \times (1 - p)^{\lambda - 1}, & \lambda > 3. \end{cases} \tag{26}$$

4.4 Calculation of Cost Rate

The calculation of the cost rate is similar to the calculation of downtime. There are only two things we need to do. Firstly, it is needed to replace the inspection time with the inspection cost. Then, just need to replace the PM/CM downtime with the PM/CM costs. Therefore, the expected maintenance cost when PM is performed perfectly at the λth inspection can be expressed as:

$$E_{1,\lambda}(\text{Cost}) = \begin{cases} (C_i + C_p) \times \Pr\{L_p < X(T_1) < L_f\} \times p, & \lambda = 1, \\ (2C_i + C_p) \times \Pr\{X(T_1) < L_p \& L_p < X(T_1 + T) < L_f\} \times p + (2C_i + 2C_p) \times \Pr\{L_p < X(T_1) < L_f \& L_p \\ < X(T_1 + T) < L_f\} \times (1 - p)p, & \lambda = 2, \\ (\lambda C_i + C_p) \times \Pr\{X(T_1 + (\lambda - 2)T) < L_p \& L_p < X(T_1 + (\lambda - 1)T) < L_f\} \times p + \sum_{n=2}^{\lambda-1} (\lambda C_i + (\lambda - n + 1)C_p) \\ \times \Pr\{X(T_1 + (n - 2)T) < L_p \& L_p < X(T_1 + (n - 1)T) < L_f \& L_p < X(T_1 + (\lambda - 1)T) < L_f\} \times (1 - p)^{\lambda - n} p \\ +(\lambda C_i + \lambda C_p) \times \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1 + (\lambda - 1)T) < L_f\} \times (1 - p)^{\lambda - 1} p, & \lambda > 2. \end{cases} \tag{27}$$

Likewise, the expected maintenance cost when CM is implemented at the λth inspection can be expressed as:

$$E_{2,\lambda}(\text{Cost})=\begin{cases}(C_i+C_f)\times\Pr\{X(T_1)>L_f\}, & \lambda=1,\\ (2C_i+C_f)\times\Pr\{X(T_1)<L_p\&X(T_1+T)>L_f\}+(2C_i+C_p+C_f)\times\Pr\{L_p<X(T_1)<L_f\&X(T_1+T) & \\ >L_f\}\times(1-p), & \lambda=2,\\ (3C_i+C_f)\times\Pr\{X(T_1+T)<L_p\&X(T_1+2T)>L_f\}+(3C_i+C_p+C_f)\times\Pr\{X(T_1)<L_p\&L_p<X(T_1 & \\ +T)<L_f\&X(T_1+2T)>L_f\}\times(1-p)+(3C_i+2C_p+C_f)\times\Pr\{L_p<X(T_1)<L_f\&L_p<X(T_1+T) & \\ <L_f\&X(T_1+2T)>L_f\}\times(1-p)^2, & \lambda=3,\\ (\lambda C_i+C_f)\times\Pr\{X(T_1+(\lambda-2)T)<L_p\&X(T_1+(\lambda-1)T)>L_f\}+(\lambda C_i+C_p+C_f)\times\Pr\{X(T_1+(\lambda & \\ -3)T)<L_p\&L_p<X(T_1+(\lambda-2)T)<L_f\&X(T_1+(\lambda-1)T)>L_f\}\times(1-p)+\sum\limits_{n=2}^{\lambda-2}(\lambda C_i+(\lambda-n)C_p & \\ +C_f)\times\Pr\{X(T_1+(n-2)T)<L_p\&L_p<X(T_1+(n-1)T)<L_f\&L_p<X(T_1+(\lambda-2)T)<L_f\&X(T_1 & \\ +(\lambda-1)T)>L_f\}\times(1-p)^{\lambda-n}+(\lambda C_i+(\lambda-1)C_p+C_f)\times\Pr\{L_p<X(T_1)<L_f\&L_p<X(T_1+(\lambda-2)T) & \\ <L_f\&X(T_1+(\lambda-1)T)>L_f\}\times(1-p)^{\lambda-1}, & \lambda>3.\end{cases} \tag{28}$$

Similar to Eq. (24), the expected cost rate can be expressed as follows:

$$EC=\frac{\sum\limits_{i=1}^{2}\sum\limits_{\lambda=1}^{\infty}E_{i,\lambda}(\text{Cost})}{T_{\text{up}}+T_{\text{down}}}. \tag{29}$$

5 Solution Algorithm

In this study, an optimization model of CBM for gamma deteriorating systems under PBC is proposed. Maximizing the expected profit is the main target of the proposed model. Therefore, it is needed to find the optimal decision variables, namely the first inspection interval, repeat inspection interval, and preventive maintenance threshold. From the equations deduced above, it is noted that the model is very difficult to be solved because the proposed model has three decision variables, and the expression is very complicated. If we use the traditional exact algorithms, such as a discrete algorithm, it will take a lot of time and the accuracy of the solution is relatively low. For this reason, the heuristic intelligence algorithm is adopted to solve the proposed model. Particle Swarm Optimization (PSO)is one of the most used heuristic intelligence algorithms which is convenient and efficient [48–50]. It can be known from Table 1 that PSO is not used in maintenance optimization under performance-based contracting, therefore, we chose the PSO as the solution algorithm. The corresponding solution steps are illustrated as follows.

Step 1: Input the parameters of the maintenance cost, maintenance time, gamma deteriorating process, and revenue function. Let $\phi\{T_1, T, L_p\}$ be the vector that has three decision variables.

Step 2: Set the initial state of each particle. For each particle s (s=1,2,..., S), set its position $\varphi_s=\{\varphi_s^{T_1},\varphi_s^{T},\varphi_s^{L_p}\}$ and velocity $\upsilon_s=\{\upsilon_s^{T_1},\upsilon_s^{T},\upsilon_s^{L_p}\}$ randomly with $\varphi_s^{T_1}\in(0,\overline{T_1})$, $\varphi_s^{T}\in(0,\overline{T})$, $\varphi_s^{L_p}\in(0,L_f)$ and $\upsilon_s^{T_1},\upsilon_s^{T},\upsilon_s^{L_p}\in[-1.5,1.5]$.

Step 3: Let$\tau(\tau=1,\ldots,\tau_{\max})$ represent the iteration time and denote the fitness value as $EP(\varphi_s(\tau))$. Calculating the $EP(\varphi_s(\tau))$ at each τ.

Step 4: Compare $EP(\varphi_s(\tau))$ with $EP\left(\varphi_s^{\text{best}}(\tau)\right)$, where $\varphi_s^{\text{best}}(\tau)$ represents the best position of each particle s. If $EP(\varphi_s(\tau))>EP\left(\varphi_s^{\text{best}}(\tau)\right)$, update $\varphi_s^{\text{best}}(\tau)$ and $EP\left(\varphi_s^{\text{best}}(\tau)\right)$ by $\varphi_s^{\text{best}}(\tau)=\varphi_s(\tau)$ and $EP\left(\varphi_s^{\text{best}}(\tau)\right)=EP(\varphi_s(\tau))$ respectively.

Step 5: Compare $EP(\varphi_s(\tau))$ with $EP\left(\varphi_g^{\text{best}}(\tau)\right)$ where $\varphi_g^{\text{best}}(\tau)$ represents the best position of all particles. If $EP(\varphi_s(\tau))>EP\left(\varphi_g^{\text{best}}(\tau)\right)$, update $\varphi_g^{\text{best}}(\tau)$ and $EP\left(\varphi_g^{\text{best}}(\tau)\right)$ by $\varphi_g^{\text{best}}(\tau)=\varphi_s(\tau)$ and $EP\left(\varphi_g^{\text{best}}(\tau)\right)=EP(\varphi_s(\tau))$.

Step 6: Update each particle's velocity and position at time τ+1 by $\upsilon_s(\tau+1)=\upsilon_s(\tau)+m_1r_1(\varphi_s^{\text{best}}(\tau)-\varphi_s(\tau))+m_1r_2(\varphi_g^{\text{best}}(\tau)-\varphi_s(\tau))$ and $\varphi_s(\tau+1)=\varphi_s(\tau)+\upsilon_s(\tau+1)$. m_1 and m_2 are the learning factors that demonstrate the ability of self-learning and group learning. r_1 and r_2 are random numbers between 0 and 1.

Step 7: Confirm the termination condition of the cycle. If the $\tau>\tau_{\max}$ or the convergence criteria are met, go to Step 8. If not, go back to Step 3.

Step 8: Output the globally optimal position $\varphi_g^{\text{best}}=\{T1_g^{best},T_g^{best},Lp_g^{best}\}$ and the corresponding fitness value $EP\left(\varphi_g^{\text{best}}\right)$.

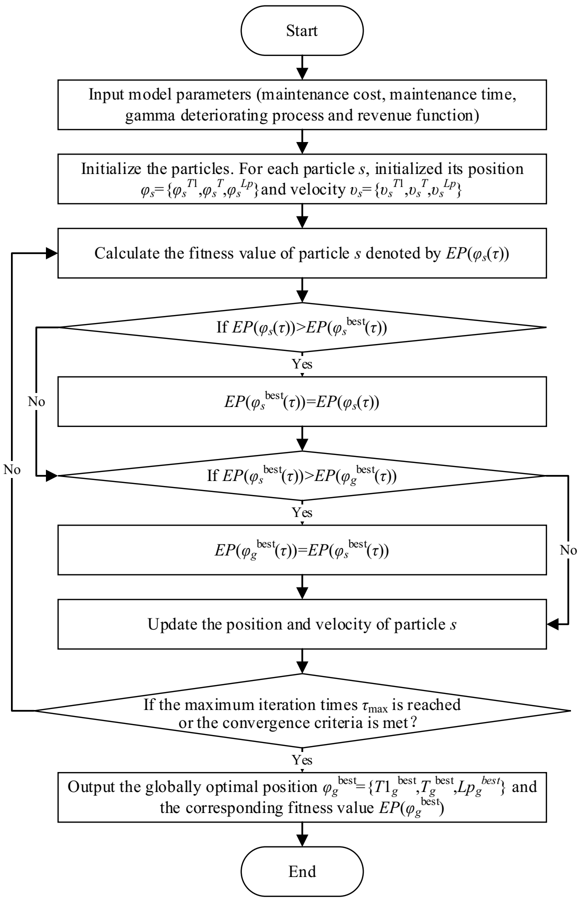

Figure 6 Flow chart of PSO algorithm

The schematic diagram of the steps is shown in Figure 6. The above is the solution algorithm and steps for the proposed model, and the benchmark model to minimize cost can be solved with reference to the proposed model. Specifically, because *EC* can be calculated during the calculation of *EP* according to Eq. (5), the solution steps of the benchmark model can be obtained just by substituting the optimization goal of maximum *EP* to minimum *EC* and adjusting relevant content and notations in the PSO solution procedure.

Table 3 The PBC parameters

θ	π	A_{min}
2	20	0.6

6 Numerical Example

In this section, we provide a numerical example in order to illustrate the superiority of the presented model. Firstly, the cost, average availability, and profit are compared between the proposed policy (i.e., profit maximization) and conventional policy (i.e., cost minimization). Then, a sensitivity analysis of different failure consequences' effects on optimization policy is investigated. Finally, a comparative analysis between the first inspection model and the traditional model (i.e., considering the first inspection or not) is presented.

6.1 Comparison of Proposed Policy and Conventional Policy

The parameters used in this part mainly refer to the existing Refs. [37, 38] with a little change. Other parameters are assumed to be typical and reasonable values. Specifically, the scale parameter and shape parameter of the degradation process is supposed to be α=1.8, β=1 respectively, and the failure maintenance threshold, L_f, is set to 50. The maintenance parameters include the probability of perfect preventive maintenance are provided in Table 2. Table 3 gives the parameters of PBC.

In Table 2, we selected different T_f and C_f to that investigating different failure consequences' effects on optimization policy.

Based on the previously proposed models and solution algorithm, MATLAB software is used to find the optimal variables according to different optimization objectives. After substituting the parameters, the calculation results of the numerical example are summarized in Table 4.

From Table 4 it can be noted that the profit from the proposed policy is more than a conventional policy. It should be known that the improvement in profit is per

Table 2 The maintenance parameters

T_i (d)	T_p (d)	T_f (d)	C_i (USD)	C_p (USD)	C_f (USD)	p
0.2	4	6 12 18 24	4	40	200 400 600 800	0.99

Table 4 Optimization results under different policy

C_i	C_p	C_f	T_i	T_p	T_f	Conventional policy						Proposed policy					
						T_1	T	L_p	EC	A	EP	T_1	T	L_p	EC	A	EP
4	40	200	0.2	4	6	19.56	4.35	36.23	2.22	0.8069	3.92	20.64	3.77	39.07	2.27	0.8141	4.01
					12	19.56	4.35	36.23	2.22	0.8013	3.81	19.81	3.63	38.51	2.23	0.8037	3.85
					18	19.56	4.35	36.23	2.22	0.7957	3.70	19.33	3.51	38.19	2.23	0.7976	3.73
					24	19.56	4.35	36.23	2.22	0.7901	3.59	18.99	3.41	37.98	2.23	0.7932	3.63
4	40	400	0.2	4	6	18.54	3.99	35.45	2.35	0.8003	3.66	19.47	3.54	38.28	2.41	0.8089	3.76
					12	18.54	3.99	35.45	2.35	0.7971	3.59	19.09	3.44	38.04	2.38	0.8022	3.66
					18	18.54	3.99	35.45	2.35	0.7939	3.53	18.82	3.35	37.89	2.37	0.7973	3.58
					24	18.54	3.99	35.45	2.35	0.7907	3.46	18.60	3.27	37.78	2.36	0.7933	3.50
4	40	600	0.2	4	6	18.08	3.67	35.29	2.44	0.7969	3.49	18.90	3.38	37.93	2.51	0.8057	3.60
					12	18.08	3.67	35.29	2.44	0.7944	3.44	18.67	3.30	37.81	2.49	0.8006	3.53
					18	18.08	3.67	35.29	2.44	0.7919	3.39	18.48	3.22	37.72	2.47	0.7965	3.46
					24	18.08	3.67	35.29	2.44	0.7893	3.34	18.33	3.15	37.66	2.47	0.7929	3.39
4	**40**	**800**	**0.2**	**4**	**6**	**17.79**	**3.38**	**35.34**	**2.52**	**0.7946**	**3.37**	**18.54**	**3.24**	**37.75**	**2.59**	**0.8034**	**3.48**
					12	17.79	3.38	35.34	2.52	0.7924	3.33	18.38	3.17	37.68	2.57	0.7992	3.41
					18	17.79	3.38	35.34	2.52	0.7902	3.28	18.24	3.10	37.63	2.56	0.7956	3.35
					24	17.79	3.38	35.34	2.52	0.788	3.24	18.12	3.04	37.59	2.55	0.7924	3.30

unit time, therefore the total profit will be greater over time. Besides, the proposed policy can also deliver better performance (availability). For example, when C_i=4, C_p=40, C_f=800, T_i=0.2, T_p=4, T_f=6 (as the bold values shown in Table 4), comparing with conventional policy, the proposed policy has increased profit by 3.25% and availability by 1.11%, while cost increased by 2.68%. Obviously, the cost will be relatively higher under the proposed policy. Nevertheless, this is understandable given keeping high system performance requires more frequent maintenance activities under the proposed policy. After all, the goal of the conventional policy is to find the optimal decision variable to minimize the cost.

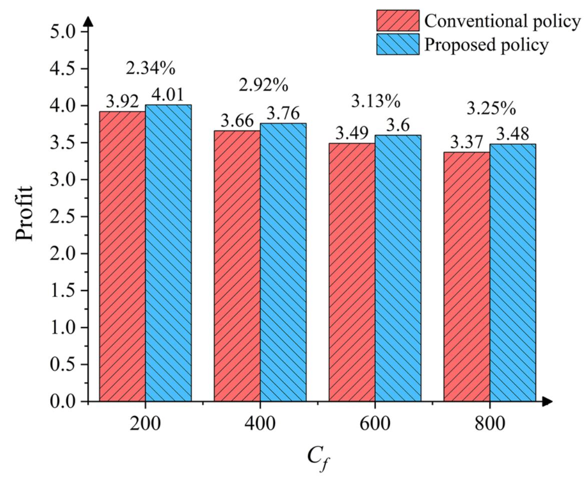

Figure 7 Sensitivity analysis of corrective maintenance cost

6.2 Sensitivity Analysis

In this part, we conduct a sensitivity analysis in order to research the effect of corrective maintenance cost and time required for corrective maintenance on optimization policy.

Firstly, a sensitivity analysis of corrective maintenance cost, C_f, is performed. We adopted a method that changing one of the parameters and fixing the others. Specifically, we fix C_i=4, C_p=40, T_i=0.2, T_p=4, T_f=6, and change the C_f. The analysis data comes from Table 4 and the comparative analysis is shown in Figure 7. Where the percentage values represent the rate of increase in profit.

It can be known from Figure 7 that the presented policy can get more profit under different values of C_f. Also, note that the profit will decrease as C_f increase. And more importantly, we noticed that the rate of increase in profit from conventional policy to the proposed policy is greater with the increase of C_f. Therefore, for the proposed policy, the higher C_f, the more obvious the profit increase.

Then, we conduct a sensitivity analysis of the time required for corrective maintenance, T_f. Similar to the sensitivity analysis of corrective maintenance cost, we fix C_i=4, C_p=40, C_f=800, T_i=0.2, T_p=4, and change the T_f. The analysis data comes from Table 4 too and the comparative analysis is shown in Figure 8.

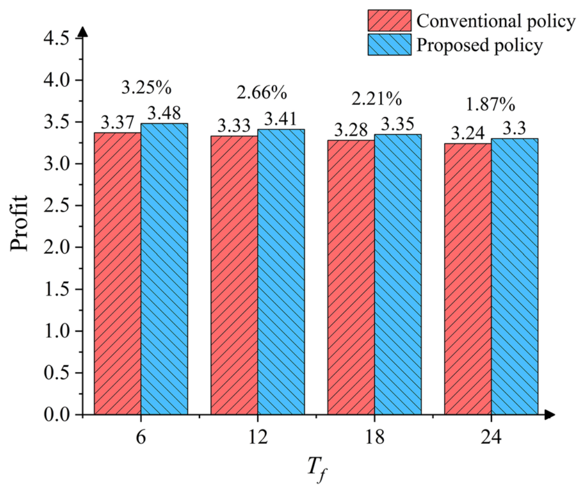

Figure 8 Sensitivity analysis of the time required for corrective maintenance

From Figure 8, it is noted that the presented policy can get more profit under different values of T_f. In addition, it can be noticed that the profit will decrease as T_f increase. We also observed that, with the increase of T_f, the rate of increase in profit from profit maximization policy to cost minimization policy is lower. Therefore, for the proposed policy, the higher T_f, the less obvious the profit increase.

6.3 Comparison of Considering First Inspection Interval or Not

Different from the traditional models, the first inspection interval is considered in our model. In this part, we conduct a comparative analysis to study the effectiveness of the proposed model involving the first inspection interval.

We select a group of parameters and then calculate the optimal decision variables under the two conditions (i.e., considering first inspection interval or not). After that, the corresponding cost rate, availability, and profit are calculated. The calculation results are summarized in Table 5. Then, the column comparison chart is made as Figure 9 shows, which comparing three aspects, namely expected cost rate (shown in Figure 9(a)), average availability (shown in Figure 9(b)), and expected profit rate (shown in Figure 9(c)). Where the percentage values represent the decrease in cost, increase in availability, and increase in profit, respectively.

After observing the data in Table 5 and the column comparison chart in Figure 9, it can be concluded that our proposed model considering the first inspection interval is better than the traditional model. Specifically, the proposed model can provide lower cost, higher availability, and higher profit under the same parameters. It demonstrates the necessity of considering the first inspection interval.

Table 5 Comparison results of considering first inspection interval or not

C_i	C_p	C_f	T_i	T_p	T_f	Not considering first inspection interval					Considering first inspection interval					
						T	L_p	EC	A	EP	T_1	T	L_p	EC	A	EP
4	40	800	0.2	4	6	5.63	33.87	3.19	0.7780	2.37	18.54	3.24	37.75	2.59	0.8034	3.48
					12	5.43	34.05	3.17	0.7721	2.27	18.38	3.17	37.68	2.57	0.7992	3.41
					18	5.33	33.96	3.15	0.7668	2.19	18.24	3.10	37.63	2.56	0.7956	3.35
					24	5.20	34.03	3.14	0.7622	2.10	18.12	3.04	37.59	2.55	0.7924	3.30

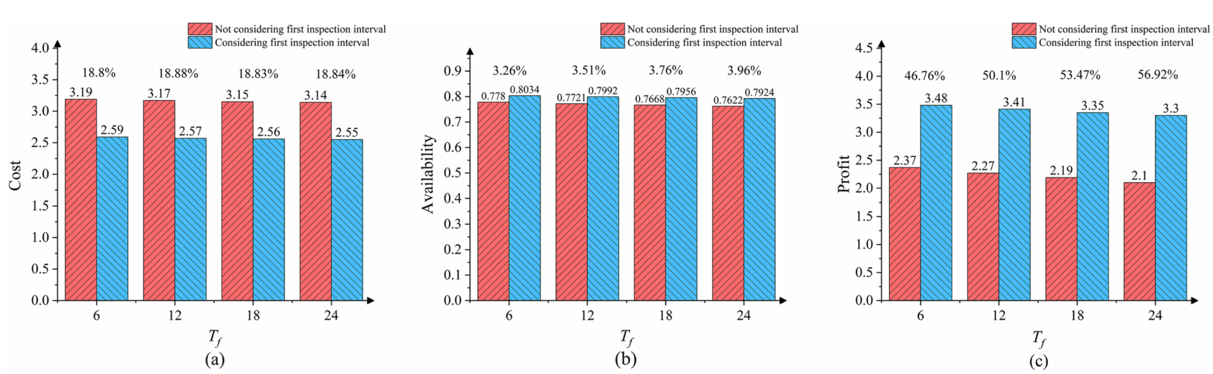

Figure 9 Column comparison chart of considering first inspection interval or not

7 Conclusions and Future Research

This paper investigated the condition-based optimization for gamma deteriorating systems under PBC. In this paper, the stepwise revenue function was adopted to link the cost and availability to profit, then a maintenance optimization model was established. To reduce the inspection frequency and the cost, the first inspection interval was incorporated into the model formulation. In addition, the PSO algorithm was used to improve the speed and accuracy of the solution. A numerical example was provided, which illustrates the superiority of the presented model. According to the comparison analysis, it can be verified that the proposed policy can increase profits and improve performance at a certain cost. In sensitivity analysis, we observed the effect of different failure consequences on optimization policy. For the proposed policy, the higher C_f, the more obvious the profit increase; the higher T_f, the less obvious the profit increase. The final comparative study showed considering the first inspection interval can provide lower cost, higher availability, and higher profit.

This work can be extended along with several interesting directions. Firstly, this paper investigates the system subject to the gamma deteriorating process, it is also worth studying another deteriorating process, like the inverse Gaussian process and Wiener process, which may be more in line with the degradation laws of some components. In addition, the proposed model is for the single-component system, there are several systems worth considering, such as series system, parallel system, and hybrid system. These two research directions need more attention, which can further expand the research scope of maintenance optimization under PBC.

Acknowledgments
Not applicable.

Authors' contributions
XZ completed the main research of the paper and wrote the first draft. QH provided research ideas and guides the completion of research and the paper writing. LW contributed to refining the ideas and programming of the solving algorithm. JL and MS assisted in the research and proofread the manuscript. All authors read and approved the final manuscript.

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Funding
Not applicable.

Availability of data and materials
The datasets supporting the conclusions of this article are included within the article.

Competing Interests
The authors declare no competing financial interests.

Received: 19 July 2021 Revised: 27 July 2022 Accepted: 11 January 2023
Published online: 14 February 2023

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