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Accuracy Analysis of Stewart Platform Based on Interval Analysis Method

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Abstract: A Stewart platform is introduced in the 500 m aperture spherical radio telescope(FAST) as an accuracy adjustable mechanism for feed receivers. Accuracy analysis is the basis of accuracy design. However, a rapid and effective accuracy analysis method for parallel manipulator is still needed. In order to enhance solution efficiency, an interval analysis method(IA method) is introduced to solve the terminal error bound of the Stewart platform with detailed solution path. Taking a terminal pose of the Stewart platform in FAST as an example, the terminal error is solved by the Monte Carlo method(MC method) by 4 980 s, the stochastic mathematical method(SM method) by 0.078 s, and the IA method by 2.203 s. Compared with MC method, the terminal error by SM method leads a 20% underestimate while the IA method can envelop the real error bound of the Stewart platform. This indicates that the IA method outperforms the other two methods by providing quick calculations and enveloping the real error bound of the Stewart platform. According to the given structural error of the dimension parameters of the Stewart platform, the IA method gives a maximum position error of 19.91 mm and maximum orientation error of 0.534°, which suggests that the IA method can be used for accuracy design of the Stewart platform in FAST. The IA method presented is a rapid and effective accuracy analysis method for Stewart platform.

Key words: Stewart platform, accuracy, interval analysis, radio telescope

1 Introduction

China is now building the largest single dish radio telescope in the world in Guizhou province, which is called 500 m aperture spherical radio telescope(FAST)[1–2].

Fig. 1(a) shows the conceptualized design of the FAST system. The feed support system of FAST includes two parts[3]: one is a six-cable driven parallel manipulator with a large span that drives the feed adjustable mechanism equipped with a coarse positioning; the other one is a feed adjustable mechanism.

Fig. 1(b) shows the feed adjustable mechanism. The diameter of the feed adjustable mechanism is about 15m. In the feed adjustable mechanism, the A-B rotator can provide proper orientation of the feed to track the celestial sources. The Stewart manipulator, fixed on the A-B rotator, is used to improve the orientation and position accuracy of the feed receivers.

Accuracy of the parallel mechanism is divided into static accuracy and dynamic accuracy. Studies have shown that the static error was the main reason leading to terminal error of the parallel mechanism, which accounts for more than 70% of the error[4–5]. Since static error can be restrained during design and compensation, we mainly studied static accuracy in this paper.

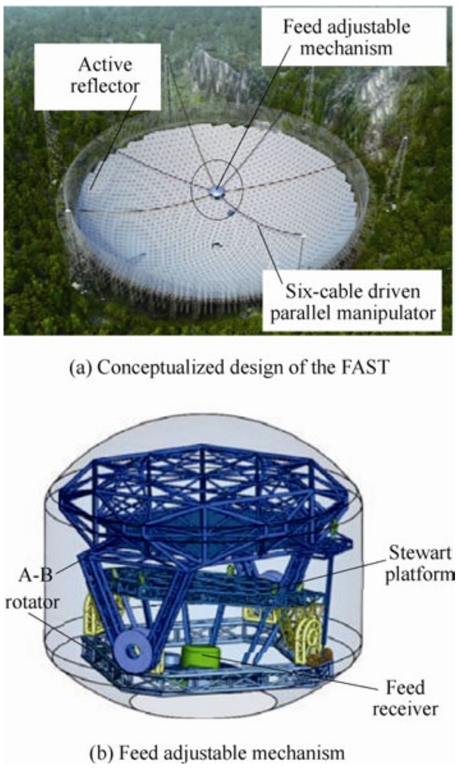

Fig. 1. Aperture spherical radio telescope(FAST)

Ensuring the accuracy of parallel mechanism in design phase requires both accuracy analysis and accuracy design,

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and accuracy analysis is the basis of the accuracy design. However, the lack of a rapid and effective accuracy analysis method remains as a main hurdle.

Accuracy analysis, also known as error analysis, comprehensively studies the terminal error and error transfer process by means of setting up a mapping model of structural parameters and terminal error of parallel mechanism. There are two main error modeling methods [6–9]: the vector method and the matrix method. The impact of errors of structural parameters on terminal error of parallel mechanism is an uncertain problem. To solve the problem, the most common mathematical methods are the Monte Carlo method (MC method) [10–11] and the Stochastic Mathematical method (SM method) [12–13].

Early studies showed that the MC method can be used to obtain more accurate terminal error bounds by sufficient sampling points in workspace and calculation numbers. However, the MC method uses a kinematics iterative solution of parallel manipulator. Because of large number of sampling points and calculation numbers, computational efficiency of the MC method is low. While computation time of the SM method is very short, it is not an accurate method because of the usage of a linear approximation model as the calculation model. In most cases, the SM method will lead to an underestimate of terminal error bounds of parallel mechanism. The interval analysis method(IA method)[14], compared to the two methods mentioned above, directly and efficiently solves the kinematic nonlinear equations with errors of structural parameters. The terminal error bounds solved by the IA method can envelope the terminal bounds solved by MC method as well.

In this paper, we explore the application of the IA method for accuracy analysis of parallel manipulator. In section 2, error model of Stewart platform is set up. Section 3 discusses the solution methods for terminal errors of the Stewart platform based on IA method. In section 4, solution results using the three accuracy solution methods are presented, as well as the terminal error of the Stewart platform obtained by the IA method.

2　Error Modeling of the Stewart Platform Based on Vector Chain Method

Fig. 2 is the Stewart platform in FAST. Two coordinate systems are in a fixed frame $\Re: O-XYZ$ located at the centre of the static platform and a reference frame $\Re': O'-X'Y'Z'$ located on the moving platform. Points $B_i (i=1, 2, \cdots, 6)$ are the spherical joint centers on the static platform and points $P_i (i=1, 2, \cdots, 6)$ are the spherical joint centers on the moving platform. α is the spherical joint distributed angle of the static platform and β is the spherical joint distributed angle of the moving platform.

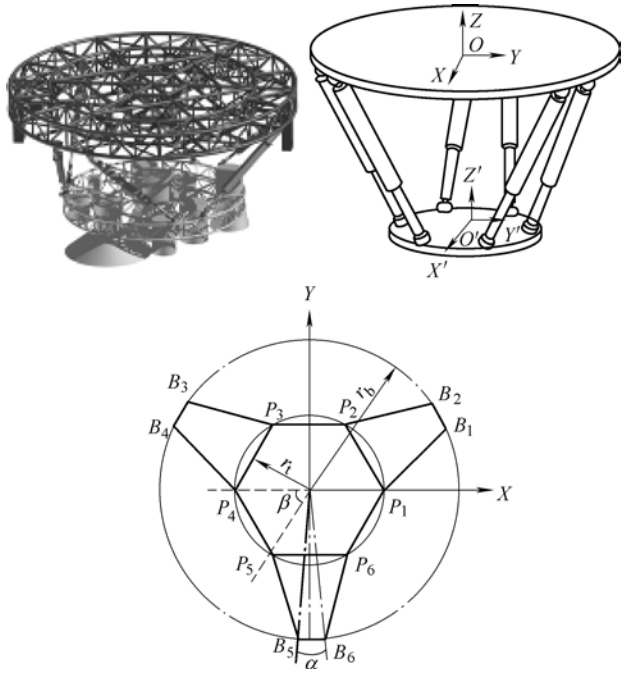


Fig. 2.　Geometric parameters of the Stewart platform

The kinematics equation of the Stewart platform can be expressed as follows:

$$\boldsymbol{l}_i = \boldsymbol{OO'} + \boldsymbol{R} \cdot \boldsymbol{O'P}_i - \boldsymbol{OB}_i, \quad i = 1, 2, \cdots, 6, \tag{1}$$

$$\boldsymbol{R} = \begin{pmatrix} \mathrm{c}\phi_a \mathrm{c}\phi_b & \mathrm{c}\phi_a \mathrm{s}\phi_b \mathrm{s}\phi_c - \mathrm{s}\phi_a \mathrm{c}\phi_c & \mathrm{c}\phi_a \mathrm{s}\phi_b \mathrm{c}\phi_c + \mathrm{s}\phi_a \mathrm{s}\phi_c \\ \mathrm{s}\phi_a \mathrm{c}\phi_b & \mathrm{s}\phi_a \mathrm{s}\phi_b \mathrm{s}\phi_c + \mathrm{c}\phi_a \mathrm{c}\phi_c & \mathrm{s}\phi_a \mathrm{s}\phi_b \mathrm{c}\phi_c - \mathrm{c}\phi_a \mathrm{s}\phi_c \\ \mathrm{s}\phi_b & \mathrm{c}\phi_b \mathrm{s}\phi_c & \mathrm{c}\phi_b \mathrm{c}\phi_c \end{pmatrix}, \tag{2}$$

where s presents the sin function, and c presents the cos function. $\boldsymbol{R}$ is a rotation martrix based on three Euler angles.

As shown in Fig. 2, the vector method is used to set up an error model of the Stewart platform. $\delta \boldsymbol{OB}_i$ are the error vectors of joints B_i, $\delta \boldsymbol{O'P}_i$ are the error vectors of joints P_i, and $\delta \boldsymbol{L}$ are the error of l_i. Differentiation of Eq. (1) gives [15]

$$\delta l_i \boldsymbol{I}_i + l_i \delta \boldsymbol{I}_i = \delta \boldsymbol{OO'} + \delta \boldsymbol{R} \cdot \boldsymbol{O'P}_i + \boldsymbol{R} \cdot \delta \boldsymbol{O'P}_i - \delta \boldsymbol{OB}_i, \tag{3}$$

where $\boldsymbol{I}_i$ is an axial unit vector,

$$\delta \boldsymbol{R} = \begin{pmatrix} 0 & -\delta\omega_z & \delta\omega_y \\ \delta\omega_z & 0 & -\delta\omega_x \\ -\delta\omega_y & \delta\omega_x & 0 \end{pmatrix} \boldsymbol{R} = \boldsymbol{R}_\Omega \cdot \boldsymbol{R}. \tag{4}$$

Substituting Eq. (4) into Eq. (3) yields

$$\delta \boldsymbol{q} = \boldsymbol{J}_0^{-1} \cdot \delta \boldsymbol{L} + \boldsymbol{J}_0^{-1} \cdot \boldsymbol{D}_1 \cdot \delta \boldsymbol{O'P}_i + \boldsymbol{J}_0^{-1} \cdot \boldsymbol{D}_2 \cdot \delta \boldsymbol{OB}_i, \tag{5}$$

where

$$\delta \boldsymbol{q} = (\delta \boldsymbol{OO'}^{\mathrm{T}}, \delta \boldsymbol{\Omega}^{\mathrm{T}})^{\mathrm{T}},$$

$$\delta \boldsymbol{L} = \left(\delta l_1\ \delta l_2\ \delta l_3\ \delta l_4\ \delta l_5\ \delta l_6\right)^{\mathrm{T}},$$

$$\boldsymbol{D}_1 = \begin{pmatrix} -\boldsymbol{I}_1^{\mathrm{T}} \cdot \boldsymbol{R} & 0 & \cdots & 0 \\ 0 & -\boldsymbol{I}_2^{\mathrm{T}} \cdot \boldsymbol{R} & \cdots & 0 \\ \vdots & \vdots & & 0 \\ 0 & 0 & \cdots & -\boldsymbol{I}_6^{\mathrm{T}} \cdot \boldsymbol{R} \end{pmatrix},$$

$$\boldsymbol{D}_2 = \begin{pmatrix} \boldsymbol{I}_1^{\mathrm{T}} & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & \boldsymbol{I}_6^{\mathrm{T}} \end{pmatrix},$$

$$\boldsymbol{J}_0 = \begin{pmatrix} \boldsymbol{I}_1^{\mathrm{T}} & \boldsymbol{I}_1^{\mathrm{T}} \times (\boldsymbol{R} \cdot \boldsymbol{O'P}_1) \\ \vdots & \vdots \\ \boldsymbol{I}_6^{\mathrm{T}} & \boldsymbol{I}_6^{\mathrm{T}} \times (\boldsymbol{R} \cdot \boldsymbol{O'P}_6) \end{pmatrix}.$$

Therefore, the error transmission function for the Stewart platform is

$$\delta \boldsymbol{q} = \boldsymbol{E} \delta \boldsymbol{\varepsilon}, \tag{6}$$

where $\boldsymbol{E}$ is the error transmission matrix,

$$\boldsymbol{E} = (\boldsymbol{J}_0^{-1}, \boldsymbol{J}_0^{-1} \cdot \boldsymbol{D}_1, \boldsymbol{J}_0^{-1} \cdot \boldsymbol{D}_2) \in \mathbf{R}^{6\times 42}, \tag{7}$$

$$\delta \boldsymbol{\varepsilon} = (\delta \boldsymbol{L}^{\mathrm{T}},\ \delta \boldsymbol{O'P}_i^{\mathrm{T}},\ \delta \boldsymbol{OB}_i^{\mathrm{T}}) \in \mathbf{R}^{42\times 1}. \tag{8}$$

3 Solution Method for Terminal Error of the Stewart Platform

Terminal error of the Stewart platform can be solved by Eq. (6). The common solution methods are the MC method and the SM method.

Accurate terminal error bound of a parallel manipulator can be obtained by MC method with more than 1 000 000 randomized computations. The SM method was used to set the structural tolerances as random variables with numerical characteristics, from which the relationship of the structural tolerances and terminal errors of the parallel manipulator can be calculated.

IA method can directly solve the kinematic nonlinear equations with errors of structural parameters quickly. This section will introduce the IA method in details.

Considering a variable x within a certain value range, it can be expressed as

$$x \in (a, b). \tag{9}$$

By using IA method, interval variable x is defined as

$$x = (\underline{x}, \overline{x}), \tag{10}$$

where $\underline{x}$ and $\overline{x}$ are the lower bound and upper bound of x respectively.

Similar with linear algebra, elements of interval vector and matrix are interval variables. For the interval variable x, the interval spacing can be described as

$$W(x) = \overline{x} - \underline{x}. \tag{11}$$

The interval center is

$$m(x) = \frac{\overline{x} + \underline{x}}{2}. \tag{12}$$

For two interval variables, the interval arithmetic can be expressed as follows:

$$-x = (-\overline{x}, -\underline{x}), \tag{13}$$

$$x + y = (\underline{x} + \underline{y}, \overline{x} + \overline{y}), \tag{14}$$

$$x - y = (\underline{x} - \overline{y}, \overline{x} - \underline{y}), \tag{15}$$

$$\frac{1}{x} = \left(\frac{1}{\overline{x}} \frac{1}{\underline{x}}\right), \quad 0 \notin x, \tag{16}$$

$$\begin{cases} \underline{x \cdot y} = \min(\underline{x} \cdot \overline{y},\ \overline{x} \cdot \underline{y}), \\ \overline{x \cdot y} = \max(\underline{x} \cdot \underline{y},\ \overline{x} \cdot \overline{y}), \end{cases} \tag{17}$$

$$x \cap y = \begin{cases} \varPhi, & \underline{x} > \overline{y} \text{ or } \underline{y} > \overline{x}, \\ (\max(\underline{x}, \underline{y}),\ \min(\overline{x}, \overline{y})), & \underline{x} \leqslant \overline{y} \text{ or } \underline{y} \leqslant \overline{x}, \end{cases} \tag{18}$$

$$x \cup y = (\min(\underline{x} \cdot \underline{y}),\ \max(\overline{x} \cdot \overline{y})). \tag{19}$$

From Eqs. (9)–(19), the core of the interval arithmetic is that the interval variable can be a collection of any number in its range, which enables easy computation.

Interval function can be defined as $F(x)$. The upper and lower bounds of the interval function $F(x)$ can be calculated by interval arithmetic. The interval function of the interval vector $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)^{\mathrm{T}}$ can be expressed as $\boldsymbol{F}(\boldsymbol{x})$. Obviously, the interval function is more complicated than a normal function.

For an interval equation set $\boldsymbol{F}(\boldsymbol{x}, \boldsymbol{T}) = \boldsymbol{0}$ (where $\boldsymbol{T}$ is parameters, namely interval variables) the Newton iteration method can be adopted as a solution. The specific flow is as follows.

(1) Setting initial value $\boldsymbol{x}_0 = (x_1, x_2, \cdots, x_n)^{\mathrm{T}}$ of the iterative calculation.

(2) Ordinary equation set $\boldsymbol{f}(\boldsymbol{x}_C, \boldsymbol{t}) = \boldsymbol{0}$, which is correspond to the interval equation set, can deduce to its derivation function $\boldsymbol{f}'(\boldsymbol{x}_C, \boldsymbol{t}) = \boldsymbol{0}$, and obtain its interval derivative matrix $\boldsymbol{F}'(\boldsymbol{x}, \boldsymbol{T}) = \boldsymbol{0}$.

(3) Iterative equation can be expressed as

$$x_{n+1} = x_n \cap (m(x) - F'^{-1}(x, T)F(m(x), T)).$$

(4) Repeat step (3) until $w(x_{n+1}) - w(x_n) < \varepsilon$.

Based on the above steps, interval solution x of $F(x,T) = 0$ can be obtained. The meaning of the interval solution is

$$x = \{x | f(x_C, t) = 0, t \in T\}.$$

However, interval analysis may over-estimate the problem, namely the range of interval solution x may be larger than the exact range.

4 Simulation

Taking the Stewart platform in FAST as an example, terminal error of the Stewart platform will be studied with structural parameters in Table 1.

Table 1. Structural parameters of the Stewart platform

Parameter	Value
Radius of the static platform r_b /m	3.4
Radius of the moving platform r_t /m	2.25
Distributed angle of joints on the static platform α /(°)	17
Distributed angle of joints on the moving platform β /(°)	21
Initial pose $(x_0, y_0, z_0, \phi_a, \phi_b, \phi_c)$ / (m, m, m, (°), (°), (°))	(0, 0, –2, 0, 0, 0)

Fig. 1(b) is the schematic picture of the feed adjustable mechanism. The structure of the feed adjustable mechanism mainly includes: mantle, mechanical and electrical equipments, A-B rotator, and Stewart manipulator. The mantle covers the entire mechanism. Equipments can be placed in the cabin. The A-B rotator can provide proper orientation of feed to track celestial sources. The Stewart manipulator, fixed on the A-B rotator, is to improve the accuracy of orientation and position of the feed receivers.

4.1 Comparison of three solving methods for terminal error

The required workspace of the Stewart platform can be described as a sphere: diameter is 0.5 m and the centre of the sphere is at (0, 0, –2 m), attitude angles ϕ_a, ϕ_b, ϕ_c are from –5° to 5°. According to the requirement of feed receivers and installation, all the related parameters and the value range of parameters are listed in Table 1. Assuming $\pm$2 mm of tolerance of all Structural parameters according to Normal distribution, terminal position error bounds at the sample pose of the Stewart platform is shown in Fig. 3.

In Fig. 3, envelope region indicated by black points is terminal error bounds at the sample pose of the Stewart platform by MC method; envelope region indicated by blue lines is obtained by IA method, and envelope region indicated by red dotted lines is produced by SM method. Computation times of the three solving methods are listed in Table 2.

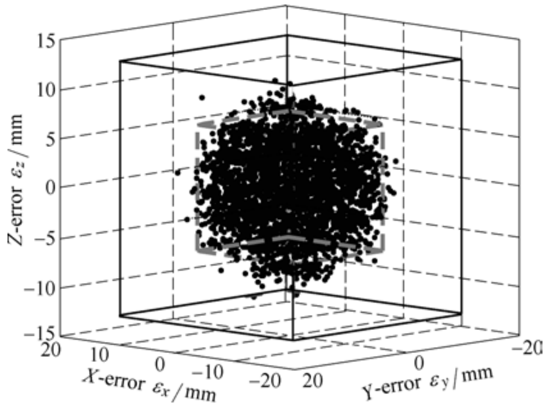


Fig. 3. Terminal position error bound of the Stewart platform

Table 2. Computing time based on the three methods

Computing method	Computing time t/s
Monte Carlo method	4 980
Stochastic mathematical method	0.078
Interval analysis method	2.203

Fig. 3 and Table 2 show that taking a large number of computing samples can produce the most accurate terminal error bounds by using the MC method. The largest position error at the sample pose of the Stewart platform by using MC method is 12.58 mm. The largest position error by using the SM method is 10.32 mm, and the largest position error by using IA method is 19.15 mm.

Compared with MC method, the SM method gives a 20% underestimate while the IA method gives a 35% overestimate.

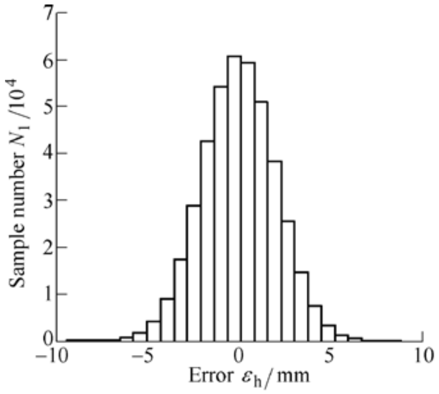
The long calculation time of the MC method makes it unrealistic in practical accuracy design. Although the SM method used shorter computational time, computation it is not an accurate method because of using a linear approximation model as the calculation model. In most cases, SM method will lead to under estimate of terminal error bounds of parallel mechanism. Therefore, the IA method is the most practical method for accuracy analysis and design because of the high calculation speed and the ability of enveloping the real error bounds.

4.2 Accuracy analysis of Stewart platform by IA method

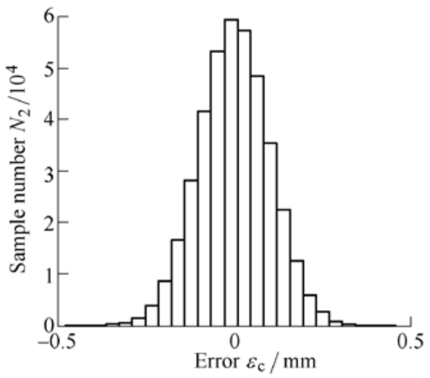
Error bounds of the Stewart platform can be estimated by the IA method. Structural errors of dimension parameters are normal as shown in Fig. 4.

Manufacture and installation error of hinges are depicted as in Fig. 4(a); manufacture and installation error of chains are depicted in Fig. 4(b).

Terminal error bounds of the Stewart platform can be calculated by IA method at any pose in its workspace.



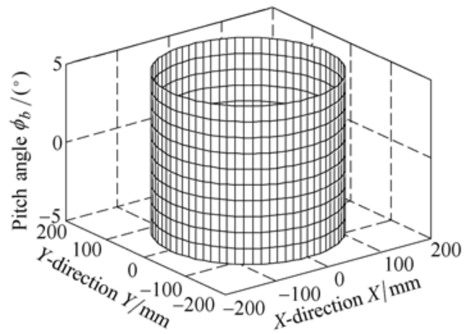
(a) Manufacture and installation error of hinges



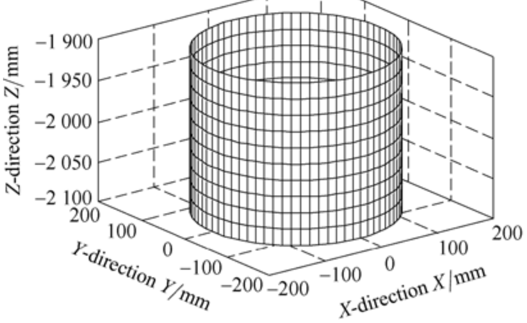
(b) Manufacture and installation error of chains

Fig. 4. Distribution of structural errors of parameters

Fig. 5 shows the working trajectories of calculating the maximum position and orientation terminal error of the Stewart platform.



(a) Working trajectory A



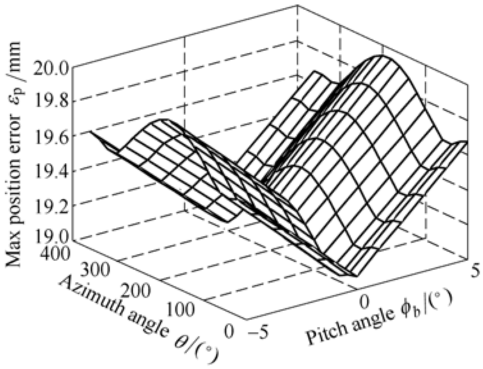
(b) Working trajectory B

Fig. 5. Working trajectories of Stewart platform

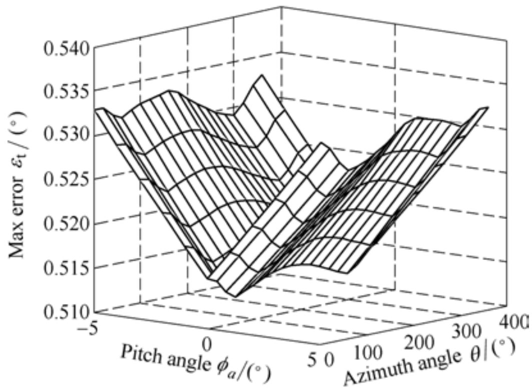
In Fig. 5(a), the working trajectory A is with working center of (0, 0, –2 m), z is a constant of –2 m, working radius is 150 mm, and pitch angle is from −5° to 5°.

In Fig. 5(b), the working trajectory B is as follows: working center is at (0, 0, –2 m), pitch angle is constant as 0°, z is from –2.1 mm to –1.9 mm, and working radius is 150 mm.

The maximum terminal position and orientation error of trajectory A is shown in Fig. 6. In Fig. 6, the maximum position error is about 19.91 mm, and the maximum orientation error is about 0.534° in trajectory A.



(a) Maximum position error



(b) Maximum orientation error

Fig. 6. Maximum terminal error of working trajectory A

The maximum terminal position and orientation error of trajectory B is shown in Fig. 7. In Fig. 7, the maximum position error is about 19.39 mm, and the maximum orientation error is about 0.528° in trajectory B.

According to the IA method, the terminal error of the Stewart platform can be obtained quickly. Moreover, the analysis results envelope the real error bounds. Therefore, the IA method can improve the solution efficiency of accuracy analysis, accuracy distribution and calibration.

5 Conclusions

(1) IA method can solve the terminal error of the Stewart platform quickly and efficiently. Comparing with the MC method and the SM method, simulation proves that the analyses result by the IA method can envelopes real error bounds.

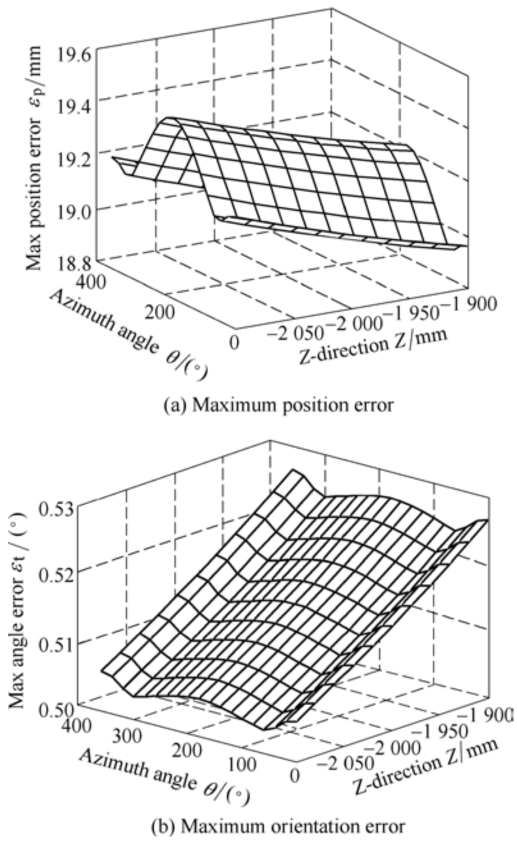


Fig. 7.　Maximum terminal error of working trajectory B

(2) Based on the given structural error of dimension parameters, the maximum position error of the Stewart platform in FAST is 19.91 mm and maximum orientation error is 0.534° by using IA method.

(3) An efficient accuracy analysis method is provided for the coming accuracy study of the Stewart platform in FAST.

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