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Applicability and Generality of the Modified Grübler-Kutzbach Criterion

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Abstract: A generally applicable criterion for all mechanism mobility has been an active domain in mechanism theory lasting more than 150 years. It is stated that the Modified Grübler-Kutzbach criterion for mobility has been successfully used to solve the mobility of many more kinds of mechanisms, but never before has anyone proven the applicability and generality of the Modified Grübler-Kutzbach criterion in theory. In order to fill the gap, the applicability and generality of the Modified Grübler-Kutzbach Criterion of mechanism mobility is systematically demonstrated. Firstly, the mobility research background and the Modified Grübler-Kutzbach criterion are introduced. Secondly, some new definitions, such as half local freedom, non-common constraint space of a mechanism and common motion space of a mechanism, etc, are given to demonstrate the correctness and broad applicability of the Modified Grübler-Kutzbach criterion. Thirdly, the general applicability of the Modified Grübler-Kutzbach criterion is demonstrated based on screw theory. The mobilities of the classical DELASSUS mechanisms and a modern planar parallel mechanism, are determined through the Modified Grübler-Kutzbach criterion, which are as examples to show the practical application of the Modified Grübler-Kutzbach criterion.

Key words: mobility, screw theory, classical mechanism, parallel mechanism

1 Introduction

The mobility analysis of mechanisms has a long history more than 150 years that can be traced to the initial studies of the works of CHEBYCHEV in 1854[1] and the works of GRÜBLER in 1883[2]. From then on, it has being remained attractive[3–5]. Clearly, the most significant development of the mobility analysis is that of the Kutzbach-Grübler Criterion[6–7]. However, it has been found that the real mobilities of some mechanisms are not consistent with the calculated results through the Kutzbach-Grübler criterion[3, 7–8].

In 1997, HUANG, et al[7], prooposed the definition of common constraint of mechanism, pointed out that a common constraint of a mechanism is a screw reciprocal to all the motion screws of the mechanism screw system, and defined the order of mechanisms. In 2002, he also brought up the concept of the redundant constraint from the limbs of parallel mechanisms, which constitute the constraint screw principle of mobility analysis, thus the idea of modified Grübler-Kutzbach criterion was proposed[9].

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Using the Modified Grübler-Kutzbach criterion, scholars have solved the mobilities of some modern parallel mechanisms[7, 10–12] comprised of CARRICATO's, CPM, DELTA, etc, and the mobilities of some classical mechanisms[7–8, 9–12], such as BENNETT, GOLDBERG, BRICARD, HERVE, SARRUS and MYARD mechanism, were also solved.

The Modified Grübler-Kutzbach criterion has no connection with the dependency[3] of the non-linear closure equations of a mechanism and is only based on the simplest part of the screw theory. Besides, using the criterion it is quite easily to judge the mobility to be instantaneous or of full-cycle. In 2005, DAI, et al[13], further theoretically elucidated the principle. In 2006, HUANG, et al, concluded eight rules[12] to help people to hold the method. In 2011, LIU, et al[14], analyzed the mobility of ALTMANN linkages by this method. Up to now, the theoretical demonstration of the generality of the Modified Grübler-Kutzbach criterion has not been given.

This paper focuses on the demonstration of the generality and practicality of the Modified Grübler-Kutzbach criterion. Section 2 introduces some fundamental concepts, the screw theory and the reciprocal screw theory for the Modified Grübler-Kutzbach Criterion, and demonstrates the general validity of the Modified Grübler-Kutzbach criterion in theory. Section 3 shows the generality of the Modified Grübler-Kutzbach Criterion through practical examples of single loop overconstrained DELASSUS linkages and the multi-loop parallel planar mechanism. Section 4 elaborates the practicality of the implementation of the Modified Grübler-Kutzbach Criterion. Section 5 presents conclusions.

2 Demonstrate the General Validity of the Modified Grübler-Kutzbach Criterion

2.1 Screw

On the basis of the screw theory[6, 15–16], each joint axis with connection one in a linkage or a mechanism can be regarded as a spiral joint axis generally, this axis can be expressed as a screw like

$$\boldsymbol{\$}=(\boldsymbol{S};\boldsymbol{S}^0)^{\mathrm{T}}, \tag{1}$$

where $\boldsymbol{S}=(l \quad m \quad n)^{\mathrm{T}}$ is the unit vector along the direction of the joint axis;

$$\boldsymbol{S}^0=h\boldsymbol{S}+\boldsymbol{r}\times\boldsymbol{S}=(p \quad q \quad r)^{\mathrm{T}}, \tag{2}$$

$\boldsymbol{S}^0$ represents the location and the pitch of the screw axis. h is the pitch of the screw $\boldsymbol{\$}$, $h=(\boldsymbol{S}^0\bullet\boldsymbol{S})/(\boldsymbol{S}\bullet\boldsymbol{S})$, and $\boldsymbol{r}$ is the position vector of any point on the screw axis.

2.2 Reciprocal screw

If there is a screw $\boldsymbol{\$}=(l \quad m \quad n;p \quad q \quad r)^{\mathrm{T}}$ and another screw $\boldsymbol{\$}'=(l' \quad m' \quad n';p' \quad q' \quad r')^{\mathrm{T}}$, the mutual moment of the two screws is

$$\boldsymbol{\$}\circ\boldsymbol{\$}'=lp'+mq'+nr'+pl'+qm'+rn'. \tag{3}$$

If the mutual moment is zero, the screw $\boldsymbol{\$}'$ is called the inverse or reciprocal screw of the screw $\boldsymbol{\$}$. On the contrary, the screw $\boldsymbol{\$}$ is the inverse screw of the screw $\boldsymbol{\$}'$ too. They are reciprocal to each other. An inverse screw of a motion screw is a constraint permitting the existence of the motion screw. If $\boldsymbol{\$}$ expresses a movement, then $\boldsymbol{\$}'$ is a constraint permitting the existence of the movement.

2.3 Terms relative to the Modified Grübler-Kutzbach criterion

Several important concepts must be proposed here in order to prove the generality of the Modified Grübler-Kutzbach criterion.

(1) Motion screw

A pair in a mechanism can be equated as one or more than one pair with connection one. Each equivalent pair with connection one in a mechanism can be expressed as a screw, which is called a motion screw of the mechanism.

(2) Motion screw system

All of the motion screws of a mechanism constitute a screw system defined as the motion screw system of the mechanism, the screw system is

$$\boldsymbol{\varphi}=\{\boldsymbol{\$}_i \,|\, \boldsymbol{\$}_i=(l_i \quad m_i \quad n_i;p_i \quad q_i \quad r_i)^{\mathrm{T}},(i=1,2,\cdots,N)\}, \tag{4}$$

where N is the total number of the joints after being replaced by the joints with connection one.

The biggest number of the rank of the system is six. The real rank of the screw system is named as r_{m}, the rank can be obtained from Ref. [7], and it is equal to the maximal number of the independent screws in the screw system.

(3) Constraint screw

If a screw is reciprocal to any a motion screw of a mechanism, this screw is called a constraint screw of the mechanism.

(4) Constraint screw system

All of the constraint screws of a mechanism constitute the constraint screw system of the mechanism. The real number of the rank of the constraint screw system is named as r_{c}, and $r_{\mathrm{c}}=6-r_{\mathrm{m}}$.

(5) Constraint space of a mechanism

A space spanned by all independent constraint screws of a mechanism is called the constraint space of the mechanism. The rank of the mechanism is r_{c}.

(6) Common constraint screw

The inverse screws of a motion screw are constraints to the motion screw. If a screw is reciprocal to all the motion screws of a mechanism, this screw is called a common constraint screw of the mechanism[7]. All the screws reciprocal to every motion screw of a mechanism are called the common constraint screws. λ is the maximal number of the independent common constraints and also named as common constraint factor. If there are w_1 common constraint screws in sum in a mechanism, then $\mu=w_1-\lambda$, here, μ is the number of the excessive common constraints of the mechanism.

(7) Common constraint screw system of a mechanism

If a screw system consists of all the common constraint screws of a mechanism, it is named as the common constraint screw system of the mechanism. The rank of the system is λ as defined above.

(8) Common constraint space of a mechanism

If a space is spanned by all the independent common constraint screws of a mechanism, it is named as the common constraint space of the mechanism. The dimension of the space is λ.

(9) Common motion screw of a mechanism

If a screw is reciprocal to all the common constraint screws of a mechanism, this screw is called a common motion screw of the mechanism. All the screws reciprocal to every constraint screw of a mechanism are called the common motion screws. The maximum number of the independent common motion screws is d and obviously in three-dimensional space it is

$$d=6-\lambda. \tag{5}$$

(10) Common motion screw system of a mechanism

All the common motion screws of a mechanism constitute a screw system; this is called a common motion screw system of the mechanism. The rank of the system is d.

(11) Common motion space of a mechanism

If a space is spanned by all the independent common motion screws of a mechanism, it is named as the common motion space of the mechanism. The dimension of the space is d called the order of the mechanism[7].

(12) Non-common constraint screw

If a screw is reciprocal to one or more than one but not all of the motion screws of a mechanism, it is called the non-common constraint screw of the mechanism. k is the maximal number of the independent non-common constraints and also named as non-common constraint factor, v is the number of the excessive non-common constraints[9]. If a mechanism has w_2 non-common constraint screws in sum, then

$$v = w_2 - k . \tag{6}$$

(13) Non-common constraint screw system

If a screw system is spanned by all the non-common constraint screws of a mechanism, it is named as the non-common constraint screw system. The rank of the system is k .

(14) Non-common constraint space of a mechanism

If a space is spanned by all the independent non-common constraint screws of a mechanism, it is called the non-common constraint space of the mechanism. The dimension of the space is k .

2.4 Grübler-Kutzbach criterion

The generally accepted theory of the degrees of freedom(DOF) of mechanisms is the Kutzbach-Grübler criterion[2, 6–7, 16–17]. There are mainly three forms of the Kutzbach-Grübler formula, the first form applicable to the planar and spherical mechanisms is

$$M = 3(n - g - 1) + \sum_{i=1}^{g} f_i , \tag{7}$$

where n is the number of the total members of the mechanism, g is the number of the total kinematic pairs, f_i is the DOF of the ith kinematic pair.

The second form is applicable for the spatial mechanisms:

$$M = 6(n - g - 1) + \sum_{i=1}^{g} f_i . \tag{8}$$

The third version of the criterion given by HUNT[6] and employed by TSAI[17] is

$$M = d(n - g - 1) + \sum_{i=1}^{g} f_i , \tag{9}$$

where d is the dimension of the motion space of all members of a mechanism, d is 3 for the planar and spherical mechanisms, d is 6 for the spatial mechanisms, and d is called as the dimension of the common motion space of the mechanism in this paper. But there was not a introduction in detail about how to calculate d systematically until HUANG gave the definition about the common constraint of a mechanism in 1997, then he presented the Modified Grübler-Kutzbach criterion.

2.5 Modified Grübler-Kutzbach criterion

The Modified Grübler-Kutzbach criterion is given[7, 9] as

$$M = d(n - g - 1) + \sum_{i=1}^{g} f_i + \nu - \xi , \tag{10}$$

where v is the parallel-redundant-constraint factor or the number of the excessive non-common constraints, and can be obtained from Eq. (6). ξ is the degrees of partial freedom or half partial freedom of links, which does not affect the motion of the other links or only affect the motion of the one-side links, d is the order of the mechanism and can be obtained from Eq. (5) or

$$d = \mathrm{rank}(\varphi) , \tag{11}$$

where φ can be gotten based on Eq. (4). In a multi-loop parallel mechanism, it is difficult to directly caculate the rank of the kinematic screw system φ for there exist quite a few kinematic pairs, whereas Eq. (5) often makes the caculation easy and feasible. In single-loop mechanisms and serial chains, the order of a mechanism can be directly obtained from Eq. (11) generally.

2.6 Demonstration of the general validity of the Modified Grübler-Kutzbach criterion

If there are n rigid bodies in the three-dimensional space, because there are six independent motion screws for each rigid body under the case of no constraints, the dimension of the common motion space of the n rigid bodies is six. Then, six independent coordinates are needed in order to describe the direction and location of a body in the three-dimensional space. So the sum of the degrees of freedom of the n bodies is $6n$. If one of the n bodies is selected as a referring body, the number of the degrees of freedom of the system is $6(n - 1)$. Whereas if the n bodies are exerted by λ independent common constraint screws, the dimension of the common motion space of the n bodies is d and $d = 6 - \lambda$. Thus, the number of the degrees of freedom of one of the n rigid bodies is d. The sum of the degrees of freedom of the n bodies is dn. If one of the n bodies is selected as a referring body, the degrees of freedom of the system is $d(n - 1)$. If the n bodies constitute a mechanism with g joints, assuming b_i binds are formed by the ith joint, the sum of the binds in the mechanism is $\sum_{i=1}^{g} b_i$, and $\sum_{i=1}^{g} b_i = w_1 + w_2$ (refer to the common constraint screw, the non-common constraint screw in section 2.3), the number of the effective constraints is $\lambda + k$; there is f_i number of degrees of freedom in the ith joint, and $b_i = 6 - f_i$ under the case of no common constraints. If we considering the common constraint space with rank λ and the common motion space with rank d, it is easy to get

$$w_{2i} = d - f_i \,, \tag{12}$$

where w_{2i} represents the number of non-common constraint screws arisen from the ith joint, and we have

$$w_2 = \sum_{i=1}^{g} w_{2i} = \sum_{i=1}^{g} (d - f_i) = k + v \,. \tag{13}$$

There are only k independent constraint screws among the w_2 non-common constraint screws in sum, and k is

$$k = \sum_{i=1}^{g} (d - f_i) - v \,. \tag{14}$$

In the common motion space, the number of the degrees of freedom of a mechanism composed of n members is

$$M = d(n-1) - k = d(n-1) - \sum_{i=1}^{g} (d - f_i) + v \,. \tag{15}$$

If the number of the degrees of the partial or half partial freedom(see section 3.2) ξ is subtracted, then

$$M = d(n-1) - \sum_{i=1}^{g} (d - f_i) + v - \xi \,, \tag{16}$$

$$M = d(n-g-1) + \sum_{i=1}^{g} f_i + \nu - \xi \,. \tag{17}$$

This is a global criterion of the degrees of freedom of mechanisms, and is called the Modified Grübler-Kutzbach Criterion.

3 Application Examples of the Modified Grübler-Kutzbach Criterion

Here the mobility of two kinds of mechanisms is analyzed using the Modified Grübler-Kutzbach Criterion, the first is the classical DELASSUS mechanisms, the second is a modern planar parallel mechanisms.

The definition of mobility and the number of inputs or motors should be introduced in order to distinguish the number of inputs from mobility before beginning the mobility analysis.

(1) The number of inputs

The number of motors or inputs of a mechanism represents the number of different finite displacements in the joints needed to define the configuration of the mechanism.

(2) Mobility

Mobility M or the degree of freedom represents the number of independent coordinates needed to define the location and the direction, namely the pose of a kinematic chain or a mechanism.

Generally, the degree of freedom of a mechanism is equal to the number of the inputs or motors. But sometimes, they are not equal, and this case only emerges in this kind of mechanism within which the inputs are dependent of each other.

3.1 Single-loop overconstrained Delassus linkages

The research on overconstrained mechanisms can be dated back to 1853 when the first overconstrained mechanism was presented by SARRUS[18]. After that time, much more attention has been paid for the study on overconstrained mechanisms and many overconstrained mechanisms have been obtained. Among the overconstrained linkages, DELASSUS linkages are four-bar linkages. The research about the mobility of DELASSUS linkages is important significance after the mobility of BENNETT, DELTA, SARRUS, BRICARD, HERVE, MYARD and GOLDENBERG had been solved by applying the Modified Grübler-Kutzbach criterion, and it is one of the main targets in this section.

The DELASSUS linkage[19] consists of a derivation of all four bar linkages in 13 different forms, whose joints are lower pairs with connectivity one[20–21]. Here we research the mobility of all the 13 DELASSUS linkages, shown in Fig. 1. The characteristic parameters of the 13 DELASSUS linkages are shown in Table 1.

Considering the first one in Table 1, a reference coordinate system o-xyz being introduced, assuming the z-axis coincides with the axis of the first screw joint, then, $\$_i = (0 \quad 0 \quad 1;\ y_i \quad -x_i \quad h_i)^{\mathrm{T}}, (i = 1, 2, 3, 4)$, here h_i is the pitch of the ith joint, (x_i, y_i) are the coordinates of the point in the axis of the ith joint. If all the h_i are equal to each other, and h_i=h≠0, then for the linkage, using Eq. (3), all the common reciprocal screws can be obtained $\$_{1\mathrm{r}} = (0 \quad 0 \quad 0;\ 1 \quad 0 \quad 0^{\mathrm{T}})$, $\$_{2\mathrm{r}} = (0 \quad 0 \quad 0;\ 0 \quad 1 \quad 0^{\mathrm{T}})$, $\$_{3\mathrm{r}} = (0 \quad 0 \quad -1;\ 0 \quad 0 \quad h)^{\mathrm{T}}$.

Based on Eq. (5), $\lambda = 3$ and $d = 6 - 3 = 3$. Here, there are three common-constraints and two independent non-common constraints, so $\nu = 0$ based on Eq. (6), and there is no partial freedom and half partial freedom in this mechanism, so $\xi = 0$. Then use the Modified Grübler-Kutzbach Criterion,

$$M = d(n-g-1) + \sum_{i=1}^{g} f_i + \nu - \xi = 3(4-4-1)+4=1.$$

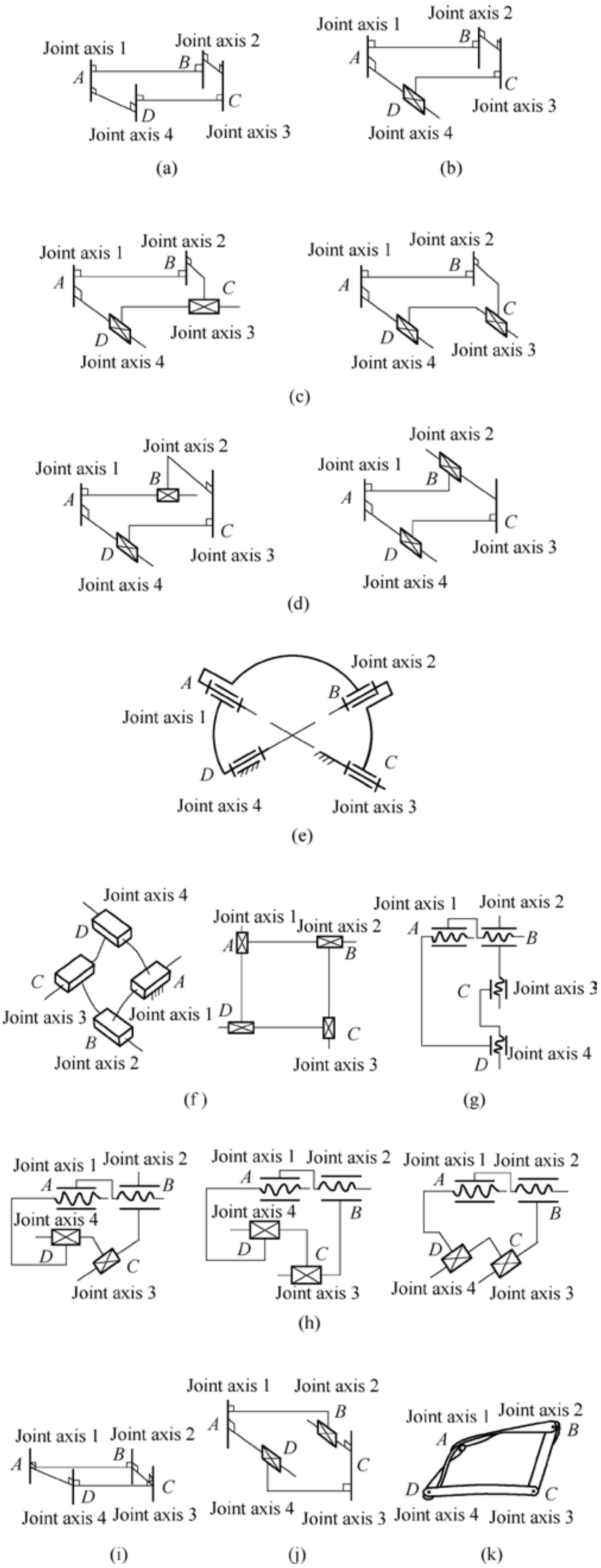

Fig. 1. DELASSUS mechanism including thirteen classic linkages

If h_i=0 (i=1, 2, 3, 4), all the common reciprocal screws are

$$\$_{1r} = (0 \quad 0 \quad 0; \quad 1 \quad 0 \quad 0)^{T},$$
$$\$_{2r} = (0 \quad 0 \quad 0; \quad 0 \quad 1 \quad 0)^{T},$$
$$\$_{3r} = (0 \quad 0 \quad 1; \quad 0 \quad 0 \quad 0)^{T}.$$

By the same way, we can get M=1.

The mobility of all the 13 DELASSUS linkages is researched by using the Modified Grübler-Kutzbach Criterion here, the similar analyzing process of each DELASSUS linkage is omitted considering the length of the paper, the data are listed in Table 1. This work is a new progress in the mobility research of the classic mechanisms or the single-loop overconstraint linkages using the Modified Grübler-Kutzbach Criterion.

3.2 Multi-loop parallel planar linkage

The mobility of the modern parallel mechanisms has been well resolved with the Modified Grübler-Kutzbach criterion [7, 9–12]. However, a new planar parallel manipulator[22] is found with special nature.

The planar parallel manipulator contains three branches, each of them with same structure, shown in Fig. 2. The moving platform is $D_1D_2D_3$, the fixed platform is the part with joints A_{ij}(i, j=1, 2, 3). All the joints are rotating pairs with parallel axes.

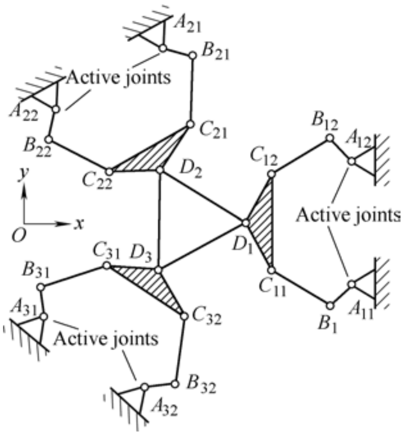

Fig. 2. Planar parallel mechanism

According to the Modified Grübler-Kutzbach Criterion, in the mechanism, the number of bodies is n=17; the number of the joints g=21. A reference coordinate system o-xyz being introduced, assuming the z-axis is parallel with the axes of these rotation joints, then $\$_i = (0 \quad 0 \quad 1; y_i \quad -x_i \quad 0)^{T} \quad (i = 1, 2, \cdots, 21)$. So, d=3 is the order of the motion of the manipulator, furthermore, there is no non-common constraints in this mechanism, so $\nu = 0$, according to Eq. (16), the mobility is

$$M = d(n-g-1) + \sum_{i=1}^{g} f_i + \nu - \xi =$$
$$3(17-21-1)+21-\xi = 6-\xi.$$

Table 1. Degree of freedom of parallel-screw linkages

Serial No.	Type	Condition		Rank d	Total No. of part n	Total No. of pair g	Total No. of pair mobility $\sum_{i=1}^{g} f_i$	Mobility of linkage M	Figure
1	H-H-H-H-	All axes parallel, pitches equal	$h_i \neq 0$	3	4	4	4	1	Fig.1(a)
			$h_i = 0$	3	4	4	4	1	
2	H-H-H-P-	Screw axes parallel, pitches equal, prism normal to screws	$h_i \neq 0$	3	4	4	4	1	Fig.1(b)
			$h_i = 0$	3	4	4	4	1	
3	H-H-P-P-	Screw axes parallel, pitches equal, prism normal to screws	$h_i \neq 0$	3	4	4	4	1	Fig.1(c)
			$h_i = 0$	3	4	4	4	1	
4	H-P-H-P-	Screw axes parallel, pitches equal, prism normal to screws	$h_i \neq 0$	3	4	4	4	1	Fig.1(d)
			$h_i = 0$	3	4	4	4	1	
5	R-R-R-R-	Spherical linkage-axes intersect in a single point		3	4	4	4	1	Fig.1(e)
6	P-P-P-P-	Spatial slider linkage		3	4	4	4	1	Fig.1(f)
		Planar slider linkage.		2	4	4	4	2	
7	H-H-H-H-	Screws form two parallel, coaxial pairs	$h_i \neq 0$	3	4	4	4	1	Fig.1(g)
			$h_i = 0$	3	4	4	4	1	
8	H-H-P-P-	Screws are coaxial and parallel to same plane as prisms	$h_i \neq 0$	2	4	4	4	2	Fig.1(h)
			$h_i = 0$	2	4	4	4	2	
9	H-H-H-H-	Screws are parallel and the common normals form a quadrilateral with two pairs of equal sides. Equal sides are adjacent. $h_1 = h_3 = (h_2 + h_4)/2$, where h_1, h_3 are the pitches of the screws lying in the symmetry plane of the quadrilateral, h_2, h_4 are the pitches of the remaining screws	$h_i = 0$	3	4	4	4	1	Fig.1(i)
			$h_i \neq 0$	3	4	4	4	1	
10	H-H-H-H-	The same as before, but the normals form a parallelogram. $h_1 + h_3 = h_2 + h_4$, numbering the axes in the usual manner		3	4	4	4	1	Fig.1(i)
11	H-H-H-H-	The same as before, but the normals form a crossed parallelogram. Pitches obey relationships $h_1 = h_3$, $h_2 = h_4$		3	4	4	4	1	Fig.1(i)
12	H-P-H-P-	The screws are parallel and have equal pitch. The directions of the prisms are symmetric with respect to the plane containing the screw axes		3	4	4	4	1	Fig.1(j)
13	R-R-R-R-	Bennett Linkage		3	4	4	4	1	Fig.1(k)

Note: H—Helical pair, P—Prismatic pair, R—Rotation pair

Here, a new definition should be proposed.

Half local freedom: If a part in a mechanism can receive the motions of its fore parts but can not transfer all the motions to its following-up parts, then the motions failed to transfer in the mechanism is called the half passive freedom. In Fig. 2, each sub-platform $D_iC_{i1}C_{i2}$ (i=1, 2, 3) has a half passive freedom for the rotation of the sub-platform $D_iC_{i1}C_{i2}$ (i=1, 2, 3) about the axis of joint D_i can not be transfered to the platform $D_1D_2D_3$. So, there are 3 degrees of half local freedom, ξ=3, M=3.

The number of active joints in the mechanism with half passive freedoms equals to the sum of the degrees of freedom of the output part and all the half passive freedoms. So, six active joints have been chosen, seen in Fig. 2. In other words, the screws of the 6 active joints are dependent, only three of them are independent. Thus, the number of the independent active joints equals to the degrees of freedom of the output part.

4 Conclusions

(1) The general validity of the Modified Grübler-Kutzbach Criterion for mobility is elaborated or demonstrated in both theory and practice.

(2) Some relative new terms, such as the half passive freedom, the non-common constraint space and the common motion space of a mechanism are proposed.

(3) The mobility of the classical over-constrained single-loop DELASSUS mechanisms classified into thirteen types is determined using the Modified Grübler-Kutzbach Criterion.

(4) The motion and active joints of a new planar parallel mechanism are obtained using the Modified G-K Criterion.

(5) The number of the independent active joints equals to the degrees of freedom of the output part, but in a comprehensive explanation, the number of total active joints equals to the degrees of freedom of the output part

plus the degrees of the half passive freedom.

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