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Type Synthesis of 4-DOF Parallel Kinematic Mechanisms Based on Grassmann Line Geometry and Atlas Method

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Abstract: Many methods are proposed to deal with the type synthesis of parallel kinematic mechanisms(PKMs), but most of them are less intuitive to some extent. Thus, to propose a concise and intuitive type synthesis method for engineering application is a very challenging issue, which should be further studied in the field. Grassmann line geometry, which can investigate the dimensions of spatial line-clusters in a concise way, is taken as the mathematic foundation. Atlas method is introduced to visually describe the degrees of freedom(DOFs) and constraints of a mechanism, and the dual rule is brought in to realize the mutual conversion of the freedom-space and constraint-space. Consequently, a systematic method based on Grassmann line geometry and Atlas method is generated and the entire type synthesis process is presented. Three type 4-DOF PKMs, i.e., 1T3R, 2T2R and 3T1R(T: translational DOF; R: rotational DOF), are classified according to the different combinations of the translational DOFs and rotational DOFs. The type synthesis of 4-DOF PKMs is carried out and the possible configurations are thoroughly investigated. Some new PKMs with useful functions are generated during this procedure. The type synthesis method based on Grassmann line geometry and Atlas method is intuitive and concise, and can reduce the complexity of the PKMs' type synthesis. Moreover, this method can provide theoretical guidance for other PKMs' type synthesis and engineering application. A novel type synthesis method is proposed, which solves the existing methods' problems in terms of complicated, not intuitive and unsuitable for practical application.

Key words: type synthesis, parallel kinematic mechanism, Grassmann line geometry, atlas method

1 Introduction

As the counterpart of serial kinematic mechanisms (SKMs), parallel kinematic mechanisms(PKMs) are featured by multi-closed-loop structure[1], i.e., the mobile platform is connected to the base through several kinematic chains and all the actuators are mounted in the base. Gough-Stewart platform is the most typical and famous PKM. Due to the multi-closed-loop structure, on one hand, PKMs possess some valuable potential such as low moving inertia, high load to weight ratio and high dynamic performance, and on the other hand, PKMs also suffer from strong kinematic coupling. For such reasons, PKMs have attracted extensive attentions from both academia and industry, and have been studied intensively from their appearance.

Theoretically, the strong coupling characteristic of PKMs will inevitably bring in complicate kinematics. This is a great challenge to the control and kinematic calibration technologies. In other words, the coupling characteristic is disadvantageous to the further improvement of accuracy. In contrast, the lower mobility PKMs[2–3], i.e., PKMs with fewer than six degrees of freedom(DOFs), are featured by relatively weak coupling performance when compared with the six-DOF PKMs. This is a great advantage for practical application. Therefore, PKMs with lower mobility are more attractive for the development of industrial robots. Their successful application cases have also demonstrated this fact.

Among the lower mobility PKMs, the 3-DOF PKMs have been extensively used in industry, such as the Delta in packaging, the Tricept, Exechon, Sprint Z3 and Hermes in machining, and the TriVariant in weld. Of note is that the successful application of Delta robot in packaging lines has greatly promoted the development and application of lower mobility PKMs. Actually, the 4-DOF PKMs, which are one of the most important classes of lower mobility PKMs, are the required mechanism configurations for the application in packaging production lines. In this application, three

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translational DOFs and one rotational DOF are generally necessary. Thus, to realize this function, a rotational DOF must be serially combined to the Delta robot. This increases the moving mass and lowers the dynamic performance.

To avoid the mentioned drawback and further improve the manipulation performance, the 4-DOF H4 PKM has been proposed. In the concept of the H4 mechanism, four kinematic chains are used and the parallelogram linages (similar to the architecture of Delta) are adopted. The H4 can realize the required function in packaging industry. However, the double-platform structure is used in H4 to increase the rotational capability. This structure increases the difficulty in manufacturing and the consequent cost, and is disadvantageous to the improvement of operation speed. Therefore, new configurations of 4-DOF PKMs should be investigated to provide more choices for industrial robots. In view of these, this paper will try to do some contribution, and deal with the type synthesis issue of 4-DOF PKMs.

Type synthesis is to derive the topological configuration[4] of a mechanism according to the requirement for DOFs. As is well known, topological configuration is fundamental and decisive to the comprehensive property of a mechanism, and will definitely affect the performance of the corresponding robots developed on the basis of this mechanism. Therefore, the type synthesis of a mechanism is the most critical step for the successful design of a robot. Moreover, an effective type synthesis method can guide the creation of more novel mechanisms with useful functions, which is a significant and challenging issue in the domain of PKMs[5–6].

The most commonly used method in this field is topological graphs(TG)[7]. A lot of work have been contributed to this area and many effective theories or methods dealing with type synthesis have been proposed, such as the methods based on screw theory[8–11], single-opened-chains units[12–13], G_F set[14], displacement group theory[15–18] using Lie Group Algebra, and theory of linear transformation and geometrical analysis[19]. All of the theories are very effective and have promoted the development of PKMs.

Based on these theories, lots of novel mechanisms have been generated and proposed. To carry out the type synthesis of PKMs under the guidance of these mentioned theories, one should grasp the relatively complicated procedures and the profound mathematical theory first. This is a disadvantage for their widespread application. This paper will try to introduce a more concise and intuitive method, and the Grassmann Line Geometry will be used as the mathematic foundation.

As is well known, Grassmann Line Geometry is a very concise mathematical theory investigating the geometrical features of spatial line-clusters. The linear dependence of line vectors can be classified and expressed by their dimensions. Under this description, Grassmann Line Geometry can be easily integrated with the relevant research of mechanism theory, especially the freedom and constraint. For such a reason, Grassmann Line Geometry has been successfully applied to singularity analysis[20–23] and flexible mechanisms' design[24–25]. In this paper, it will be used in the type synthesis of PKMs.

In order to integrate with the description method of Grassmann Line Geometry and to establish an intuitive and concise type synthesis method, a graphic expression approach will be introduced and used in this paper. Under this approach, the DOFs and constraints of a mechanism can be visually expressed. This graphic method is intuitive and also easy to understand for designer. Based on the mathematic foundation of Grassmann Line Geometry and the graphic expression method, a new type synthesis method will be proposed and the corresponding procedure will be presented in detail. On this basis, the type synthesis of 4-DOF PKMs will be carried out and the results will be listed sequentially.

The remainder of this paper is organized as follows. In section 2, the basic criterions of Grassmann Line Geometry are introduced first, the atlas method and the Blanding rules are summarized sequentially, thereafter, a type synthesis method based on Grassmann line geometry and atlas method is introduced, and the detailed procedure is also presented. In section 3, the type synthesis of 4-DOF PKMs is carried out on the basis of the proposed method and the different combinations of translational DOFs and rotational DOFs, and detailed synthesis process and the derived mechanism configurations are also provided. Conclusions are given in section 4.

2 Type Synthesis Method

2.1 Basic criterions and typical line-clusters of Grassmann Line Geometry

Using Grassmann Line Geometry to carry out the type synthesis and express the freedoms and constraints of PKMs, the dimensions of the corresponding line-clusters should be identified first. Some basic criterions, which lay the theoretical basis in the processing of this issue, are listed as follows:

(1) In a plane, there are at most three independent lines or only two independent parallel lines;

(2) Among all of the spatially parallel lines, there are only three independent parallel lines;

(3) Among all of the coplanar and concurrent lines, there are only two independent lines;

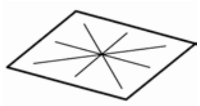
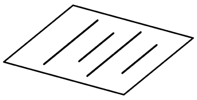
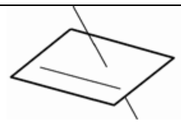
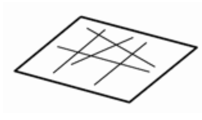
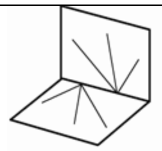
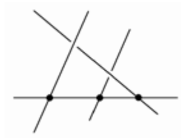
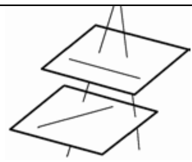
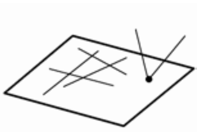
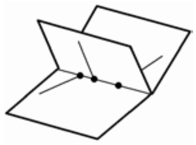
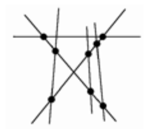
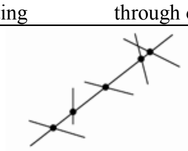
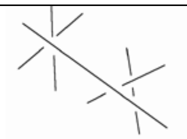
(4) Among all of the concurrent lines in space, there are only three independent lines;

(5) For two sets concurrent lines (or one set concurrent lines and one set parallel lines) in two different planes, if the intersections lie in the intersecting line of the two planes, there are only three independent lines;

(6) There are at most five independent lines in two or more parallel planes or planes that intersect at one line.

According to these criterions, the space line-clusters can be classified and expressed by their dimensions. Some typical line-clusters of Grassmann Line Geometry under different dimension are listed in Table 1.

Table 1. Typical line-clusters of Grassmann line geometry

Dimension	Typical line-clusters			
1				
2	 Planar concurrent	 Parallel and coplanar	 Non-coplanar	
3	 Spatial concurrent	 Coplanar	 Concurrent in two different planes	 Regulus of lines
4	 Non-parallel and non-intersecting	 Coplanar or passing through one point	 Concurrent with one line	 Concurrent with two lines
5	 Intersect with one line	 Non-singular line-cluster		

2.2 Atlas method and Blanding rules

Based on Grassmann Line Geometry, the basic physical meanings relevant to the type synthesis of a mechanism, i.e., constraint and freedom, should be brought in to the line-clusters, and the corresponding definitions are as follows: line without arrows denotes vector, line with double arrows denotes couple; red color denotes constraint; blue color denotes freedom. Then, four basic elements, which can be used to describe the motion and constraint in a mechanism, are generated and listed in Table 2.

Table 2. Basic elements in a line graph and their meanings

Basic element	Mathematic meaning	Physical meaning
	Vector	Rotational DOF
	Vector	Constraint force
	Couple	Translational DOF
	Couple	Constraint couple

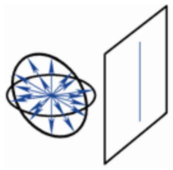
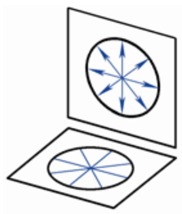
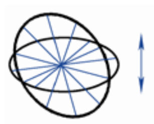
These elements can be expanded into a line graph[26], which can express an n-dimensional freedom space or constraint space[27]. The corresponding space line graph is the mathematic description of a mechanism's motions or constraints. This description is intuitive and concise, and the line graph also has clear and definite physical meaning. Therefore, the qualitative analysis of a mechanism's motions and constraints can be carried out in an intuitive way. In this paper, the process using freedom- or constraint-space line graph to describe mechanisms' motions or constraints is called Atlas Method.

This paper mainly focuses on the type synthesis of 4-DOF PKMs. To illustrate the application of the Atlas Method, some typical four-dimensional freedom spaces representing the DOFs or motions of 4-DOF mechanisms are presented in Table 3. The corresponding physical meanings are also included in this table.

In a mechanism, there exists close relationship between DOFs of the mobile platform and the constraints of all limbs. In other words, a close relationship exists between the corresponding freedom-space and constraint-space line graphs. The identification of this relationship is fundamental to the type synthesis of a mechanism. Based on Grassmann Line Geometry and Atlas Method, this paper brings in dual rule to uncover this issue and to deal with the type synthesis of PKMs on this basis.

Blanding proposed a basic dual rule that can be used to investigate the relationship between the freedom-space and constraint-space line graphs. This dual rule can be summarized as: provided that a line graph $\mathfrak{R}$ contains n independent or non-redundant lines, then the corresponding dual line graph $\mathfrak{R}'$ contains 6−n independent or non-redundant lines, and each line in $\mathfrak{R}$ intersects with all lines in $\mathfrak{R}'$.

Table 3. Some four-dimensional freedom spaces

Atlas	Physical meaning
	Three-dimensional translations and one-dimensional rotation
	Two-dimensional translations and two-dimensional rotations
	One-dimensional translations and three-dimensional rotation

According to the Blanding rule, the dual constraint-space line graph can be uniquely identified when the freedom-space line graph is given, and vice versa. Taking the physical meanings of motion or force into consideration, a generalized Blanding rule[24] directly reflecting the relationship between DOFs and constraints of a mechanism can be summarized as follows:

(1) The axes of rotational DOFs of a mechanism are orthogonal to the axes of all constraint couples and intersect with the lines of all constraint forces;

(2) The axes of translational DOFs of a mechanism are orthogonal to the lines of all constraint forces and are arbitrary to the axes of all constraint couples.

Using this rule can identify the dual relationship between the freedom-space line graph and the constraint-space line graph in a concise and intuitional way and realize the mutual conversion between freedom-space atlas and constraint-space atlas. Thereafter, the analysis upon DOFs and constraints of a mechanism can be carried out based on the two kinds of atlases. All of these efforts have laid important foundation for the type synthesis of mechanisms.

2.3 Type synthesis procedure

On the basis of the rules presented in the previous section, the technological process of the type synthesis using Grassmann Line Geometry and Atlas Method can be summarized as follows:

(1) Generate the freedom-space atlas according to the DOFs of the PKM to be designed;

(2) Identify the constraint-space atlas from the freedom-space atlas by using the dual rule;

(3) Determine the number of limbs (the number of limbs is not less than that of DOFs of the mechanism to be designed, actually, they are the same in most cases), then decompose the constraint space into constraint subspaces of limbs;

(4) Identify the corresponding freedom spaces for limbs from the constraint subspaces derived in last step using the dual rule;

(5) Configure the kinematic joints for each limb based on the derived freedom spaces and generate the kinematic chains of all limbs, then constitute the mechanism using all limbs;

(6) Check the movement continuity of the synthesized mechanism;

(7) If the generated mechanism has continuous movements, the type synthesis based on this subspace is finished; if the generated mechanism does not have continuous movements, the synthesis stage should be back to Step 5.

The entire technological process of the type synthesis is presented in Fig. 1.

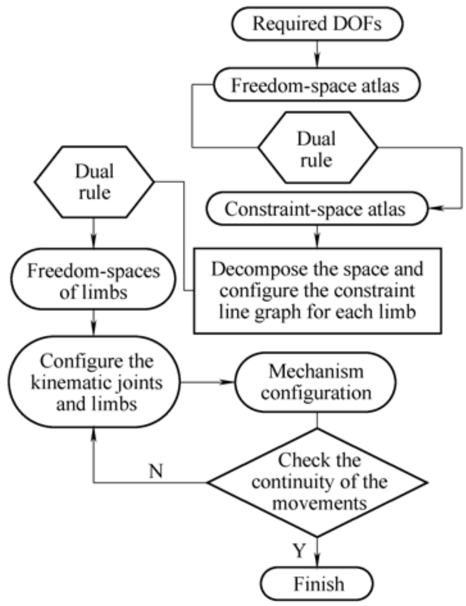


Fig. 1. Technological process of the type synthesis

3 Type Synthesis of 4-DOF PKMs

According to the difference of DOF-type, 4-DOF PKMs can be generally classified into 1T3R-type, 2T2R-type and 3T1R-type, where, T represents translational DOF and R represents rotational DOF. Based on this classification, this section will investigate the type synthesis of these mechanisms.

Here, the 2T2R-type PKMs are taken into consideration first. For this type, the translational plane and the rotational plane can be vertical or parallel, but the parallel situation is rarely used in practice. Therefore, the type synthesis of 2T2R-type PKMs discussed here is only focused on the vertical situation, and the specific type synthesis process is as follows.

For the vertical situation, the DOFs can be represented by the freedom-space atlas as shown in Fig. 2(a). Using the dual rule, the corresponding constraint-space atlas can be derived as shown in Fig. 2(b). This atlas represents one-dimensional force and one-dimensional couple, and the axes of the force and the couple are perpendicular to each other.

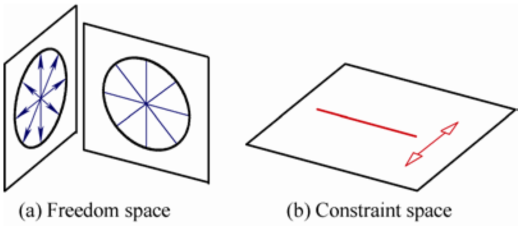
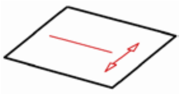
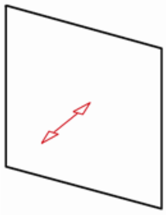
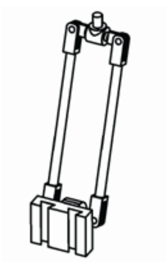
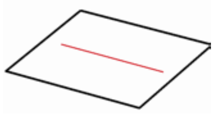
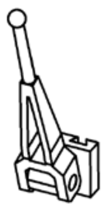
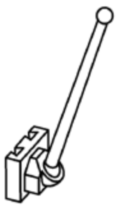


Fig. 2. Atlases of freedom and constraint space

For the constraint-space atlas presented in Fig. 2(b), two one-dimensional constraint spaces(i.e., one-dimensional force and one-dimensional couple) can be decomposed and assigned to the limbs. In this synthesis process, the configurations of four limbs and an active prismatic joint in each limb are used. Then, there is one-dimensional couple constraint for one limb, one-dimensional force constraint for another limb, and no constraint for the other limbs. The constraint-space atlas of each limb is presented in Table 4.

Table 4. Type synthesis process

	Constraint-space atlas of each limb	Model of each limb
1th limb	 One-dimensional couple constraint	 PR(Pa)RR limb
3th limb	 One-dimensional force constraint	 PRS limb
2th and 4th limbs	Null No constraint	 PSS limb

According to the dual rule, the freedom-space atlas of the first limb is composed of three-dimensional translations and two-dimensional rotations. Such a freedom-space can be realized by a PR(Pa)RR(P: prismatic joint; R: revolute joint; Pa: parallelogram linkage) limb. Similarly, the freedom-space atlas of the third limb is composed of two-dimensional translations and three-dimensional rotations and can be realized by a PRS(S: spherical joint) limb. The second and the fourth limbs are six-dimensional freedom spaces and can be realized by PSS or PUS(U: universal joint) limbs. The CAD models of all limbs are also presented in Table 4.

The four limbs can constitute a 4-DOF spatial parallel mechanism as shown in Fig. 3(a), and the kinematic scheme is presented in Fig. 3(b). Based on the CAD model and the kinematic scheme, it can be confirmed that this mechanism has continuous movements.

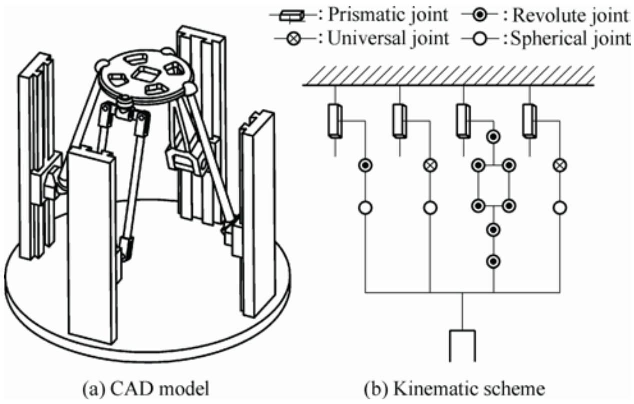


Fig. 3. Type synthesis results

Note that, the CAD model shown in Fig. 3(a) is not the unique configuration for the kinematic scheme presented in Fig. 3(b). When the directions of the constraint force and couple are different from that presented in Table 4, the arrangement of the limbs should be different accordingly. The mechanism shown in Fig. 4 is an example. The mechanisms in Fig. 3(a) and Fig. 4 have the same kinematic scheme as shown in Fig. 3(b).

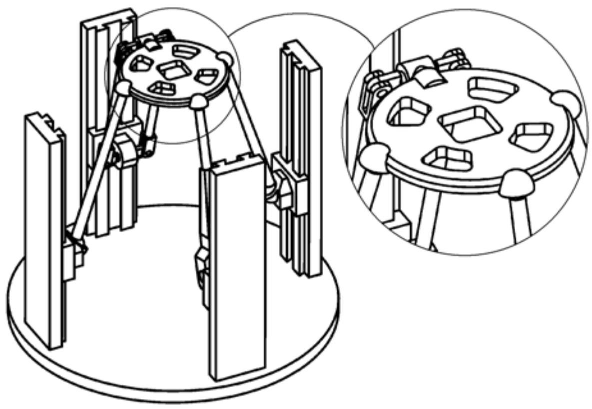
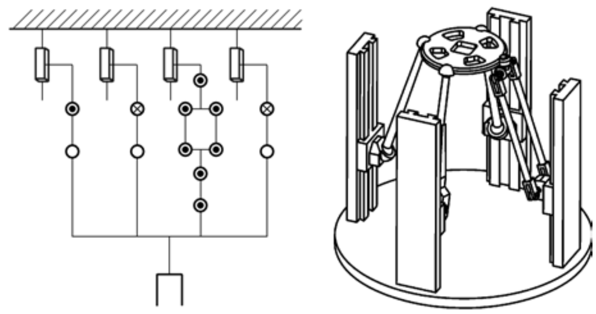


Fig. 4. New mechanism with different configuration

The arrangement order of the limbs can also affect the result. For example, the kinematic scheme shown in Fig. 5(a) has the same kinematic chains with that in Fig. 3(b), but the order of the chains are changed, and then a different mechanism as shown in Fig. 5(b) can be generated.

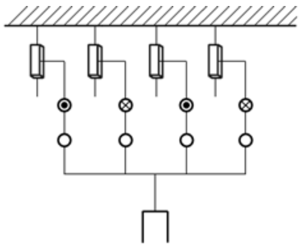
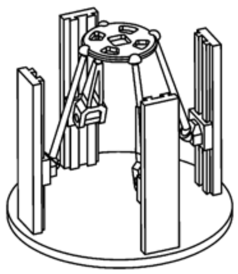
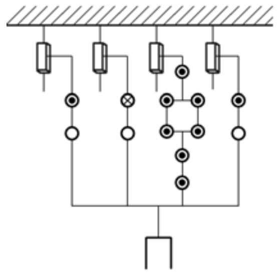
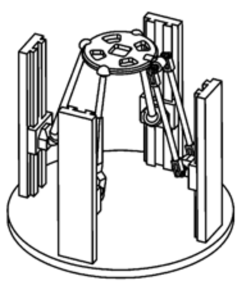
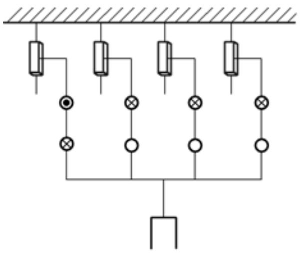
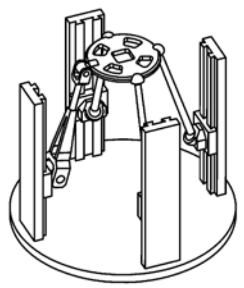


(a) Kinematic scheme (b) CAD model

Fig. 5. Type synthesis results

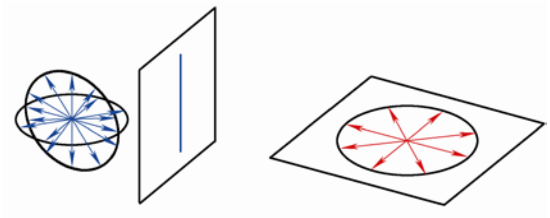
Similarly, other 2T2R-type PKMs can be synthesized according to the same process. Some mechanisms and their kinematic scheme and models are presented in Table 5, in which, P represents that this prismatic joint is active.

Table 5. 2T2R-type mechanisms

Kinematic Scheme	Model
 PRS-PSS-PRS-PSS	
 PRS-PRPaRR-2PSS	
 PRU-3PSS	

The type synthesis of 3T1R-type PKMs can be carried out in a similar process. For this situation, the freedom-space and the corresponding constraint-space atlases can be represented by the graphs provided in Figs. 6(a) and 6(b).

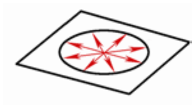
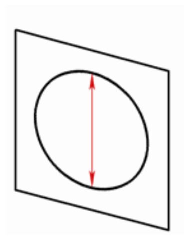
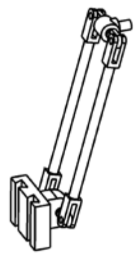
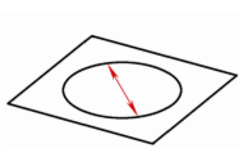
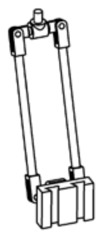
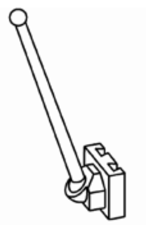
The atlas shown in Fig. 6(a) represents one-dimensional rotational DOF and three-dimensional translational DOFs, and the atlas shown in Fig. 6(b) represents the corresponding two-dimensional constraint couples. Such a constraint atlas can be composed into two one-dimensional constraint couples and the directions of the two couples are perpendicular to each other. As shown in Table 6, a vertical couple constraint is assigned for the first limb and the a horizontal couple constraint is assigned for the second limb, and no constraint exert is exerted upon the other two limbs. The freedom spaces of the first two limbs are two five-dimensional freedom spaces and can be realized by PR(Pa)RR limbs, respectively. The last two limbs can be realized by PSS limbs. The corresponding CAD models are also included in Table 6.



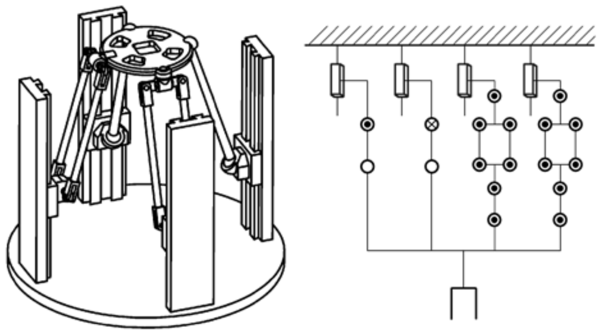
(a) Freedom space (b) Constraint space

Fig. 6. Atlases of 3T1R-type PKMs

Table 6. Type synthesis process of 3T1R-type PKMs

	Constraint-space atlas of each limb	Model of each limb
1th limb	 One-dimensional couple constraint	 PR(Pa)RR limb
2th limb	 One-dimensional couple constraint	 PR(Pa)RR limb
3th and 4th limbs	Null No constraint	 PSS limb

By combining the four limbs, a new mechanism can be generated as shown in Fig. 7(a). The corresponding kinematic scheme of this mechanism is presented in Fig. 7(b). The derived mechanism has continuous movements.

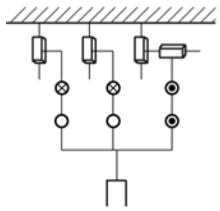
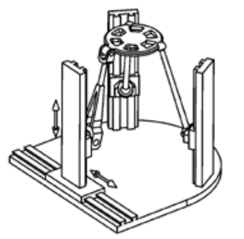
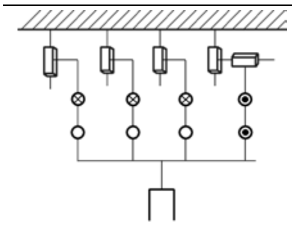
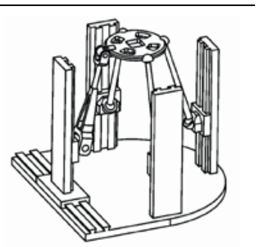
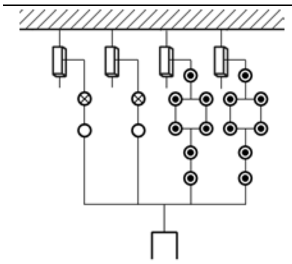
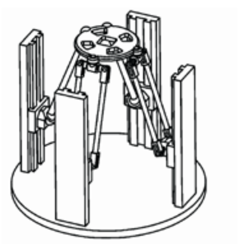


(a) CAD model (b) Kinematic scheme

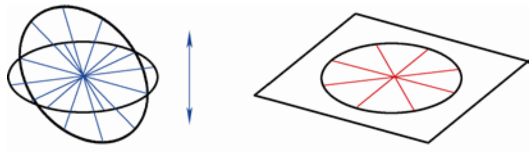
Fig. 7. 3T1R-type parallel kinematic mechanism

Some other 3T1R-type PKMs can also be generated in this process, the specific synthesis process will not be provided here. Some typical mechanisms are listed in Table 7, and the kinematic schemes of the synthesized mechanisms are given in this table.

Table 7. 3T1R-type mechanisms

Kinematic Scheme	Model
 $\underline{P}_v\underline{P}_h$RR-2$\underline{P}$SS	
 $\underline{P}_v\underline{P}_h$RR-3$\underline{P}$SS	
 2$\underline{P}$RPaRR-2$\underline{P}$SS	

For the 1T3R-type PKMs, the freedom space can be represented by the graph shown in Fig. 8(a). It is a spatial space composed of three-dimensional rotations and one-dimensional translation. Then, the constraint space can be generated as shown in Fig. 8(b), this is a two-dimensional force constraint.

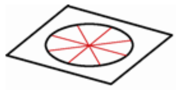
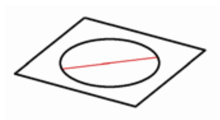
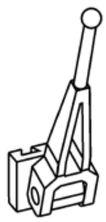
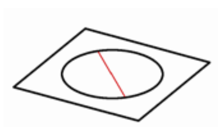
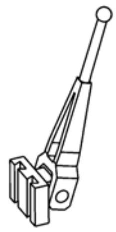
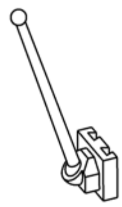


(a) Freedom space (b) Constraint space

Fig. 8. Atlases of 3T1R-type PKMs

For the constraint space presented in Fig. 8(b), it can be directly composed into two one-dimensional forces with their directions perpendicular to each other. By using the first two limbs to realize the two forces, the type synthesis process can be summarized as shown in Table 8. When the two forces are assigned to one limb and other limbs are exerted no constraint, the process is presented in Table 9.

Table 8. Type synthesis process of 1T3R-type PKMs (case I)

	Constraint-space atlas of each limb	Model of each limb
1th limb	 One-dimensional force constraint	 PRS limb
2th limb	 One-dimensional force constraint	 PRS limb
3th and 4th limbs	Null No constraint	 PSS limb

As shown in Table 8, the one-dimensional force constraint can be realized by PRS limb. The four limbs can constitute a new PKM as given in Table 10, and this mechanism can be represented by 2$\underline{P}$RS-2$\underline{P}$SS.

For the type synthesis process given in Table 9, the first limb is exerted two-dimensional forces, and can be realized by a PS kinematic chain. By combining the four limbs listed in Table 9, a 3$\underline{P}$SS-$\underline{P}$S PKM can be derived which is also included in Table 10.

As previously discussed, the results presented in this paper are just some typical mechanisms, and the list is not exhaustive.

Table 9. Type synthesis process of 1T3R-type PKMs (case II)

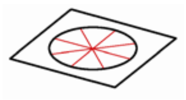
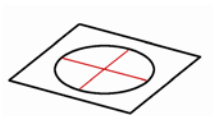
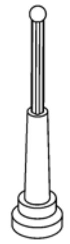
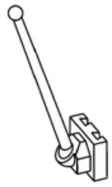
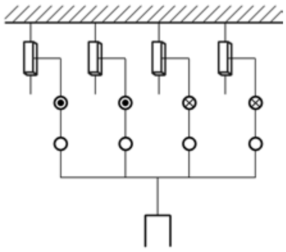
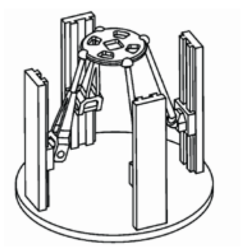
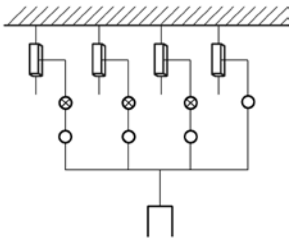
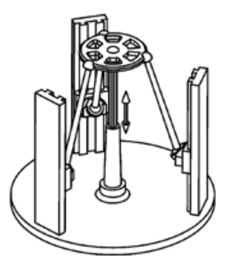
	Constraint-space atlas of each limb	Model of each limb
1th limb	 One-dimensional force constraint	 PS limb
2th, 3th and 4th limbs	Null No constraint	 PSS limb

Table 10. 1T3R-type mechanisms

Kinematic Scheme	Model
 2PRS-2PSS	
 3PSS-PS	

For the limitation of the presented paper, the application of the generated mechanisms will not be discussed here. But, some typical applications have been proved in industry, such as the 3T1R-type PKMs have promising prospects in packing production line to realize the rapid pick-and-place manipulation, and the 1T3R-type PKMs are suitable to be used to develop the segment assembly robot in shield tunneling machine. Therefore, the discussion on the use of the PKMs listed in this paper will be presented in our future work.

4 Conclusions

(1) Based on Grassmann line geometry and atlas method, a systematic type synthesis method for PKMs is proposed, and the technological process of the type synthesis is presented in detail.

(2) The DOFs and constraints of a mechanism are visually described by atlas method, and the mutual conversion of freedom-space atlas and constraint-space atlas is realized by bringing in the dual rule.

(3) The type synthesis of 4-DOF PKMs is carried out and some derived typical 4-DOF PKMs are presented. The corresponding CAD models and kinematic schemes are also provided. The presented method can be used in the type synthesis of other DOF-type PKMs.

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