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Novel Mobility Formula for Parallel Mechanisms Expressed with Mobility of General Link Group

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Abstract: The determination of virtual constraints is always one of the key and difficult problems in traditional mobility calculation. To make mobility calculation simple, considering avoiding virtual constraints, some new formulae have been presented, however these formulae can hardly intuitively reflect general link group's restrictions on output member and its influences on independence of output parameters, which is premise to the judgment of the properties of mobility. Towards the problem to reveal the intrinsic relationship between the degree of freedom(DOF) of a mechanism, the link group, and the dimension of output parameters, also to avoid determination of virtual constraint, based on the new concepts of the "DOF of general link group" and "node parameters", a new formula in the calculation of the mobility of mechanisms is presented that is expressed with DOFs of the general link groups and rank of motion parameters of base point of the output link. It is named GOM(mobility of groups and output parameter) formula. On the basis of new concepts of "effective parameters" and "invalid parameters", a rule is put forward for solving the DOF of mechanisms with invalid parameters by GOM formula, that is, the base point parameters are the subset of effective parameters of link group. Thereafter, several examples are enumerated and the results coincide with the prototype data, which proves the validity of the proposed formula. Meanwhile, it is obtained that the necessary and sufficient condition for the judgment of output parameters independence is that each of the DOF of the link group is not less than zero. The proposed formula which is simple in calculation provides theoretical basis for the judgment of independence of output parameters and provides references for type synthesis of novel parallel mechanisms with independence requirements of their output parameters.

Key words: parallel mechanism, degree of freedom, general link group, output parameters

1 Introduction

The research on the calculation of the mechanism mobility has been done over the past 150 years, and as many as 35 different formulas/approaches are presented[1]. To get the degree of freedom(DOF) of a mechanism, the number of overconstraint must be determined correctly. As early as 1929 KUTZBACH expended GRÜBLER's formula for mobility of the planar mechanisms, and gave a new mobility formula for spatial mechanisms, of which the order, the number of members and joints are the same for all independent loops. It is well known as "G-K criterion"[2–3]. It has successfully solved overconstraint of both single loop bodies and some multi-loop mechanisms. In 1954, MOROSKINE[4] proposed a formula with the rank of linear homogeneous set of equations defining the kinematic constraints and the number of scalar kinematic parameters of the mechanism to calculate the mobility of a planar or spatial mechanism. This method has great generality in the abstract, however, it is comparatively difficult to get the rank. VOINEA, et al[5], and HUNT[6] proposed a formula for mobility calculation of mechanisms having independent closed loops with different ranks in 1960. But this formula does not work for some complex mechanisms, because of the existence of a special overconstraint, also named virtual constraint, see for example shown in Fig. 9. In order to solve the problem, BAGCI, et al[7–12], proposed formulae in different forms containing virtual constraint items. Among these methods, screw theory selected as the mathematical tool to calculate the mobility and to analyze the singular configuration has attracted great interest, and for many years, scholars have made important contributions[13–22]. In this domain HUANG,

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et al, proposed the "modified G-K criterion", which has been successfully applied to solve the mobility of parallel mechanisms, type synthesis and some other problems based on screw theory with the support by National Natural Science Foundation of China[23]. And in 2010 HUANG received "IFToMM" award. However this method requires a complex mathematical tool, screw theory, to deal with virtual constraints. In recent years, many scholars shifted to the formulae for quick calculation of mobility, and presented the following formulae with which determination of virtual constraints can be avoided.

In 2003, RICO extended the works of HERVE, et al, and proposed a formula, containing the displacement parameters of output member, to calculate the mobility of a single-loop closed mechanism. Later, in 2006, the following formula for multi-loop parallel mechanisms was presented by RICO[24–26], which renders determination of virtual constraints unnecessary:

$$F=\sum f_i-\sum_{j=1}^{k}\dim(\boldsymbol{A}_j^{m/f})+\dim(\boldsymbol{A}_a^{m/f}), \tag{1}$$

where F is the number of degrees of freedom, $\boldsymbol{A}_j^{m/f}$ is the closure algebra of the moving platform with respect to the fixed platform when they are connected only by the jth connector serial chain, $\boldsymbol{A}_a^{m/f}$ is the absolute closure algebra of the moving platform with respect to the fixed platform.

In 2005, GOGU, et al[27–28], proposed a new method for mobility calculation of multi-loop mechanisms by the method of linear transformation and the formula is expressed as

$$F=\sum f_i-\left[\sum_{i=1}^{k}\dim(\boldsymbol{R}_{Gi})-\dim(\bigcap_{i=1}^{k}\boldsymbol{R}_{Gi})\right], \tag{2}$$

where $\boldsymbol{R}_{Gi}$ denotes the limb motion parameters. With this method, the structural synthesis for many robotic manipulators has been done.

In 2008, a general DOF formula which can avoid calculating virtual constraints was presented[29–30]:

$$\begin{cases}F=\sum_{i=1}^{m}f_i-\sum_{j=1}^{v}\xi_{L_j},\\ \xi_{L_J}=\operatorname{rank}\left[\left(\bigcap_{i=1}^{j}\boldsymbol{M}_{bi}\right)+\boldsymbol{M}_{b(j+1)}\right],\end{cases} \tag{3}$$

where F is the DOF of a single-loop chain, $\sum_{i=1}^{m}f_i$ is the total DOF of all the kinematic pairs of the single-loop chain, ν is the number of independent loops of a mechanism, ξ_{L_j} is the rank of the jth independent loop composed of the (j+1)th branch and the equivalent single-open chain of sub-parallel mechanism with first j branches, $\boldsymbol{M}_{bi}$ is the position and orientation characteristic matrix of ith branch and $\boldsymbol{M}_{b(j+1)}$ is the position and orientation characteristic matrix of the (j+1)th branch.

In 2012, YANG, et al[31–32], gave another form of expression on the basis of Eq. (3). It was extended in the form:

$$F=\sum_{i=1}^{m}f_i-\sum_{k=1}^{(\nu+1)}\dim(\boldsymbol{M}_{b_k})+\dim(\bigcap_{k=1}^{(\nu+1)}\boldsymbol{M}_{b_k}), \tag{4}$$

where $\boldsymbol{M}_{b_k}$ is the position and orientation characteristic set of movable platform relative to the fixed platform of the kth branch when the two platforms are connected only by the kth branch.

On the basis of this method, some research about type design also has been done[33].

The authors have proposed the new concepts of expanded link group, virtual-loop rank and virtual-loop constraints in 2010 and presented two novel general DOF formulae[34–35]:

$$F=\sum_{i=1}^{P}f_i-\sum_{j=1}^{L}d_j^X, \tag{5}$$

where $\sum_{i=1}^{P}f_i$ is the sum of all pair freedoms in the ith link group, d_j^X is the reduced freedoms when the jth link group and generalized pair are closed, L is the number of independent virtual loops.

$$F=6n-\sum_{i=1}^{P_Z}p_i+\sum_{j=1}^{L}m_j^X, \tag{6}$$

where $\sum_{i=1}^{P_Z}p_i$ is the sum of all pair constraints in the zth link group, m_j^X is the number of constraints of the jth virtual loop.

In a word, these earlier works mentioned above are the gratifying achievements made in mobility calculation in recent years. The common merit of Eqs. (1), (2) and (4) is that the displacement parameters and output parameters are all contained. However, the displacement parameters of general link group being not considered by each single link group, the influencing action of each link group on the mobility and output parameters has not been reflected intuitively. To more fully reflect the relationship between the output parameters, the DOFs of mechanisms and of link groups, a new formula, named GOM formula(mobility of groups and output parameter), is proposed in this paper. It is expressed with the mobility of general link group and the rank of output parameters.

2 Displacement Output Parameter of the Output Link and Its Rank

The general motion of a structural member M can be decomposed into translation with a reference point and rotation about the point. Reference point usually represents the character of kinematic pair in the end of the link group, and can be on the extended axis of the kinematic pair. It is referred to as node. At the given coordinate, the displacement parameters(linear and angular displacement parameters) of link group where the node locates are short for node parameters. It is the total displacement parameters of link group before its end part is restricted by other link groups at the given coordinate. Points C, D and K of the mechanism shown in Fig. 1 are all called nodes. Node parameter of the kth link group expressed as $\boldsymbol{G}_k^{gz}$ is named displacement matrix. Superscript symbol, gz, is the meaning of "link group", which can also be replaced by the real name of a link group. For example, the matrix of link group AB can be written as $\boldsymbol{G}_k^{AB}$. $\boldsymbol{G}_k^{gz}=(\alpha\ \beta\ \gamma, x\ y\ z)$, where $\alpha\ \beta\ \gamma$, $x\ y\ z$ are formal parameters, and represent the rotations around and translations along x-, y-, z-axes, respectively. The value of the formal parameter can be either 0 or 1. 0 denotes nonexistence of the movement while 1 denotes existence. Dimension of $\boldsymbol{G}_k^{gz}$ is expressed by d_k^{gz}, namely, $d_k^{gz}=\dim(\boldsymbol{G}_k^{gz})=\dim(\alpha\ \beta\ \gamma, x\ y\ z)$, which can also be written as $d_k^{gz}(\alpha\ \beta\ \gamma, x\ y\ z)$ for short. In order to distinguish the different link groups in the same mechanism, rank d_k^{gz} of link group AB can be written as d_k^{AB}.

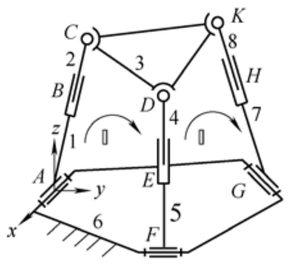

Fig. 1. 3-RPS mechanism

A particular size and a relative position of the pairs will make some of the motion parameters of link group insignificant, and such useless parameters are called invalid parameters, or otherwise effective parameters.

In the reference system, the point, determined by the intersection of the effective parameters of link group, is named base point, which is usually located in the node with the least number of motion parameters. For different mechanisms the base point may be the only one or different ones. Base point parameters can be expressed with a matrix ${}^N\boldsymbol{O}_B^{M,J}$, where M denotes the output link, B the base point, J the fixed link, N the location of the node. ${}^N\boldsymbol{O}_B^{M,J}=(\alpha\ \beta\ \gamma, x\ y\ z)$, where $\alpha\ \beta\ \gamma$, $x\ y\ z$ are all formal parameters used to represent the rotations around and translations along x-, y-, z-axes, respectively. The value of the formal parameter also is either 0 or 1. ${}^N d_B^{M,J}$ is used to represent the dimension of this matrix(in short, rank), i.e. ${}^N d_B^{M,J}=\dim({}^N\boldsymbol{O}_B^{M,J})$, and can be written simply as ${}^N d_B^{M,J}(\alpha\ \beta\ \gamma, x\ y\ z)$.

The output parameters of output member can be all independent ones or partly independent. We call the parameters including both independent and dependent ones mixing output parameters(mixing parameters for short).

In order to reflect the difference of motion parameters between different types of output links, the first-class output parameter is defined if the parameters of each node have the same number and character, or otherwise, the second-class one. For example member 3 in the mechanism shown in Fig. 1 is selected as the output member, and its output parameter belongs to the second-class and C is the base point, $d_1^{AC}(\alpha\ \beta\ \gamma, 0\ y\ z)=5$, and its node parameters of D and K are $d_2^{DF}(\alpha\ \beta\ \gamma, x\ y\ z)=6$ and $d_3^{KG}(\alpha\ \beta\ \gamma, x\ y\ z)=6$, respective. In Fig. 2, link 2, the connecting rod is taken as the output one. As the displacement parameters of nodes B and C are $(\alpha\ 0\ 0, 0\ y\ z)$, both of them belonging to the first-class output parameters. Thus point B can be selected as the base point, and ${}^N d_B^{M,J}(\alpha\ 0\ 0, 0\ y\ z)=3$, certainly, point C can also be selected as the base one.

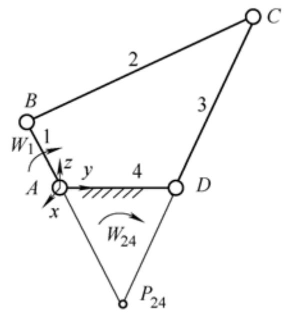

Fig. 2. Planar 4R mechanism

For the kth link group, its base point parameters ${}^N\boldsymbol{O}_B^{M,J}$ are included in node parameters $\boldsymbol{G}_k^{gz}$, namely

$$\boldsymbol{G}_k^{gz} \supseteq {}^N\boldsymbol{O}_B^{M,J}. \tag{7}$$

Base point parameters ${}^N\boldsymbol{O}_B^{M,J}$ are the subset of effective parameters of $\boldsymbol{G}_k^{gz}$ of link group, which is in accord with

$${}^N\boldsymbol{O}_B^{M,J}=\bigcap_{k=1}^{L+1}\boldsymbol{G}_k^{gz}. \tag{8}$$

The expression is called the intersection rule of effective parameters of link group, or simply intersection rule. The dimension of base point parameters is

$${}^N d_B^{M,J}=\dim\left(\bigcap_{k=1}^{L+1}\boldsymbol{G}_k^{gz}\right). \tag{9}$$

It is noteworthy that for the second-class output parameter, the establishment of the coordinate system must follow the principle of "the least motion parameters", or else it may become first-class output parameter. The result of the DOFs will not be changed, but the calculation process may be affected and the judgment of output type of the links also will be confused.

3 Formula Expressed with DOF of Link Group for Mechanism Mobility

For the kth link group in a parallel mechanism, the mobility, denoted as F_k, can be calculated as

$$F_k = \sum_{i=1}^{P_k} f_i - d_k^{gz}, \tag{10}$$

where P_k is the total number of kinematic pairs in the kth link group, and f_i is the DOF of the ith pair.

There are L+1 link groups in a parallel mechanism with L independent loops. The relationship between F, F_k, and ${}^N d_B^{M,J}$, can be expressed as

$$F = \sum_{k=1}^{L+1} F_k + {}^N d_B^{M,J}. \tag{11}$$

It is named GOM formula.

4 Several Applications of GOM Formula

Through an analysis of the following mechanisms, the functions and implication of GOM formula will be explained.

Example 1. For the 4R mechanism shown in Fig. 2, in which BC is the output member and B is the base point, the rank is ${}^N d_B^{M,J} = {}^B d_B^{2,4}\,(\alpha\ 0\ 0, 0\ y\ z)$=3. The rank of link group AB is $d_1^{AB}\,(\alpha\ 0\ 0, 0\ y\ z)$ =3, F_1= $\sum_{i=1}^{P_1} f_i - d_1^{AB}$ =2 – 3=– 1. For link group CD d_2^{CD} $(\alpha\ 0\ 0,\ 0\ y\ z)$ =3, F_2=$\sum_{i=1}^{P_2} f_i - d_2^{CD}$=2 – 3=– 1.

$\dim\left[\boldsymbol{G}_1^{AB}\ (\alpha\ 0\ 0, 0\ y\ z) \cap \boldsymbol{G}_2^{CD}\ (\alpha\ 0\ 0, 0\ y\ z)\right] = {}^N d_B^{M,J}$ $(\alpha\ 0\ 0, 0\ y\ z)$=3 can be obtained by Eq. (9), which is in accord with the intersection rule of output parameters. F=F_1+F_2+ ${}^N d_B^{M,J}$ =(– 1)+(– 1)+3=1. It means of the output parameters, α, y, z, only one of which can be selected as the independent parameter, and the rest are related ones. This example shows that when member 2 is the output one, only by taking the number of the mixing parameters as the dimension of output parameters can the right result by Eqs. (11), (1), (2) and (4) be obtained.

Next this example will be analyzed with independent output displacement parameters. Let R and T be the motions of rotation and translation respectively, n_R and n_T be the number of R and T respectively, d^D the dimension of the independent motion, $d^D = n_R + n_T$. For this mechanism, there is only an independent rotation around point P_{24}, that is n_R=1, $d^D = n_R + n_T$=1+0=1. Substituting this result into Eq. (11), the result is $F = F_1$+ F_2+ d^D = – 1+(– 1)+1= – 1. That is wrong without exception. Clearly, the dimension of output parameter calculated by Eq. (11) is the dimension of mixing parameters, rather than that of independent parameters.

Therefore, the motion of the link should be expressed with the mixed output parameters rather than with independent parameters, which is of great importance in the context of structure synthesis theory. With this concept, more types of mechanisms will be synthesized in structure design.

Example 2. A 6R mechanism is depicted in Fig. 3, where $AB//CD//EF$, AB=CD=EF, and member 2 is taken as the passive one. During the motion of the mechanism, BE always keeps parallel with and equals to AF, thus member 2 makes a translational movement, the output parameters of which belong to the first-class. Point B can be selected as a base point and ${}^N d_B^{M,J} = {}^B d_B^{2,5}\,(0\ 0\ 0, 0\ y\ z)$=2. The rank of link group AB is gotten as $d_1^{AB}\,(\alpha_1\ 0\ 0, 0\ y\ z)$=3. The actual displacement parameters of link groups CD and EF are $d_2^{CD}\,(\alpha_2\ 0\ 0, 0\ y\ z)$ =3 and $d_3^{EF}\,(\alpha_3\ 0\ 0, 0\ y\ z)$ =3, respectively. The intersection obtained with their actual parameters is ${}^N d_B^{M,J}$ =$\dim\left[\boldsymbol{G}_1^{AB}\ (\alpha_1\ 0\ 0, 0\ y\ z) \cap \boldsymbol{G}_2^{CD}\ (\alpha_2\ 0\ 0, 0\ y\ z) \cap \boldsymbol{G}_3^{EF}\ (\alpha_3\ 0\ 0, 0\ y\ z)\right] = {}^N d_B^{M,J}$ $(\alpha\ 0\ 0, 0\ y\ z)$=3. Obviously, this result does not match with the real movement of member 2. This wrong result is ascribed to the following reason: since α_1, α_2, α_3 are equal to each other, namely, α_1=α_2=α_3=α, only one of them is independent, the other two are invalid parameters. That is to say, the rank of effective parameters of link group CD is $d_2^{CD}\,(0\ 0\ 0, 0\ y\ z)$=2. Similarly, $d_3^{EF}\,(0\ 0\ 0, 0\ y\ z)$=2. A result is achieved that F_1=$\sum_{i=1}^{P_1} f_i - d_1^{AB}$ =2 – 3=– 1, F_2= $\sum_{i=1}^{P_2} f_i - d_2^{CD}$ =2 – 2=0, F_3= $\sum_{i=1}^{P_3} f_i - d_3^{EF}$ =2 – 2=0. The mobility of the mechanism is F=F_1+F_2+F_3+ ${}^N d_B^{M,J}$ =– 1+0+0+2=1.

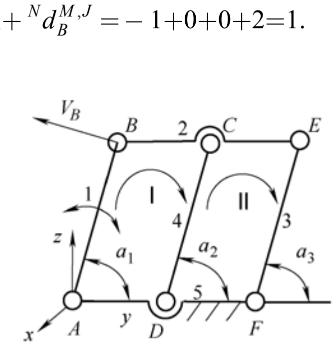

Fig. 3. Planar 6R mechanism

Through the above analysis, a conclusion can be drawn: base point parameters are the subset of effective parameters, rather than actual parameters of link groups, which is called the intersection rule of effective parameters.

It is noteworthy that because of a translational movement member 2 makes, it only has an instantaneous one-dimensional independent displacement, namely, $n_T=1$, thus $d^D=n_R+n_T=1$. Substituted into Eq. (11), a result is achieved. $F=F_1+F_2+F_3+d^D=-1+0+0+1=0$. This value is clearly not correct. It is obvious that, only using mixing parameters, ${}^{B}d_B^{2,4}\left(0\,0\,0,0\;y\;z\right)=2$, can we obtain the right result.

It is proved once again that using mixing output parameters is of great significance and application value.

Example 3. Fig. 1 is the 3-RPS mechanism. Component 3 is the output member, of which the output parameters belong to the second-class. Taking C as the base point, the following can be obtained: ${}^{N}d_B^{M,J}={}^{C}d_B^{3,6}\left(\alpha\;\beta\;\gamma,0\;y\;z\right)=5$. The rank of link group AC is $d_1^{AC}\left(\alpha\;\beta\;\gamma,0\;y\;z\right)=5$, meanwhile those of DF and KG are $d_2^{DF}\left(\alpha\;\beta\;\gamma,x\;y\;z\right)=6$ and $d_3^{KG}\left(\alpha\;\beta\;\gamma,x\;y\;z\right)=6$, respectively. $\dim\left[\boldsymbol{G}_1^{AC}\left(\alpha\;\beta\;\gamma,0\;y\;z\right)\cap\boldsymbol{G}_2^{DF}\left(\alpha\;\beta\;\gamma,x\;y\;z\right)\cap\boldsymbol{G}_3^{KG}\left(\alpha\;\beta\;\gamma,x\;y\;z\right)\right]={}^{N}d_B^{M,J}\left(\alpha\;\beta\;\gamma,0\;y\;z\right)=5$ is drawn from Eq. (9), which meets the intersection rule of output parameters. $F_1=\sum_{i=1}^{P_1}f_i-d_1^{AC}=5-5=0$, $F_2=\sum_{i=1}^{P_2}f_i-d_2^{DF}=5-6=-1$, $F_3=\sum_{i=1}^{P_3}f_i-d_3^{KG}=5-6=-1$. Thus, $F=F_1+F_2+F_3+{}^{N}d_B^{M,J}=0+(-1)+(-1)+5=3$ is attained.

It can be seen that the two link groups, AC and DF, are with the identical structure, but with different DOFs($F_1=0$, $F_2=-1$). This result shows that the DOF of the link group is not only related to the structure, but also related to their relative position in the coordinate system.

Example 4. The 2R/3R/RP planar mechanism is shown in Fig. 4, in which member 2 is the output link and C is the base point, the rank is ${}^{N}d_B^{M,J}={}^{C}d_B^{M,J}\left(\alpha\;0\;0,0\;y\;0\right)=2$, which belongs to the second-class output parameter. The rank of link group AB, CD, EG is $d_1^{AB}\left(\alpha\;0\;0,0\;y\;z\right)=3$, $d_2^{CD}\left(\alpha\;0\;0,0\;y\;0\right)=2$, $d_3^{EG}\left(\alpha\;0\;0,0\;y\;z\right)=3$ respectively. Furthermore, $F_1=\sum_{i=1}^{P_1}f_i-d_1^{AB}=2-3=-1$, $F_2=\sum_{i=1}^{P_2}f_i-d_2^{CD}=2-2=0$, $F_3=\sum_{i=1}^{P_3}f_i-d_3^{EG}=3-3=0$, $F=F_1+F_2+F_3+{}^{N}d_B^{M,J}=-1+0+0+2=1$.

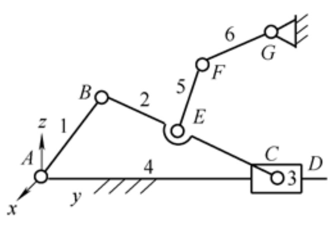

Fig. 4. Planar 2R/3R/RP mechanism

Thus, $\dim\left[\boldsymbol{G}_1^{AB}\left(\alpha\;0\;0,0\;y\;z\right)\cap\boldsymbol{G}_2^{CD}\left(\alpha\;0\;0,0\;y\;0\right)\cap\boldsymbol{G}_3^{EG}\left(\alpha\;0\;0,0\;y\;z\right)\right]={}^{N}d_B^{M,J}\left(\alpha\;0\;0,0\;y\;0\right)$, and the intersection rule is satisfied.

Example 5. Link 2 of the RRRHR mechanism shown in Fig. 5 is taken as the output one, and its loop rank is 4 For link group AB, the rank of which is $d_1^{AB}\left(\alpha\;0\;0,0\;y\;z\right)=3$, $F_1=\sum_{i=1}^{P_1}f_i-d_1^{AB}=2-3=-1$. And for CE, $d_2^{CE}\left(\alpha\;\beta\;0,0\;y\;0\right)=3$, $F_2=\sum_{i=1}^{P_2}f_i-d_2^{CE}=3-3=0$. $\dim\left[\boldsymbol{G}_1^{AB}\left(\alpha\;0\;0,0\;y\;z\right)\cap\boldsymbol{G}_2^{CE}\left(\alpha\;\beta\;0,0\;y\;0\right)\right]={}^{N}d_B^{M,J}\left(\alpha\;0\;0,0\;y\;0\right)=2$, and the intersection rule is satisfied. It can be obtained that $F=F_1+F_2+{}^{N}d_B^{M,J}=-1+0+2=1$.

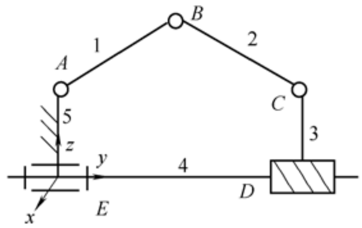

Fig. 5. RRRHR mechanism

Example 6. The 3-P3R/P5R mechanism proposed in Ref. [28], is considered in Fig. 6. In this mechanism, A_i// x-axis, B_i//D_5//D_6//y-axis, C_i//D_1//D_2//D_3//D_4//z-axis and link 4 is the output component. It belongs to first-class output parameter. The rank of base point A_4 is ${}^{N}d_B^{M,J}={}^{A_4}d_B^{M,J}\left(0\,0\,0,x\;y\;z\right)=3$. The rank of link group A_i is $d_1^{A(1-4)}=\left(\alpha\;0\;0,x\;y\;z\right)=4$, then $F_1=\sum_{i=1}^{P_1}f_i-d_1^{A(1-4)}=4-4=0$. The rank of B_i is $d_2^{B(1-4)}\left(0\;\beta\;0,x\;y\;z\right)=4$, and $F_2=\sum_{i=1}^{P_2}f_i-d_2^{B(1-4)}=4-4=0$. $d_3^{C(1-4)}\left(0\;0\;\gamma,x\;y\;z\right)=4$, $d_4^{D(1-6)}\left(0\;\beta\;\gamma,x\;y\;z\right)=5$ are the ranks of link groups C_i and D_i, respectively. $F_3=\sum_{i=1}^{P_3}f_i-d_3^{C(1-4)}=4-4=0$, $F_4=\sum_{i=1}^{P_4}f_i-d_4^{D(1-6)}=6-5=1$. Moreover, $\dim\left[\boldsymbol{G}_1^{A(1-4)}\left(\alpha\;0\;0,x\;y\;z\right)\cap\boldsymbol{G}_2^{B(1-4)}\left(0\;\beta\;0,x\;y\;z\right)\cap\boldsymbol{G}_3^{C(1-4)}\left(0\;0\;\gamma,x\;y\;z\right)\cap\boldsymbol{G}_4^{D(1-6)}\left(0\;\beta\;\gamma,x\;y\;z\right)\right]={}^{N}d_B^{M,J}\left(0\,0\,0,x\;y\;z\right)=3$, which meets the intersection rule. Hence, the mobility is $F=F_1+F_2+F_3+F_4+{}^{N}d_B^{M,J}=0+0+0+1+3=4$.

Example 7. Member 5 is selected as the output part of the mechanism depicted in Fig. 7[31]. Point E, the crossing point of A_4, B_4, C_4, D_4, A_5, B_5, C_5 and D_5, is the base point. ${}^{N}d_B^{M,J}={}^{E}d_B^{M,J}\left(\alpha\;\beta\;\gamma,0\;0\;z\right)=4$. The rank of point E in link group A_i is $d_1^{A(1-5)}\left(\alpha\;\beta\;\gamma,0\;y\;z\right)=5$, $F_1=\sum_{i=1}^{P_1}f_i-d_1^{A(1-5)}=5-5=0$. The rank of point E in link group B_i is $d_2^{B(1-5)}\left(\alpha\;\beta\;\gamma,0\;y\;z\right)=5$, $F_2=\sum_{i=1}^{P_2}f_i-d_2^{B(1-5)}=5-5=0$, and its rank in link group C_i is $d_3^{C(1-5)}\left(\alpha\ \beta\ \gamma, x\ 0\ z\right)=5$, $F_3=\sum_{i=1}^{P_3} f_i - d_3^{C(1-5)}=5-5=0$, and its rank in link group D_i is $d_4^{D(1-5)}\left(\alpha\ \beta\ \gamma, x\ 0\ z\right)=5$, $F_4=\sum_{i=1}^{P_4} f_i - d_4^{D(1-5)}=5-5=0$.

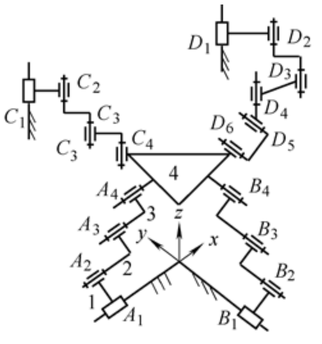

Fig. 6. 3-P3R/P5R mechanism

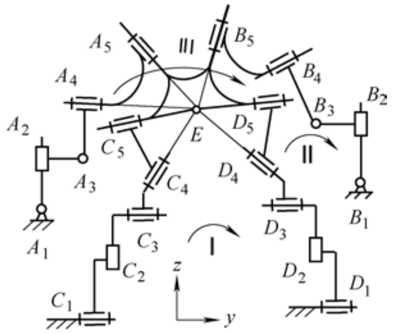

Fig. 7. 2-$R^xPR^x\overline{R^yR}$/2-$R^yPR^y\overline{R^xR}$ mechanism

$\dim\left[\boldsymbol{G}_1^{A(1-5)}\ \left(\alpha\ \beta\ \gamma, 0\ y\ z\right) \cap \boldsymbol{G}_2^{B(1-5)}\ \left(\alpha\ \beta\ \gamma, 0\ y\ z\right) \cap \boldsymbol{G}_3^{C(1-5)}\ \left(\alpha\ \beta\ \gamma, x\ 0\ z\right) \cap \boldsymbol{G}_4^{D(1-5)}\ \left(\alpha\ \beta\ \gamma, x\ 0\ z\right)\right] = {}^N d_B^{M,J}\ \left(\alpha\ \beta\ \gamma, 0\ 0\ z\right)=4$, which meets the intersection rule. $F=F_1+F_2+F_3+F_4+{}^N d_B^{M,J}=0+0+0+0+4=4$.

Example 8. Fig. 8 is the two-loop Saruss mechanism. The rank of link group AC is $d_1^{AC}\ \left(\alpha\ 0\ 0, 0\ y\ z\right)=3$ and the mobility of AC is $F_1=\sum_{i=1}^{P_1} f_i - d_1^{AC}=3-3=0$. Similarly, $d_2^{DF}\ \left(0\ \beta\ 0, x\ 0\ z\right)=3$, $F_2=\sum_{i=1}^{P_2} f_i - d_2^{DF}=3-3=0$. $d_3^{GK}\ \left(\alpha\ 0\ 0, 0\ y\ z\right)=3$, $F_3=\sum_{i=1}^{P_3} f_i - d_3^{GK}=3-3=0$. Eq. (9) becomes $\dim\left[\boldsymbol{G}_1^{AC}\ \left(\alpha\ 0\ 0, 0\ y\ z\right) \cap \boldsymbol{G}_2^{DF}\ \left(0\ \beta\ 0, x\ 0\ z\right) \cap \boldsymbol{G}_3^{GK}\left(\alpha\ 0\ 0, 0\ y\ z\right)\right] = {}^N d_B^{M,J}\left(0\ 0\ 0, 0\ 0\ z\right)=1$. We have $F=F_1+F_2+F_3+{}^N d_B^{M,J}=0+0+0+1=1$. $F={}^N d_B^{M,J}=1$ shows the motion of the platform is the translation along the z-axis.

Example 9. Fig. 9 illustrates the PCM mechanism with the axis of link groups AD, EH, LI parallel with x-, y-, z-axes, respectively. The rank of link group AD is $d_1^{AD}\left(\alpha\ 0\ 0, x\ y\ z\right)=4$, and the ranks of EH and LI are $d_2^{EH}\left(0\ \beta\ 0, x\ y\ z\right)=4$ and $d_3^{LI}\left(0\ 0\ \gamma, x\ y\ z\right)=4$, respectively. In addition, $F_1=\sum_{i=1}^{P_1} f_i - d_1^{AD}=4-4=0$, $F_2=\sum_{i=1}^{P_2} f_i - d_2^{EH}=4-4=0$, $F_3=\sum_{i=1}^{P_3} f_i - d_3^{LI}=4-4=0$. The rank of base point can be obtained.

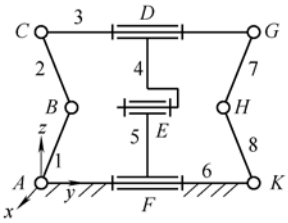

Fig. 8. Saruss mechanism

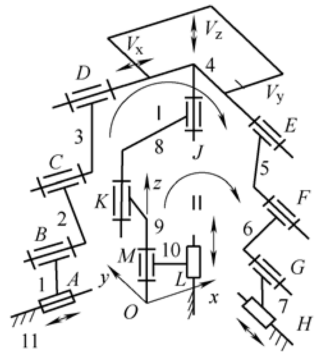

Fig. 9. 3-PRRR mechanism

$\dim\left[\boldsymbol{G}_1^{AD}\ \left(\alpha\ 0\ 0, x\ y\ z\right) \cap \boldsymbol{G}_2^{EH}\left(0\ \beta\ 0, x\ y\ z\right) \cap \boldsymbol{G}_3^{LI}(0\ 0\ \gamma, x\ y\ z)\right] = {}^N d_B^{M,J}\ \left(0\ 0\ 0, x\ y\ z\right)=3$. The mobility of this mechanism is $F=0+0+0+3=3$.

5 Implication and Intrinsic Relations between GOM Formula and Other Mobility Formulae

5.1 Intrinsic relations between GOM formula and other mobility formulae

The parallel mechanism mobility formulae, in accord with the different calculation process, can be grouped in two categories.

(1) The co-constrained approach: The mobility of mechanism is calculated with all the general link groups imposing constraints on the motion platform at the same time.

(2) The sequence-constrained approach: The mobility of mechanism is calculated with an order, that is, the moving platform is constrained by each link group in turn.

The methods proposed in Eqs. (3), (5) and (6) are sequence-constrained approaches, and the calculation process can reflect the changes of mechanism mobility with the increase of link groups.

The essence of Eqs. (1), (2), (4) and (11) are the same, for they are all co-constrained approaches.

Eq. (11) can be proved equivalent with (2) through simple derivation. Substitute Eq. (10) into Eq. (11),

$$
\begin{aligned}
F = \sum\nolimits_{k=1}^{L+1} F_k + {}^N d_B^{M,J} = \\
\sum\nolimits_{k=1}^{L+1}\left(\sum\nolimits_{i=1}^{P_k} f_i - d_k^{P_k}\right) + {}^N d_B^{M,J} = \\
\sum\nolimits_{k=1}^{L+1}\sum\nolimits_{i=1}^{P_k} f_i - \sum\nolimits_{k=1}^{L+1} d_k^{P_k} + {}^N d_B^{M,J} = \\
\sum\nolimits_{k=1}^{P} f_i - \left(\sum\nolimits_{k=1}^{L+1} d_k^{P_k} - {}^N d_B^{M,J}\right) = \\
\sum\nolimits_{k=1}^{P} f_i - \left(\sum\nolimits_{k=1}^{L+1} d_k^{P_k} - {}^N d_B^{M,J}\right) = \\
\sum\nolimits_{k=1}^{P} f_i - \left[\sum\nolimits_{k=1}^{L+1} \dim(\boldsymbol{G}_k^{gz}) - \dim\left(\bigcap\nolimits_{k=1}^{L+1} \boldsymbol{G}_k^{gz}\right)\right].
\end{aligned}
$$

Thus the GOM formula expressed in another form is obtained, expressed with the difference between the sum of dimensions of link groups' motion parameters and the dimensions of the intersection of link groups' motion parameters, namely

$$
F = \sum\nolimits_{k=1}^{P} f_i - \left[\sum\nolimits_{k=1}^{L+1} \dim(\boldsymbol{G}_k^{gz}) - \dim\left(\bigcap\nolimits_{k=1}^{L+1} \boldsymbol{G}_k^{gz}\right)\right]. \tag{12}
$$

$\boldsymbol{G}_k^{gz}$ in Eq. (12) is corresponding to $\boldsymbol{R}_{Gi}$ in Eq. (2), so Eq. (2) will be obtained by replacing $L+1$ in Eq. (11) with k. It can be proved that Eq. (11) and (2) are equivalent.

$$
\begin{aligned}
F = \sum\nolimits_{k=1}^{L+1} F_k + {}^N d_B^{M,J} = \\
\sum\nolimits_{k=1}^{P} f_i - \left[\sum\nolimits_{i=1}^{k} \dim(\boldsymbol{R}_{Gi}) - \dim\left(\bigcap\nolimits_{i=1}^{k} \boldsymbol{R}_{Gi}\right)\right].
\end{aligned} \tag{13}
$$

It can also be proved that Eq. (11) is equivalent to Eq. (1) and (4).

5.2 Characteristics and the importance of GOM formula

The important features of the GOM formula are follows

(1)The mobility for general link group, F_k, is calculated with the difference between the number of joints of general link group and the number of motion parameters.

(2) DOF of a mechanism is calculated with the number of all the output parameters as the ranks both for independent and mixing output parameters.

(3) The independence of the output parameters can be judged by the value of F_k.

(4) The proposed novel mobility formula reveals the intrinsic relationship between the mobility of a mechanism, and the link group, and the dimension of output parameters intuitively.

(5) At the given coordinate, the motion of link group and the output member are expressed with the displacement parameter matrix of the link group and the base point, respectively, which is simple and intuitive.

The values of ${}^N d_B^{M,J}$, F_k, $\sum\nolimits_{k=1}^{L+1} F_k$ and F of mechanisms shown in Fig. 1–Fig. 9 are listed in Table 1. From the results of the mechanisms shown in Fig. 1–Fig. 5, it can be seen that all the values of F_k is less than or equal to 0, namely $F_k \leqslant 0$. Moreover, there is at least one link group of each machine satisfying $F_k < 0$. Thus $\sum\nolimits_{k\ 1}^{L+1} F_k < 0$ can be obtained, and the DOF of the mechanism is less than the dimension of output parameters, that is $F < {}^N d_B^{M,J}$. The output parameters of these mechanisms are not all independent. However, for the link groups of mechanisms shown in Fig. 6–Fig. 9, with F_k is greater than or equal to 0, that is $F > {}^N d_B^{M,J}$, and the output parameters are all independent.

Table 1. Relationship between independence of the output parameters and the mobility of link group

Name of the mechanisms	Dimension of base point parameters ${}^N d_B^{M,J}$	DOFs of link group F_k	Sum of DOFs of link group $\sum\nolimits_{k\ 1}^{L+1} F_k$	DOFs of mechanism F	Whether output parameters are independent
Plane 4R mechanism	${}^N d_B^{M,J}(\alpha\, 0\, 0, 0\, y\, z) = 3$	$F_1 = F_2 = -1$	$F_1 + F_2 = -2$	$F = 1$	N
Plane 6R mechanism	${}^N d_B^{M,J}(0\, 0\, 0, 0\, y\, z) = 2$	$F_1 = -1$, $F_2 = 0$, $F_3 = 0$	$F_1 + F_2 + F_3 = -1$	$F = 1$	N
Plane 2R/3R/RP mechanism	${}^N d_B^{M,J}(\alpha\, 0\, 0, 0\, y\, 0) = 2$	$F_1 = -1$, $F_2 = 0$, $F_3 = 0$	$F_1 + F_2 + F_3 = -1$	$F = 1$	N
3-RPS mechanism	${}^N d_B^{M,J}(\alpha\, \beta\, \gamma, 0\, y\, z) = 5$	$F_1 = 0$, $F_2 = F_3 = -1$	$F_1 + F_2 + F_3 = -2$	$F = 3$	N
RRRHR mechanism	${}^N d_B^{M,J}(\alpha\, 0\, 0, 0\, y\, 0) = 2$	$F_1 = -1$, $F_2 = 0$	$F_1 + F_2 = -1$	$F = 1$	N
Saruss mechanism	${}^N d_B^{M,J}(0\, 0\, 0, 0\, 0\, z) = 1$	$F_1 = F_2 = F_3 = 0$	$F_1 + F_2 + F_3 = 0$	$F = 1$	Y
3-PRRR mechanism	${}^N d_B^{M,J}(0\, 0\, 0, x\, y\, z) = 3$	$F_1 = F_2 = F_3 = 0$	$F_1 + F_2 + F_3 = 0$	$F = 3$	Y
3-P3R/P5R mechanism	${}^N d_B^{M,J}(0\, 0\, 0, x\, y\, z) = 3$	$F_1 = F_2 = F_3 = 0$, $F_4 = 1$	$F_1 + F_2 + F_3 + F_4 = 1$	$F = 4$	Y
2-$R^xPR^x\,\overline{R^yR}$ /2-$R^yPR^y\,\overline{R^xR}$ mechanism	${}^N d_B^{M,J}(\alpha\, \beta\, \gamma, 0\, 0\, z) = 4$	$F_1 = F_2 = F_3 = F_4 = 0$	$F_1 + F_2 + F_3 + F_4 = 0$	$F = 4$	Y

On the basis of these results, some conclusions about output parameters can be drawn:

(1) The dimension of output parameters is the number of mixing output parameters of base point, including both independent and dependent parameters.

(2) If the mobility of some of the link group, F_k, is less than 0, and DOFs of other link groups are less than or equal to 0, not all the output parameters are independent. Correspondingly, the mobility of each link group is not less than 0, that is, $F \geqslant {}^N d_B^{M,J}$, and all the output parameters are independent.

(3) The value of the mobility of the link group can be used to judge how the link groups constrain the motion platform, also to reflect the degree of association among output parameters.

The sufficient and essential condition for all the independent output parameters is that the mobility of each link group is greater than or equals to zero.

Another implication of GOM formula is that based on the known mobility of mechanism F and output parameter ${}^N d_B^{M,J}$, the $\sum_{k=1}^{L+1} F_k$ can be calculated by the GOM formula. When conducting type synthesis, the value of F_k can be determined firstly, then according to Eq. (10) combined with other factors, $\sum_{i=1}^{P_k} f_i$ and d_k^{gz} will be obtained. Synthesis of the mechanisms required can be completed.

6 Conclusions

(1) The presented GOM formula is equivalent to Eqs. (1), (2) and (4). All of them are effective and correct.

(2) The dimension of output parameters is the number of total output parameters of the base point, no matter whether the output parameters are independent or mixing ones.

(3) Base point parameters of output member are the subset of effective parameters.

(4) F_k can be used to judge how the link group constraints on the motion platform, and to analyze the independence of the output parameters.

(5) The sufficient and essential condition for all independent output parameters is that the mobility of each link group is greater than or equal to zero.

(6) Many mechanisms, containing non-independent output parameters, can be synthesized to expand the type of mechanisms based on GOM formula.

(7) The methodology presented will provide an important theoretical foundation for the development of mechanism theory, and also will have broad application prospects.

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