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Modelica-based Object-orient Modeling of Rotor System with Multi-Faults

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Abstract: Modelica-based object-orient method is proved to be rapid, accurate and easy to modify, which is suitable for prototype modeling and simulation of rotor system, whose parameters need to be modified frequently. Classical non-object-orient method appears to be inefficient because the code is difficult to modify and reuse. An adequate library for object-orient modeling of rotor system with multi-faults is established, a comparison with non-object-orient method on Jeffcott rotor system and a case study on turbo expander with multi-faults are implemented. The relative tolerance between object-orient method and non-object-orient is less than 0.03%, which proves that these two methods are as accurate as each other. Object-orient modeling and simulation is implemented on turbo expander with crack, rub-impact, pedestal looseness and multi-faults simultaneously. It can be conclude from the case study that when acting on compress side of turbo expander separately, expand wheel is not influenced greatly by crack fault, the existence of rub-impact fault forces expand wheel into quasi-periodic motion and the orbit of expand wheel is deformed and enhanced almost 1.5 times due to pedestal looseness. When acting simultaneously, multi-faults cannot be totally decomposed but can be diagnosed from the feature of vibration. Object-orient method can enhance the efficiency of modeling and simulation of rotor system with multi-faults, which provides an efficient method on prototype modeling and simulation.

Key words: rotor system, multi-faults, object-orient, modeling, Modelica

1 Introduction

Nowadays, machinery consisting of rotor system has been dominant in most of heavy equipment such as gas turbine, turbo expander, aero engine and so forth. Rotor system is always the key component of these equipment and the dynamic properties of which can determine the performance of the whole system. Before manufacturing, the modeling and simulation of rotor system can reflect the dynamic properties, which provide the engineers references to decide whether this design should be modified or which parameter should be altered in order to optimize the whole system so that a better performance can be obtained. The modeling and simulation of rotor system can trace back to JEFFCOTT[1], who is the very first scientist gave a plain interpretation of the rotor model proposed by FÖPPL[2]. As modeling and simulation of rotor system develops, there appear two typical methods in dealing with this problem. One is transfer matrix method proposed by PROHL[3] and improved by HORNER, et al[4], based on lumped parameter method, the other is finite element method which is first applied on rotor system by RUHL, et al[5], based on distributed parameter method.

In the earlier days of rotor system research using finite element method, researchers must code by themselves to solve the problems[6]. It is also useful method in the following years if the rotor system is not too complex[7]. As the structure of rotor system gets more complicated and the commercial software of finite element method develops, such as Ansys, Abaqus, etc., the modeling and simulation based on finite element method is always executed under these integrated computational platforms[8]. On the other hand, users must code by themselves according to the structure of rotor system based on transfer matrix method[9–12] The code for one certain rotor system cannot be applied on another rotor system, which means it is difficult to modify and not reusable. Furthermore, classical program is causal, which means the calculating direction and order are determined, it is difficult for users to modify so that optimization cannot be implemented easily. These disadvantages cause an inefficient modeling and simulation, which cannot fulfill the requirement of prototype modeling and its optimization: to obtain characteristics of rotor system rapidly.

Modelica is an object-oriented language for modeling of large, complex and heterogeneous physical systems, which has the characteristics of multi-domain, non-causal and object-oriented[13].The draft version of Modelica language and its program syntax are proposed in the doctoral degree thesis of Dr. ELMQVIST[14] and improved by the nonprofit organization Modelica Association. In Modelica, algorithm of modeling is described by equations instead of value assignment statement. DASSL[15] and other algorithms are selected to solve the complicated system described by differential equations, algebra equations and discrete equations. The compilation is based on generalized

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Kirchhoff's law, which is a reduction of the inconvenience of user's programming and a promotion of modeling efficiency. Modelica now has been widely used in engineering fields[16–20] .

Modelica has the characteristic of object-oriented and non-causal, which can be used to fulfill the requirement of easy to modify and reusable. Object-oriented means rotor system can be decomposed into several basic components, all kinds of rotor system can be built by these reusable basic components, and their parameters can be simply modified. Non-causal implies that the program sequence should never be settled by users, all the equations and components do not have the order of input and output, which also leads a system to be easy to modify. All these advantages lead to a rapid and easy to modify prototype modeling.

Any rotor system may suffer from different kinds of faults and malfunctions. A proper modeling and simulation of rotor system with common faults should be a significant contribution that engineers can understand the performance of faults well, so that faults can be discovered or diagnosed in the early stage, which can prevent malfunction even failure of machinery consisting of rotor system. Common faults of rotor system including crack, rub-impact and pedestal looseness should be considered in prototype modeling in order to acquire the feature and prevent the damage. The entire components construct a "library" of rotor system, which is a terminology implies that all the related components are packaged for the researchers to use it.

2　Modeling of Rotor System

Before detailed parametric design, a prototype model of rotor system should be built in order to obtain the basic dynamical behavior. A prototype model should be simple but accurate and the modeling method of prototype should be rapid and easy to modify. Modelica seems to be an appropriate choice for prototype modeling.

Fig. 1 shows how to model rotor system prototype with Modelica via MWorks[21]. Left column shows *RotorDynamics Library*, which contains basic parts and faults of rotor system. When modeling the prototype of a certain rotor system, users need to drag parts and faults into modeling window, connect them and assign values for every parameter. 0 shows an example. First arrow shows the parts of bearing is dragged into modeling window, and second arrow shows the parameter value assignment. This modeling method leads to be simple, rapid and easy to modify.

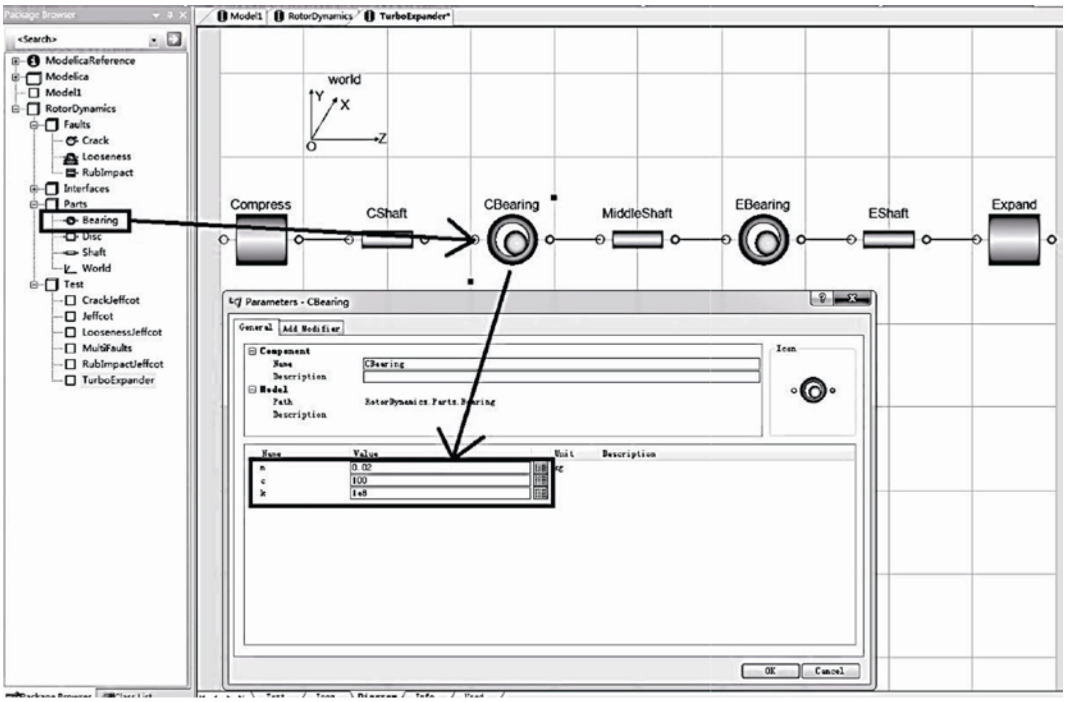


Fig. 1.　Prototype modeling with Modelica

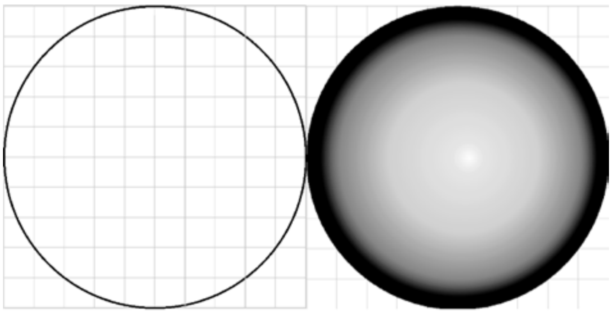
2.1　Interfaces

As indicated in the introduction, the compilation of system using Modelica is based on generalized Kirchhoff's law. Kirchhoff's law is originally used in circuit analysis, which does not only hold in electric circuit analysis, but can be spread and generalized into other domains such as hydraulic, heat transfer, mechanics and so forth. Any system of these domains can be simplified into "circuits" similar with electric circuit, which means, a model of such system can be extract so that the system is described by components connecting to each other like circuits. The connection may be substantial like pipe in hydraulic or just abstract like mechanics. The connections construct nodes, and generalize Kirchhoff's law hold: for one certain domain, the chosen potential variables are equal and the sum of chosen flow variables equals to zero at the nodes.

The term “interface” in Modelica language covers two kinds of meanings. The first meaning of interface is the same with connecting node. In the study of rotor system modeling using Modelica, two radial displacements of axle center in Cartesian coordinate x and y are chosen as potential variables, and two radial forces F_x and F_y along direction of x and y are chosen as flow variables. So, at the connecting node,

$$\begin{gathered} x_i = x_j, \quad y_i = y_j, \quad i, j \in \mathbf{Z}^+; \\ \sum F_{xi} = 0, \quad \sum F_{yi} = 0, \quad i, j \in \mathbf{Z}^+. \end{gathered} \tag{1}$$

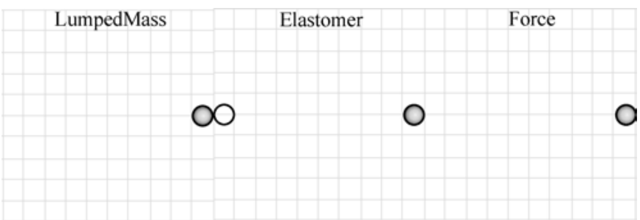
In order to make the direction of rotor system clear, two types of connecting node, named left flange and right flange are designed as illustrated in Fig. 2.



(a) Left (b) Right

Fig. 2. Connecting nodes of rotor system

The second meaning of interface resembles Virtual Base Class in object-oriented programming languages. Components of domains where generalized Kirchhoff’s law holds can be classified into several basic styles. For example, in the rotor system domain, components can be classified into three styles: lumped mass, elastomer and force, as shown in Fig. 3. Components of one style may have different control equations but the same behavior. Bearings and discs belong to lumped mass, which act as rigid body in lumped parameter method modeling of rotor system; Shaft is classified in elastomer, the deformation of which affects the behavior of rotor system greatly; Faults, like crack and external force, like sealing force should be cataloged in force style because they have unidirectional influence on rotor system exclusively.



(a) Lumped mass (b) Elastomer (c) Force

Fig. 3. Virtual base classes of rotor system

Interfaces must be modeled in the very beginning of modeling and simulation using Modelica. It is the foundation of all the components, sub systems and systems. Fig. 4 shows the basic conception of modeling using Modelica, the components of target domain should be enumerated. Then the listed components should be classified into several basic interfaces. After the construction of interfaces, the procedure should be reversed, using the interfaces to model components, and build the whole domain using components.

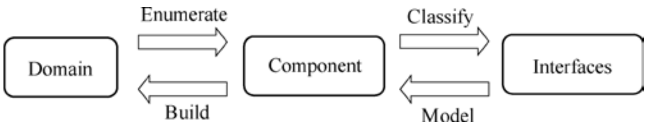


Fig. 4. Basic conception of modeling using Modelica

2.2 Components

There are four fundamental components in rotor system analysis under lumped parameter method: disc, shaft, bearing and world. The most important component is world, as shown in Fig. 5. This component defines the working environment of the rotor system: rotating angular speed ω_{r} and the gravity constant g, which are the most important parameters: the speed decides whether the working frequency has reached the natural frequency of rotor system or not, and the gravity can decide the stable position of rotor system.

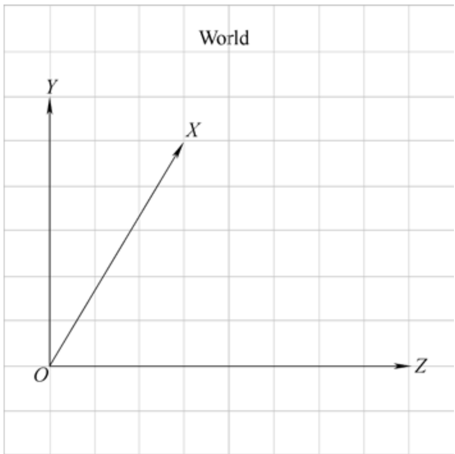


Fig. 5. Icon of component: world

The other three components disc, bearing and shaft can be classified to interfaces of lumped mass or elastomer. Fig. 6 shows the force diagram of lumped mass and elastomer. For lumped mass, the dominant parameter is the mass m, length l and bending stiffness EI have the same status with mass for the elastomers. Lumped mass interface and elastomer interface both has two nodes, L and R, so F^{L} and F^{R} stand for the force at the left node and the right node respectively. For lumped force interfaces, there exists the body force F_{b} which acts directly on the lumped mass, it may be eccentric force, gravity, oil-film force and etc. For potential variables, the lumped mass interface is assumed that radial displacements of axle center of left node and right node are equal, $x = x^{\mathrm{L}} = x^{\mathrm{R}}, y = y^{\mathrm{L}} = y^{\mathrm{R}}$; the left and right radial displacements of axle center for elastomer interface have certain relations which must be defined in model of shaft.

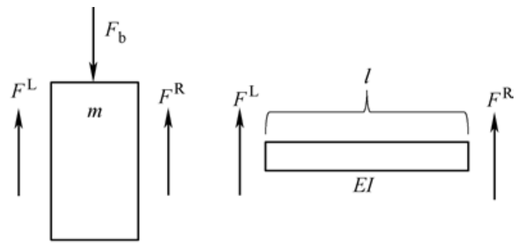


Fig. 6. Force diagram of lumped mass and elastomer

Disc simulates the wheel or impeller in practical rotating machinery, which is the mainly working part and is treated as lumped mass where external loads act. The basic mathematical equations are shown in Eq. (2), and the icon of encapsulation component using Modelica language is illustrated in Fig. 7:

$$\begin{cases} m\ddot{x} = F_x^L + F_x^R + me\omega_r^2 \cos \omega_r t, \\ m\ddot{y} = F_y^L + F_y^R + me\omega_r^2 \sin \omega_r t - mg. \end{cases} \tag{2}$$

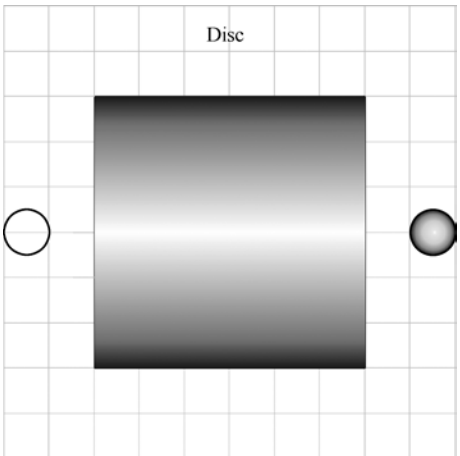


Fig.7. Icon of component: disc

In Eq. (2), e stands for eccentricity, which is a common excitation in rotor system analysis.

Fig. 8 illustrates the icon of component of bearing, which is an important supporting part in rotor system. Bearings provide support to rotor so that the system can revolute without strike or friction to the workbench. It is also the important source of stiffness and damping to the system except for structure stiffness and damping. Assume the oil-film force is linearized to $c\dot{x} + kx$ and there is no eccentricity in bearings, it has a similar equation with disc:

$$\begin{cases} m\ddot{x} = F_x^L + F_x^R - c\dot{x} - kx, \\ m\ddot{y} = F_y^L + F_y^R - mg - c\dot{y} - ky. \end{cases} \tag{3}$$

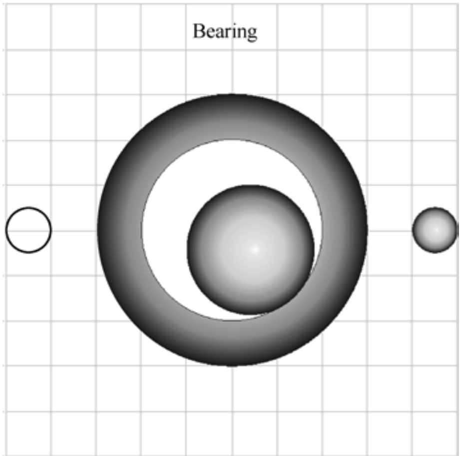


Fig. 8. Icon of component: bearing

Shaft is the only component belongs to elastomer interface in basic rotor system model, the icon of which is illustrated in Fig. 9. The traditional model of shaft in rotor system coincides with the theory of Euler beam in mechanics of materials:

$$\begin{cases} x^R = x^L + \dfrac{l^3}{6EI} F_x^R, \\ y^R = y^L + \dfrac{l^3}{6EI} F_y^R, \\ F_x^L + F_x^R = 0, \\ F_y^L + F_y^R = 0. \end{cases} \tag{4}$$

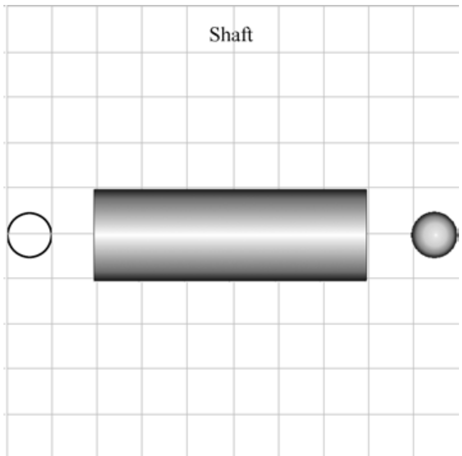


Fig. 9. Icon of component: shaft

2.3 Example: modeling and simulation of Jeffcott rotor system

Jeffcott rotor system is the simplest model in rotor dynamic analysis, the schematic of which is illustrated in Fig. 10. For the sake of testing the applicability and validity of this object-orient modeling method proposed in this paper, construction and numerical experiment of a model of Jeffcott rotor system may be the best choice(see Fig. 11). Table 1 lays the parameters of each component of Jeffcott rotor system, and the model of this system constructed by Modelica is shown in 0.

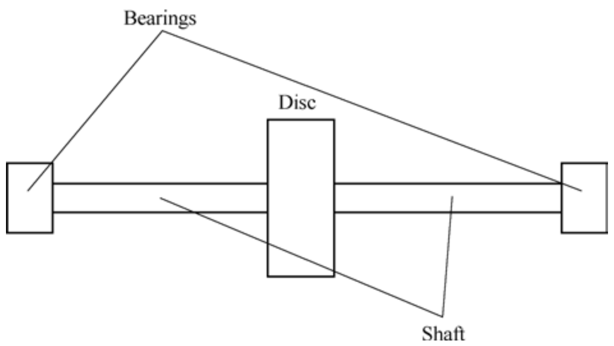


Fig. 10. Schematic of Jeffcott rotor system

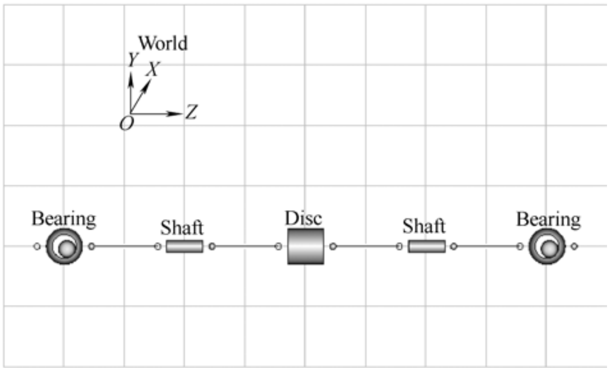


Fig. 11. Model of Jeffcott rotor system

Table 1. Parameters of Jeffcott rotor system

Component	Parameter	Value
World	Gravity $g/(\mathrm{m \cdot s^{-2}})$	9.81
	Rotating velocity $\omega_r/(\mathrm{r \cdot min^{-1}})$	1200
Left bearing	Mass m/kg	0.02
	Stiffness $k/(\mathrm{N \cdot m^{-1}})$	10^8
	Damping $c/(\mathrm{N \cdot s \cdot m^{-1}})$	100
Left shaft	Length l / m	0.03
	Diameter d/m	0.008
	Elastic module E/GPa	205
Disc	Mass m /kg	0.15
	Eccentricity e /m	0.005
Right shaft	Length l / m	0.03
	Diameter d/m	0.008
	Elastic module E /GPa	205
Right bearing	Mass m /kg	0.02
	Stiffness $k/(\mathrm{N \cdot m^{-1}})$	10^8
	Damping $c/(\mathrm{N \cdot s \cdot m^{-1}})$	100

The Modeling procedure is quite easy for this method. Users first drag the components into the modeling area, then assign values for every parameter of each component, then connect them according to the real coordination relationship. It is quite easier than traditional programming way. Simulation can be carried out using the built-in compiler of Modelica, one of the advantages of simulation using Modelica is that all the variables of the components can be extracted for further treatment. For Jeffcott rotor system, the displacements of disc is extracted and the interesting curves including vibration signal, orbit of rotor center and spectrum of frequency are illustrated in Fig. 12.

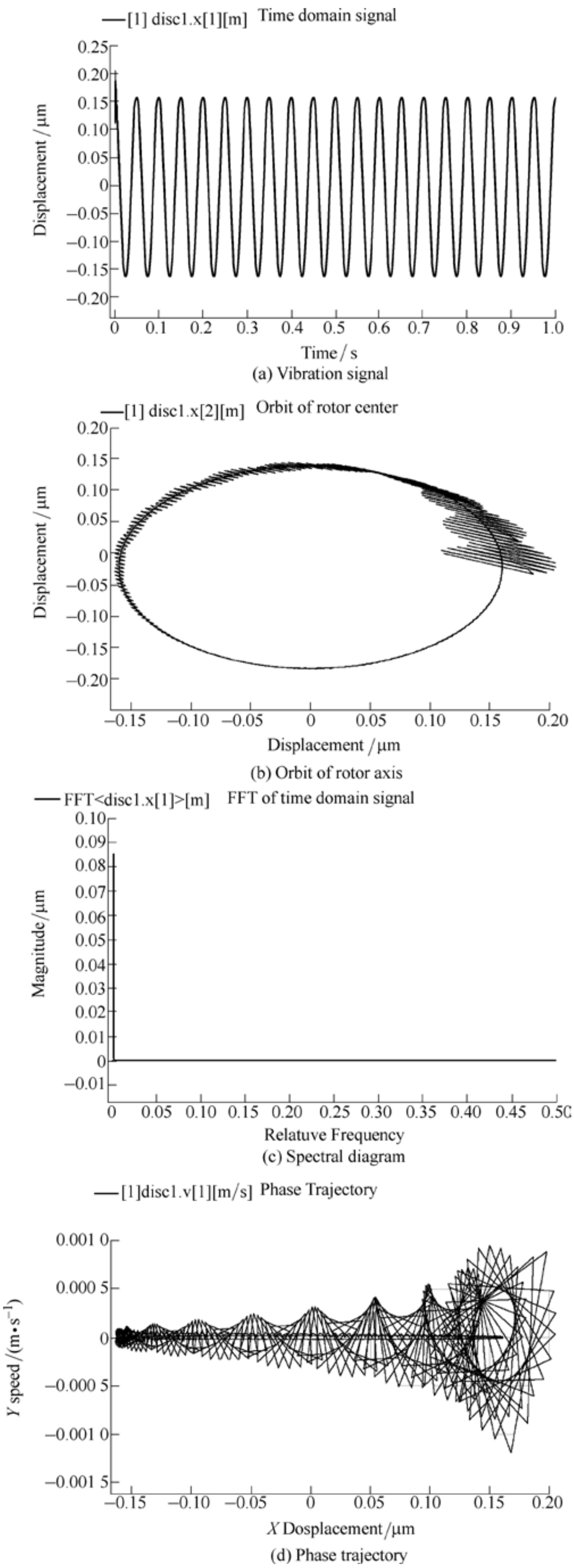


Fig. 12. Response of Jeffcott rotor system using built-in compiler of Modelica

However, the build-in complier of Modelica has some disadvantages. For examples, the influence of numerical starting stage cannot be rid of (especially the phase trajectory), the horizontal axis of FFT cannot be adjusted and so forth. It is quite natural for users to export result data to another software for analyzing, which is more powerful in data processing than Modelica, see Fig. 13.

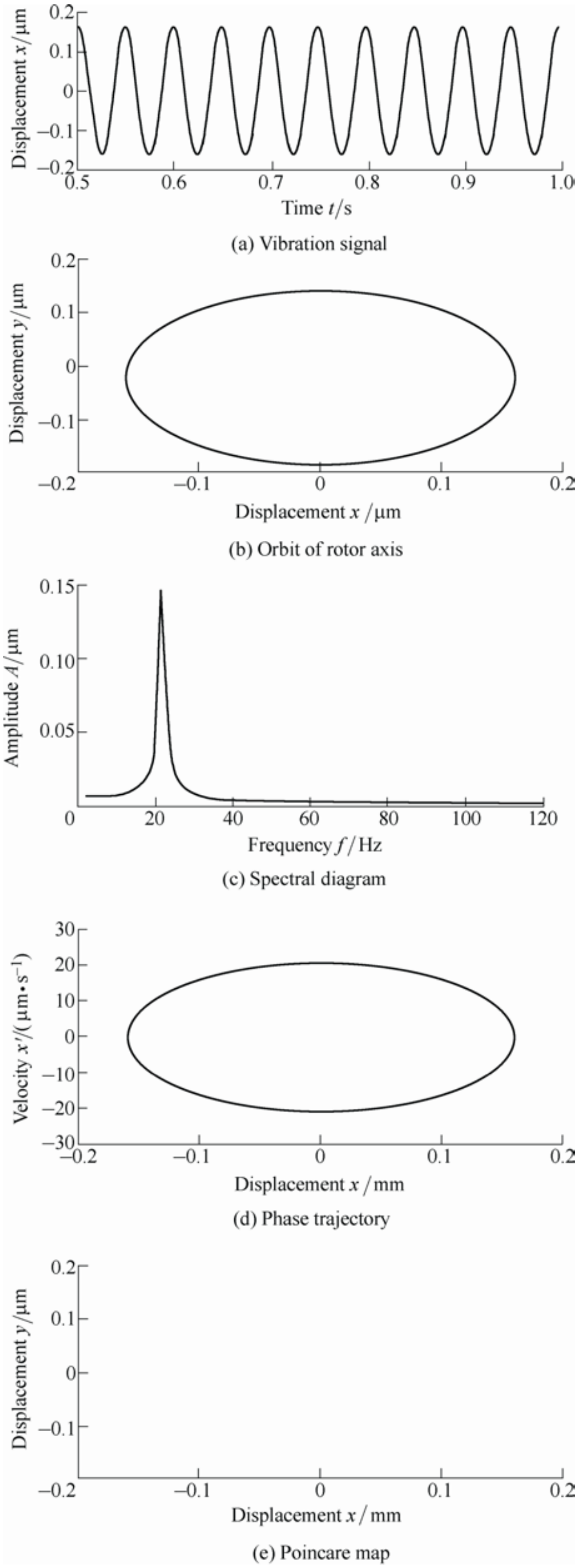


Fig. 13. Response of Jeffcott rotor system using other data processing software

2.4 Comparison with non-object-orient method

In order to evaluate the accuracy of object-orient method, it is necessary to compare results of object-orient method and non-object-orient method. Fig. 14 illustrates the error between two methods. It can be seen that the error between two methods is less than 6×10^{-11} m , and the corresponding relative tolerance is less than 0.03% . Conclusions that the results of two methods coincide with each other and object-orient method is as accurate as non-object-orient method can be drawn.

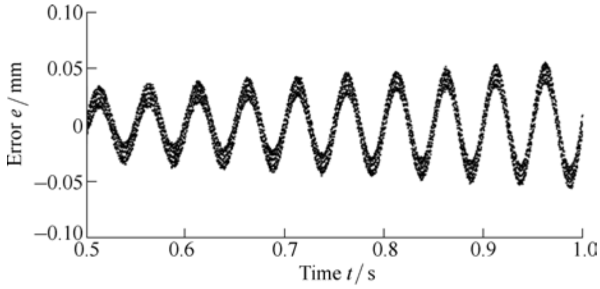


Fig. 14. Error between object-orient method and non-object-orient method

3 Modeling of Faults

3.1 Crack

Common equipment consists of rotor system like turbo expander works under bad circumstance, so that the fault of crack happens constantly and grows because of the alternating stress caused by revolution. Generally, the existence of crack reduces the stiffness of rotor system. This situation can be seen as an external crack force related to the stiffness acting at the crack position equivalently:

$$\frac{1}{2}f\left(\omega_{\mathrm{r}}t\right)\Delta k\begin{pmatrix}1+\cos\left(2\omega_{\mathrm{r}}t\right) & \sin\left(2\omega_{\mathrm{r}}t\right)\\ \sin\left(2\omega_{\mathrm{r}}t\right) & 1-\cos\left(2\omega_{\mathrm{r}}t\right)\end{pmatrix}, \tag{5}$$

where Δk is the variation of stiffness because of the crack fault. Assume l stands for the total length of the shaft, the crack locates and the length of x measured from the left side of the shaft. As the theory of material mechanics tells, the stiffness variation is defined as

$$\Delta k=\frac{48E}{x\left(3l^2-4x^2\right)}\bullet\frac{D^4}{64}\bullet \left[\frac{\pi}{2}-\arcsin\left(1-2\overline{h}\right)-2\left(1-2\overline{h}\right)\left(1-8\overline{h}+8\overline{h}^2\right)\sqrt{\overline{h}\left(1-\overline{h}\right)}\right], \tag{6}$$

where $D=2R$, $\overline{h}=T/D$. $f\left(\omega_{\mathrm{r}}t\right)$ is called switch function, which is used to simulate the opening and closing course of crack. According to the dynamic deflection caused by revolution and static deflection caused by gravity, there are three kinds of selection of switch function.

Firstly, the static deflection exceeds the dynamic deflection greatly, which is called gravity dominance. It was researched firstly by PAPADOPOULOS, et al[22], and GASCH[23]. Under this situation, the crack is assumed never switching[24], $f\left(\omega_{\mathrm{r}}t\right)$ is usually set to be a constant such as 0.5.

Secondly, the dynamic deflection exceeds the static deflection greatly, which is called non-gravity dominance. MENG[25] took whirling into consideration and proposes a revision to traditional open crack assumption. Square function[26] and cosine function[27] are always used to simulate switching generally:

$$\begin{aligned}&f\left(\omega t\right)=\begin{cases}1, & 2n\pi-\pi/2\leqslant\omega t\leqslant 2n\pi+\pi/2,\\ 0, & 2n\pi+\pi/2\leqslant\omega t\leqslant 2n\pi+3\pi/2,\end{cases}\\ &n\in\mathbf{Z}, f\left(\omega t\right)=\frac{1}{2}\left(1+\cos\omega t\right).\end{aligned}\tag{7}$$

Thirdly, the general situation. SAWICKI, et al[28], put a theory forward that the practical switching behavior of crack is between square function and cosine function. In 1992, GAO, et al[29], proposed a new switching function which is adequate for gravity dominance, non-gravity dominance and the general situation. Assume R is the radius of rotor, T is the depth of crack, $\theta=\omega t-\varphi+\dfrac{\pi}{2}-2k\pi \quad k\in\mathbf{N}$, $\cos\alpha=R-T/R$ and $\tan\varphi=\dfrac{y}{x}$, the switching function is shown in Eq. (8):

$$f\left(\theta\right)=\begin{cases}1, & -\dfrac{\pi}{2}+\alpha_0\leqslant\theta<\dfrac{\pi}{2}-\alpha_0,\\ \dfrac{1}{2}\left(1+\cos\dfrac{\theta-\dfrac{\pi}{2}+\alpha_0}{2\alpha_0}\pi\right), & \dfrac{\pi}{2}-\alpha_0\leqslant\theta<\dfrac{\pi}{2}+\alpha_0,\\ 0, & \dfrac{\pi}{2}+\alpha_0\leqslant\theta<\dfrac{3\pi}{2}-\alpha_0,\\ \dfrac{1}{2}\left(1+\cos\dfrac{\theta-\dfrac{3\pi}{2}-\alpha_0}{2\alpha_0}\pi\right), & \dfrac{3\pi}{2}-\alpha_0\leqslant\theta<\dfrac{3\pi}{2}+\alpha_0,\end{cases}\tag{8}$$

In this paper, GAO's switching function is chosen for the modeling of crack. The icon of crack model is illustrated in Fig. 15.

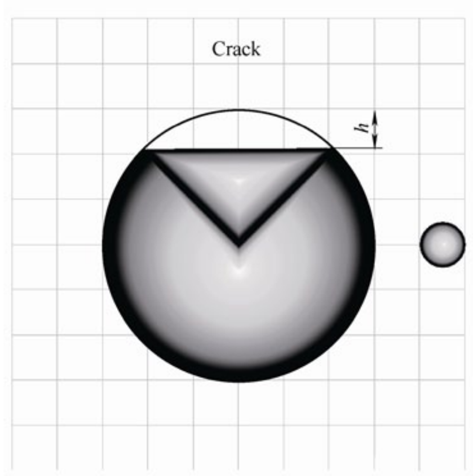


Fig. 15. Icon of fault model: crack

3.2 Rub-impact

If the rotor is installed improperly or becomes bent due to lack of support when being halt, the rotating part will contact the static part of rotational machinery such as the shell. This contact is called rub-impact because the rotor will collide and have friction to the shell. MUSZYNSKA[30–31] ignored the elastic force and friction force when collision happens and proposes the first model of rub-impact, then an elastic recover coefficient is introduced to the model by him for a revision. In this paper, a rub-impact model raised by CHU, et al[32], is used to simulate the force. Assume δ_0 is the clearance of shell and axis of rotor, k_c and f are the collision stiffness and friction coefficient between shell and rotor respectively. Suppose $e=\sqrt{x^2+y^2}$,

$$\begin{aligned}&\begin{pmatrix}F_x\\ F_y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}, & e<\delta_0,\\ &\begin{pmatrix}F_x\\ F_y\end{pmatrix}=-\frac{\left(e-\delta_0\right)k_c}{e}\begin{pmatrix}1 & -f\\ f & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}, & e\geqslant\delta_0.\end{aligned}\tag{9}$$

Rub-impact will happen when the radial displacement exceeds the clearance. The icon of rub-impact is illustrated in Fig. 16. This model can also be seen as external force acting at the disc of rotor system.

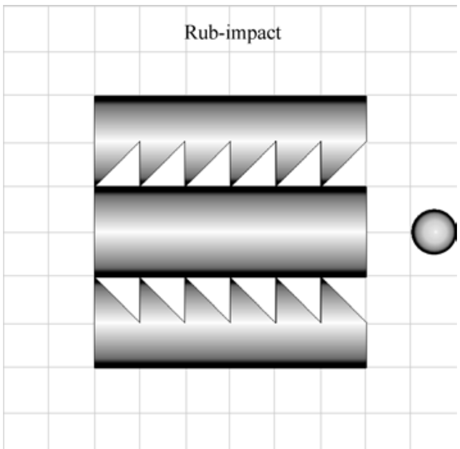


Fig. 16. Icon of fault model: rub-impact

3.3 Pedestal looseness

The rotor system must be fixed to the ground or the frame connected to the ground in order to make a stable operation without unnecessary vibrations. However, if the rotor is not installed or fixed well, pedestal looseness happens. Looseness will bring impact to the rotor system and arouse unnecessary vibration, which may be amplified because of the existence of alternating stress. This fault always happens at the position of screw connecting bearing box and the ground, and it can cause abnormal condition and is very dangerous to the machine and the operators. MUSZYNSKA, et al[33], first studies this fault. WEN, et al[34], studies the dynamic behavior of rotor with looseness. CHU, et al[35], set a piecewise linearity equation for this fault and studies the nonlinear behavior of the rotor system with pedestal looseness such as bifurcation and chaos. In this paper, the equation aroused by Chu is brought in. Assume b is the length of looseness, c is bias of looseness, $d = mg/k$ is the static deflection and k is the collision stiffness. The looseness force is shown in Eq. (10):

$$\begin{aligned} F_x &= \begin{cases} k(x+b-c), & x \leqslant -b-c, \\ 0, & -(b-c) < x \leqslant b+c, \\ k(x-b-c), & x > b+c, \end{cases} \\ F_y &= \begin{cases} k(y-d), & y \leqslant d, \\ 0, & d < y \leqslant 2b+d, \\ k(y-2b+d), & y > 2b+d, \end{cases} \end{aligned} \tag{10}$$

The icon of pedestal looseness is illustrated in Fig. 17. Because this fault always happen at the bearing box, looseness model is designed as a bearing enclosed with the force of pedestal looseness in order to simulate looseness effect on bearing box. Different from crack and rub-impact, users should replace the normal bearing with the bearing with pedestal looseness fault, instead of connecting to the system.

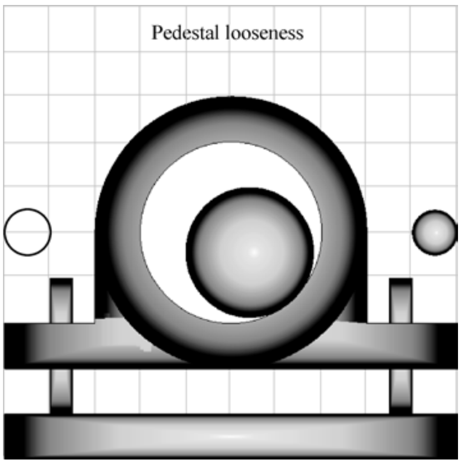


Fig. 17. Icon of fault model: pedestal looseness

4 Case Study: Turbo Expander Rotor System with Multi-faults

Turbo expander is an important part of air separation system, which produces majority part of refrigerating output of the air separation system. Schematic of turbo expander is illustrated in Fig. 18. It can be seen that turbo expander rotor system is constructed by two wheels, two bearings and three pieces of shaft. The wheels are called expand wheel and compress wheel according to their function separately. This rotor system can be as combination of two cantilever rotors: compress side and expand side.

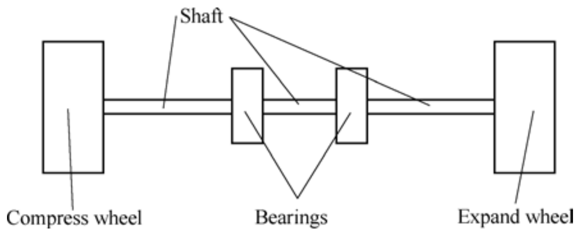


Fig. 18. Schematic of turbo expander rotor system

Rotor system of turbo expander determines the behavior of turbo expander, even the whole air separation system greatly. A prototype model is necessary for engineers to obtain the basic behavior of turbo expander rotor system under normal condition. Furthermore, it is also important to obtain the characteristic of turbo expander rotor system when different faults occur separately or simultaneously. In this case study, we focused on how will faults, including crack, rub-impact and pedestal looseness influence the vibration of expand wheel when they occurs around compress side separately and simultaneously. Therefore, expand wheel is chosen as the components of interest in this case study.

4.1 Turbo expander under normal condition

The model of turbo expander rotor system built by Modelica is illustrated in Fig. 19, which includes two discs, two bearings and three shafts. The parameters of every component are listed in Table 2.

Fig. 19. Model of turbo expander under normal condition

Table 2. Parameters of turbo expander

Component	Parameter	Value
World	Gravity $g/(\mathrm{m \cdot s^{-2}})$	9.81
	Rotating velocity $\omega_r/(\mathrm{r \cdot min^{-1}})$	6×10^4
Compress disc	Mass m /kg	0.06
	Eccentricity e/m	5×10^{-5}
Compress shaft	Length l / m	0.025
	Diameter d/m	0.01
	Elastic module E/GPa	205
Compress bearing	Mass m /kg	0.015
	Stiffness $k/(\mathrm{MN \cdot m^{-1}})$	1
	Damping $c/(\mathrm{N \cdot s \cdot m^{-1}})$	100
Middle shaft	Length l / m	0.03
	Diameter d/m	0.008
	Elastic module E/GPa	205
Expand bearing	Mass m / kg	0.02
	Stiffness $k/(\mathrm{MN \cdot m^{-1}})$	1
	Damping $c/(\mathrm{N \cdot s \cdot m^{-1}})$	100
Expand shaft	Length l / m	0.025
	Diameter d/m	0.014
	Elastic module E/GPa	205
Expand disc	Mass m/ kg	0.07
	Eccentricity e/m	5×10^{-5}

The vibration data of displacement and velocity of expand wheel are extracted after simulation and be imported to other data processing software for further processing and analyzing. In Fig. 20, five kinds of important curves in study of rotor dynamics including vibration signal, orbit of rotor axis, frequency spectrum, phase trajectory and Poincare map are illustrated. In these figures it can be concluded that the rotor system of turbo expander under normal condition has a stable 1-period rotating motion: one basic frequency, circular orbit of rotor axis and phase trajectory, and 1-period Poincare map.

4.2 Turbo expander with crack

Crack happens at the root of bearing besides compress wheel, as modeled in Fig. 21. Crack is treated as external force acting at the root of bearing, the only difference between normal condition and crack fault is the addition and connection of crack fault.

In this model, the depth of crack is assumed to be $T = 0.004$ m , the vibration response and other interesting curves of expand wheel are illustrated in Fig. 22, all the five diagrams resemble the ones without crack fault in Fig. 20. It can be concluded that the existence of crack fault around compress side hardly affects the vibration of expand wheel, the motion of expand wheel keeps to be 1-period motion as the normal condition.

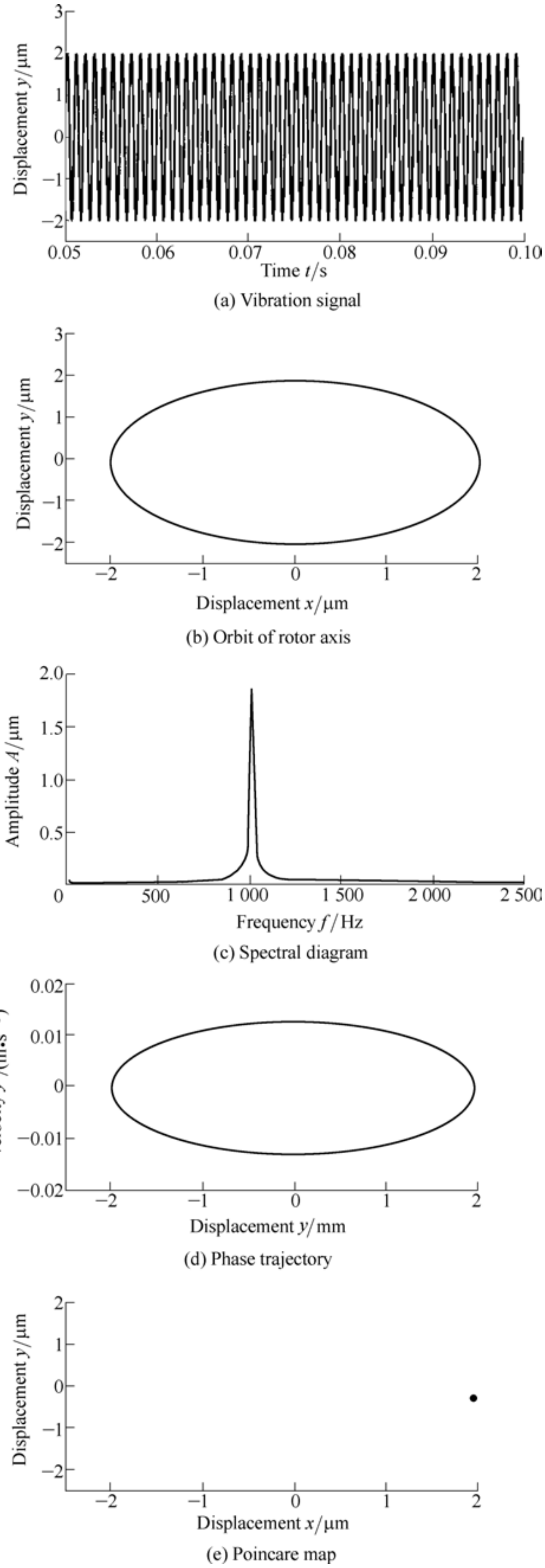


Fig. 20. Response of turbo expander under normal condition

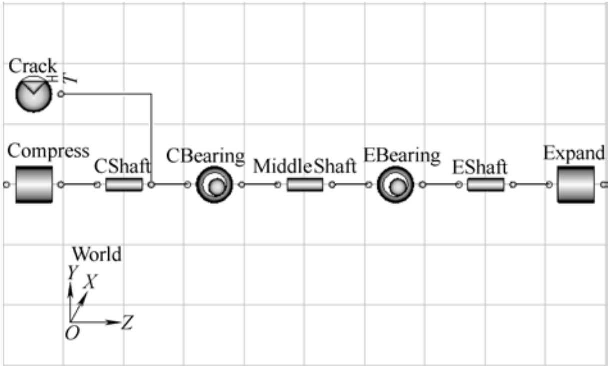
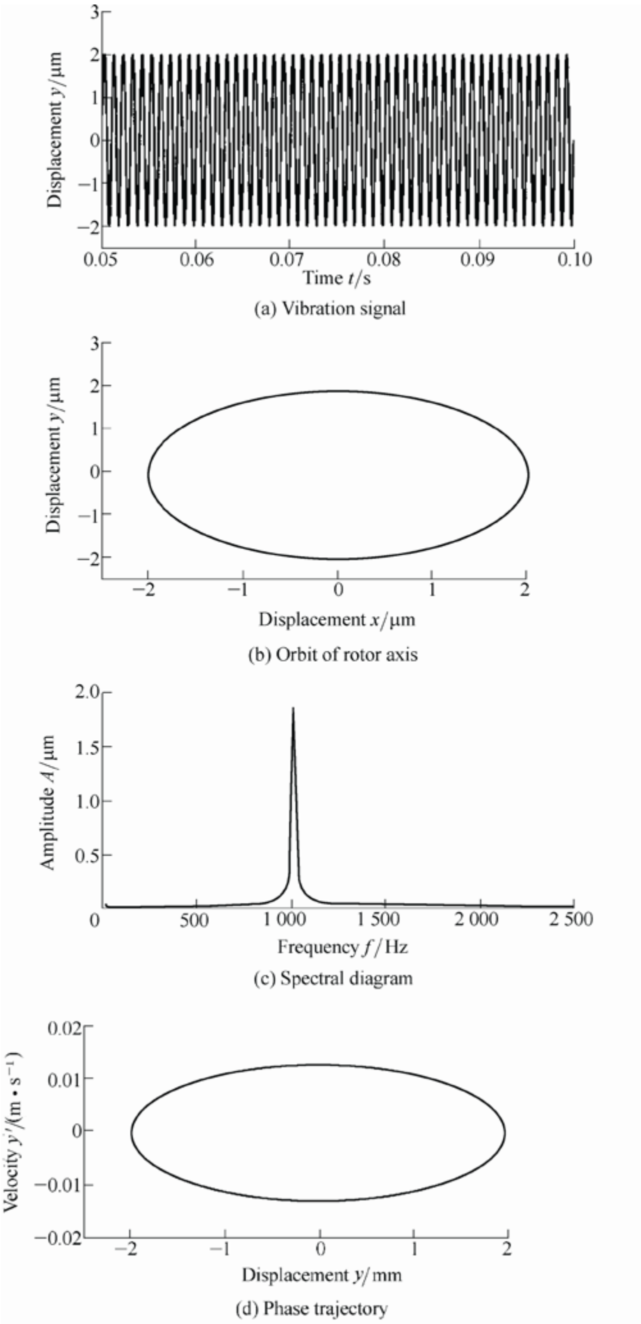


Fig. 21.　Model of turbo expander with crack

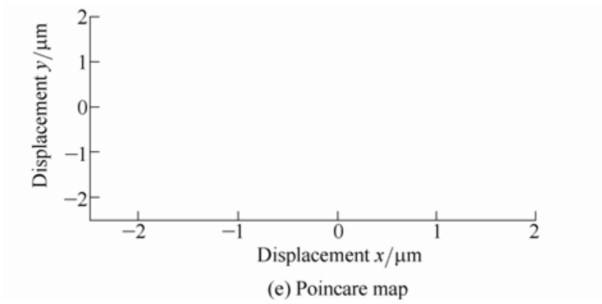


(a) Vibration signal

(b) Orbit of rotor axis

(c) Spectral diagram

(d) Phase trajectory



(e) Poincare map

Fig. 22.　Response of turbo expander with crack

4.3　Turbo expander with rub-impact

Similar with crack fault, the rub-impact fault is also treated as an external force. In this case study, the rub-impact happens at the compress wheel, and the vibration responses of expand wheel are of interest. The clearance, collision stiffness and friction coefficient are assumed to be c=1 μm, $k_c = 2.5\ \text{MN} \bullet \text{m}^{-1}$ and $f = 0.7$. The model of turbo expander with rub-impact is illustrated in Fig. 23, and the diagrams are shown in Fig. 24.

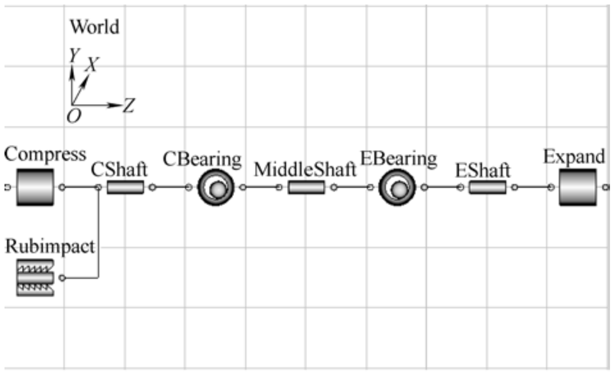
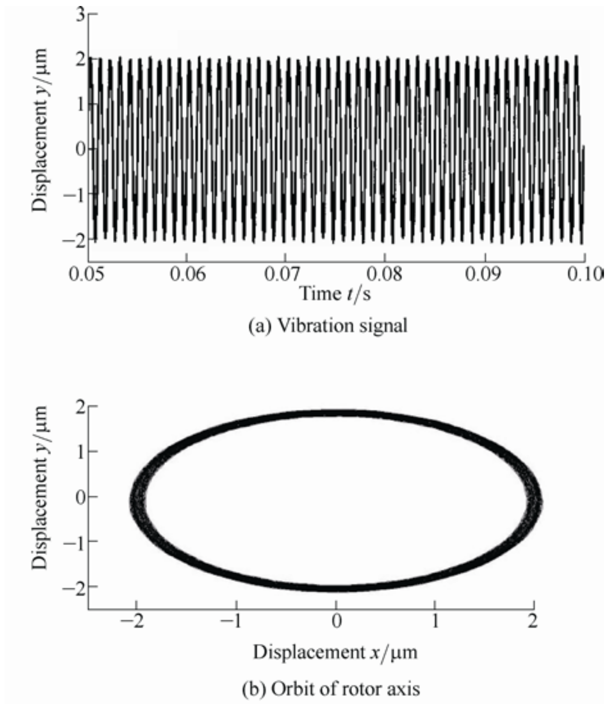


Fig. 23.　Model of turbo expander with rub-impact



(a) Vibration signal

(b) Orbit of rotor axis

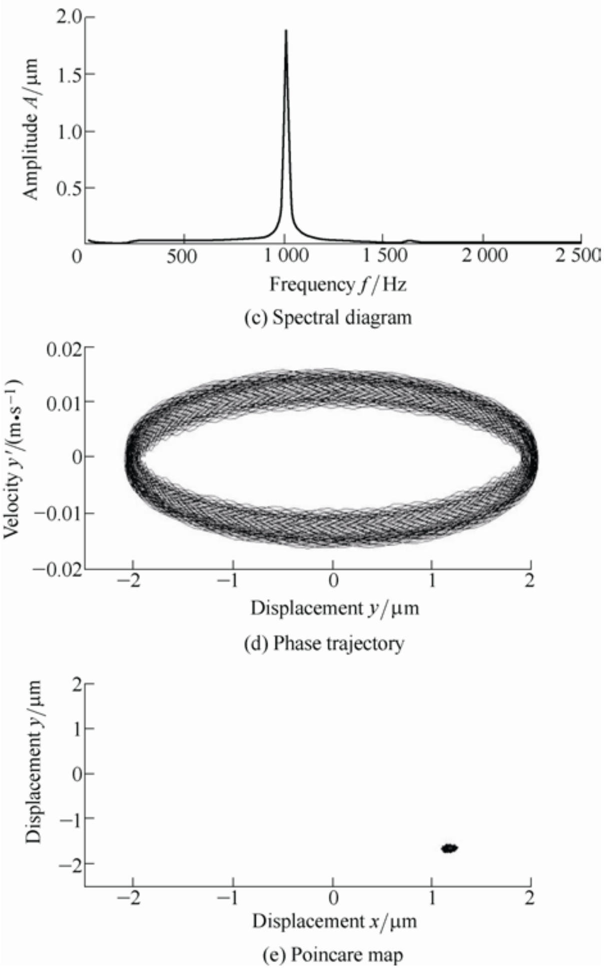


Fig. 24. Response of turbo expander with rub-impact

It can be acquired from Fig. 24 that the occurrence of rub-impact fault on compress wheel forces the vibration of expand wheel into quasi-periodic motion: orbit of axis and phase trajectory turns into overlapping circles and Poincare map shows a closed trajectory. There also appear two frequency components with relatively low amplitudes in the frequency diagram.

4.4 Turbo expander with pedestal looseness

In modeling of pedestal looseness, the bearing with looseness must replace the normal one to stimulate the fault situation, as shown in Fig. 25. The bearing beside compress wheel is changed into fault of pedestal looseness. The length of looseness, bias of looseness and collision stiffness are set to be $b = 0.01$ m, $c = 0$ m, and $k = 10^6$ N/m.

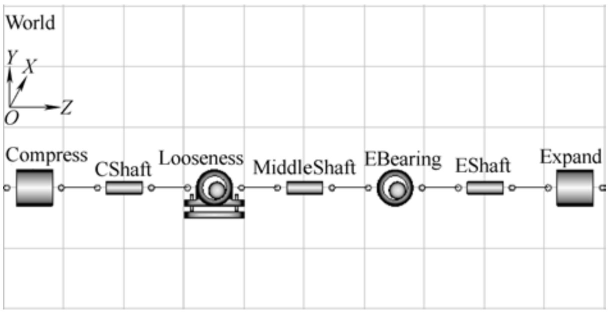
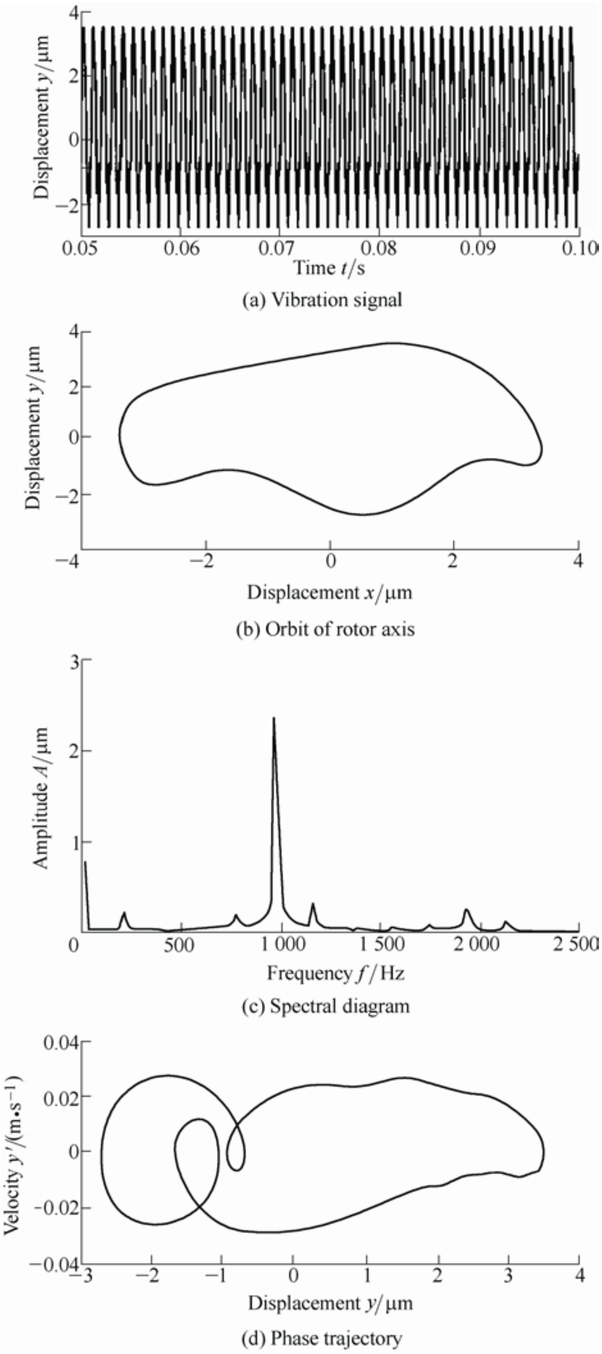


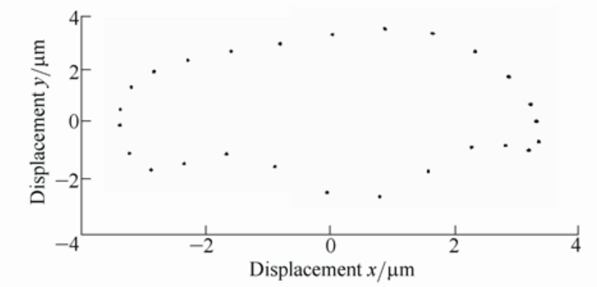
Fig. 25. Model of turbo expander with pedestal looseness

The curves are shown in Fig. 26. It can be seen from vibration signal that the existence of pedestal looseness enhances the vibration amplitude. The orbit of axis and phase trajectory are deformed greatly. There appears to be several frequency components in the frequency diagram. Poincare map seems that the expand wheel becomes multi-periodic motion. The pedestal looseness around compress side influences the vibration of expand wheel greatly.

4.5 Turbo expander with multi-faults

In Fig. 27, a model of turbo expander with crack, rub-impact and pedestal looseness is illustrated. Crack and rub-impact faults are treated as external force and pedestal looseness fault replaces normal bearing. All the faults act around compress side of turbo expander.





(e) Poincare map

Fig. 26. Response of turbo expander with pedestal looseness

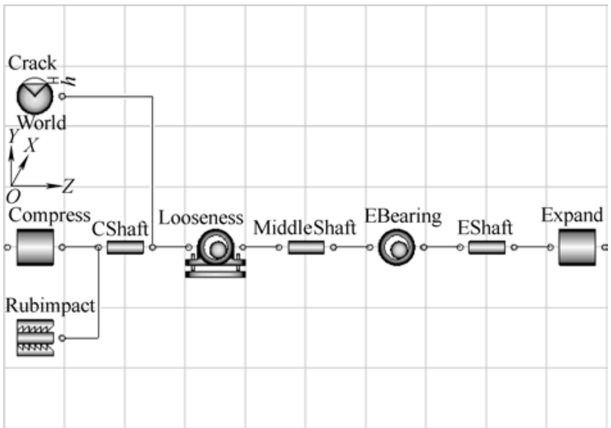
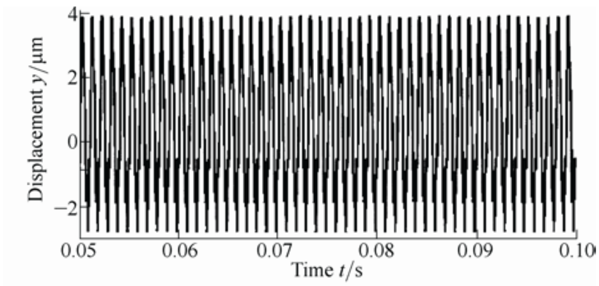
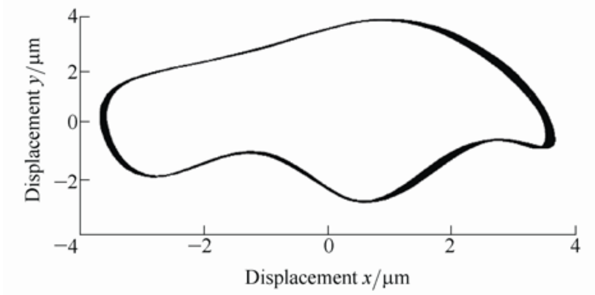


Fig. 27. Model of turbo expander with multi-faults

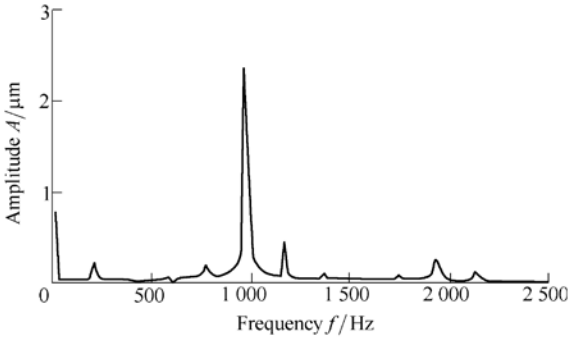
Fig. 28 shows five diagrams of turbo expander with multi-faults. It can be seen that the vibration of expand wheel becomes complicated because of the multi-faults acting simultaneously. It is difficult to decompose three faults, however, it can be concluded that the influence to turbo expander by pedestal looseness is dominant and the quasi-periodic motion indicates the existence of rub-impact, but crack fault cannot be diagnosed from these diagrams.



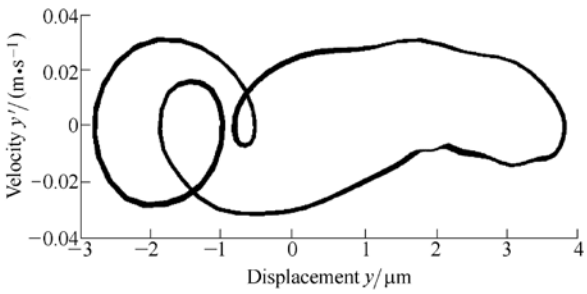
(a) Vibration signal



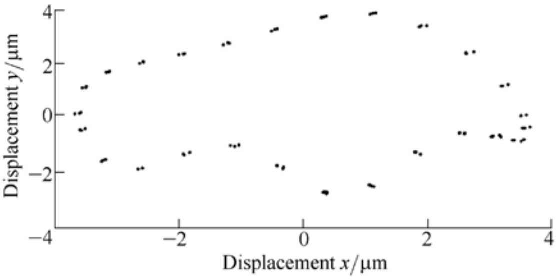
(b) Orbit of rotor axis



(c) Spectral diagram



(d) Phase trajectory



(e) Poincare map

Fig. 28. Response of turbo expander with multi-faults

5 Conclusions

(1) Object-orient method for rotor system is as accurate as non-object-orient method. This method has advantages of rapid, accurate and easy to modify, which is suitable for prototype modeling.

(2) For faults exist around compress side of turbo expander, the existence of crack fault does not influence the vibration of expand wheel greatly; expand wheel enters quasi-periodic motion because of the existence of rub-impact fault; the orbit of expand wheel axis is deformed seriously due to the existence of pedestal looseness fault.

(3) When turbo expander is affected by faults of crack, rub-impact and pedestal looseness simultaneously, it is hard to decompose them. However, the influence to turbo expander by pedestal looseness is dominant; the quasi-periodic motion indicates the existence of rub-impact.

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