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Frequency Analysis of Multiple Layered Cylindrical Shells under Lateral Pressure with Asymmetric Boundary Conditions

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Abstract: Natural frequency characteristics of a thin-walled multiple layered cylindrical shell under lateral pressure are studied. The multiple layered cylindrical shell configuration is formed by three layers of isotropic material where the inner and outer layers are stainless steel and the middle layer is aluminum. The multiple layered shell equations with lateral pressure are established based on Love's shell theory. The governing equations of motion with lateral pressure are employed by using energy functional and applying the Ritz method. The boundary conditions represented by end conditions of the multiple layered cylindrical shell are simply supported-clamped(SS-C), free-clamped(F-C) and simply supported-free(SS-F). The influence of different lateral pressures, different thickness to radius ratios, different length to radius ratios and effect of the asymmetric boundary conditions on natural frequency characteristics are studied. It is shown that the lateral pressure has effect on the natural frequency of multiple layered cylindrical shell and causes the natural frequency to increase. The natural frequency of the developed multilayered cylindrical shell is validated by comparing with those in the literature. The proposed research provides an effective approach for vibration analysis shell structures subjected to lateral pressure with an energy method.

Keywords: multiple layered, ritz method, lateral pressure, shell, boundary conditions

1 Introduction

Thin-walled structures such as shells are generally used as constructive components in many engineering applications. The shell structures are usually flat plates and cupola. In comparison with plates and beams, shells are usually exposed to more different dynamic behaviours because they can carry applied loads effectively by their curvatures. The dynamic characteristic of shells has been reported by many researchers. It was first recommend by LOVE[1]. He was the first scholar to use Kirchhoff hypothesis for shell structures and named love's shell theory. Then ARNOLD and WARBURTON[2] used this shell theory for cylindrical shell based vibration analysis.

A particular type of shell structure is cylindrical shell. Cylindrical shells are constructions with different materials for various applications in engineering from large aerospace, naval construction, civil and mechanical structures to small electrical components[3]. Mechanical and dynamics behaviours, including vibration, buckling and impact, are reasons for the popularity of cylindrical shell structures in engineering. They are used as structures in aircrafts, ships, rockets, missiles, pressure vessels, oil tanks[4–5]. Thin-walled multiple layered cylindrical shells with lateral pressure subjected to vibration is significant for a successful usage of these structures. There are group of papers being reported on vibration of cylindrical shells that include factors such as boundary conditions, thicknesses, stresses, thermal loads but the vibration of multiple layered cylindrical shells subjected to lateral pressure using energy method is restricted. Some works on vibration analysis of cylindrical shells have been reported. ARNOLD and WARBURTON[6] and ZHANG, et al[7], studied the walls of cylindrical shells and derived the equations of motion. FORSBERG[8], NAJAFIZADEH and ISVANDZIBAEI et al[9], ISVANDZIBAEI, et al[10–11], PRADHAN, et al[12], and DAI, et al[13], presented numerical analysis to prove the importance of edge conditions on free vibration. Vibration of cylindrical shells were analysed by SHARMA[14] and BAKHTIARI, et al[15], using Sander's thin-walled shell theory. SOEDEL[16], CHUNG[17], HUA, et al[18], and JUNG, et al[19], worked on circular cylindrical shells. LAM and LOY[20–21] presented works on rotating cylindrical shell based on vibration analysis. Differential quadrant method has been used by BERT and MALIK[22] to study the frequency behaviour of cylindrical shell. Some authors used functionally graded material(FGM) in shell structures and analysed the vibration behaviours. Some of them are LOY, et al[23], NAJAFIZADEH and ISVANDZIBAEI[24], GHAFAR, et al[25–26], ARSHAD, et al[27], and XIANG, et al[28].

The effect of buckling on cylindrical shell subjected to

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temperature was presented by KADOLI and GANESAN[29]. Similarly SHI-RONG and BATRA[30] worked on buckling of circular cylindrical shells. MALEKZADEH, et al[31], used composite material in cylindrical shells for dynamic action.

A method of analysing a multiple layered cylindrical shell is to determine the natural frequencies, when subjected to lateral pressure. Lateral pressure is often an important loading condition for these multiple layered cylindrical shells and causes high natural frequencies due to vibration. The behavior of multiple layered cylindrical shells under lateral pressure is dependent on the strength of structure. For pressurized cylindrical shells filled by fluid and subjected to lateral pressure, problem may arise if vibration developed as a consequence of interaction between lateral pressure, liquid and deformation of the shell. Therefore potential energy of the fluid is considered as variable at the multiple layered cylindrical shell elements where its motion is expressed in terms of displacement at the fluid. For fluid filled multiple layered cylindrical shell with lateral pressure, the fluid boundaries are divided into two sections, such as free surface boundary and fluid-multiple layered shell interaction boundary. Reported works on vibration of multiple layered cylindrical shells composed of stainless steel and aluminum subjected to lateral pressure could not be found in the literature.

This paper presents the study on the natural frequency characteristics of a multiple layered cylindrical shell under lateral pressure with asymmetric boundary conditions. The analysis is carried out based on Love theory. The governing equations with lateral pressure are derived using energy method by solving Ritz technique. The multiple layered cylindrical shell is made-up of isotropic three layers where the inner and outer layers are made of stainless steel and the middle layer is aluminum. The analysis of the natural frequency characteristics of the multiple layered cylindrical shell is presented with asymmetric boundary conditions by using beam functions as the axial modal functions. The boundary conditions of the multiple layered cylindrical shell considered are the combination of simply supported-clamped(SS-C), clamped-free(C-F) and free-simply supported(F-SS). The influence of different lateral pressures, different thickness to radius ratios, different length to radius ratios and effect of the asymmetric boundary conditions on natural frequencies characteristics are discussed. The results obtained from this method are validated by comparing with the results for cylindrical shells without lateral pressure reported in the literature.

2 Theoretical Formulation

Consider a thin-walled multiple layered cylindrical shell with the thickness h, radius of the shell R, length L, mass density ρ, modulus of elasticity E, and Poisson's ratio ν, as shown in Fig. 1. An orthogonal coordinate system is established at the mid-surface of the multiple layered shell along x, θ and z, the axial, circumferential and radial directions respectively. The corresponding displacement deformations from the multiple layered shell mid-surface are defined by u, v and w respectively. Thickness of the thin-walled multiple layered cylindrical shell is divided into three layers where the inner and outer layers are of stainless steel and the middle layer is aluminum.

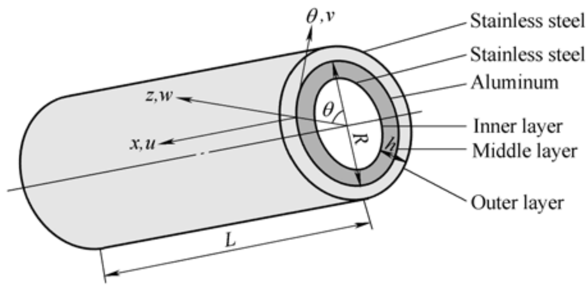

Fig. 1. Geometry of multiple layered cylindrical shell with the coordinate system

The stress-strain relations is given by Hooke's law as

$$\bar{\boldsymbol{\sigma}} = \bar{\boldsymbol{Q}}\bar{\boldsymbol{e}} . \tag{1}$$

where $\bar{\boldsymbol{\sigma}}$, $\bar{\boldsymbol{e}}$ are the corresponding stress and strain vectors respectively and $\bar{\boldsymbol{Q}}$ is the reduced stiffness matrix with Kirchhoff hypothesis expressed as follows:

$$\bar{\boldsymbol{\sigma}}^{\mathrm{T}} = \{\ \bar{\sigma}_x \quad \bar{\sigma}_\theta \quad \bar{\sigma}_{x\theta}\ \}, \tag{2}$$

$$\bar{\boldsymbol{e}}^{\mathrm{T}} = \{\ \bar{e}_x \quad \bar{e}_\theta \quad \bar{e}_{x\theta}\ \}, \tag{3}$$

$$\bar{\boldsymbol{Q}} = \begin{pmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 \\ 0 & 0 & \bar{Q}_{66} \end{pmatrix}. \tag{4}$$

Then Eq. (1) can be expressed as

$$\begin{Bmatrix} \bar{\sigma}_x \\ \bar{\sigma}_\theta \\ \bar{\sigma}_{x\theta} \end{Bmatrix} = \begin{pmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 \\ 0 & 0 & \bar{Q}_{66} \end{pmatrix} \begin{Bmatrix} \bar{e}_x \\ \bar{e}_\theta \\ \bar{e}_{x\theta} \end{Bmatrix} . \tag{5}$$

For cylindrical shells, the stiffness $\bar{Q}_{ij}$ are defined as

$$\bar{Q}_{11} = \frac{E}{1-\nu^2},\ \ \bar{Q}_{12} = \frac{\nu E}{1-\nu^2},\ \ \bar{Q}_{22} = \frac{E}{1-\nu^2},$$
$$\bar{Q}_{66} = \frac{E}{2(1+\nu)}, \tag{6}$$

where E is modulus of elasticity, and ν is Poisson's ratio. According to Love shell theory[32], the strain components in the strain vector $\bar{\boldsymbol{e}}$ are defined as linear functions of the thickness coordinate z as

$$\bar{e}_x = \bar{e}_1 + z\bar{\boldsymbol{k}}_1,\ \ \bar{e}_\theta = \bar{e}_2 + z\bar{k}_2,\ \ \bar{e}_{x\theta} = \bar{\gamma} + 2z\bar{\tau} . \tag{7}$$

where $\overline{e}_1$, $\overline{e}_2$ and $\overline{\gamma}$ are the area strains and $\overline{k}_1$, $\overline{k}_2$ and $\overline{\tau}$ are the surface curvatures and defined as follows:

$$\overline{e}_1 = \frac{\partial u}{\partial x},\ \overline{e}_2 = \frac{1}{R}\left(\frac{\partial \nu}{\partial \theta} + w\right),\quad \overline{\gamma} = \frac{\partial \nu}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta},$$

$$\overline{k}_1 = -\frac{\partial^2 w}{\partial x^2},\ \overline{k}_2 = -\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial \nu}{\partial \theta}\right), \tag{8}$$

$$\overline{\tau} = -\frac{1}{R}\left(\frac{\partial^2 w}{\partial x \partial \theta} - \frac{\partial \nu}{\partial x}\right). \tag{9}$$

From Eqs. (5) and (7), the stress vector $\overline{\boldsymbol{\sigma}}$ is defined as follows:

$$\overline{\sigma}_x = (\overline{e}_1 + z\overline{k}_1)\overline{Q}_{11} + (\overline{e}_2 + z\overline{k}_2)\overline{Q}_{12}, \tag{10}$$

$$\overline{\sigma}_\theta = (\overline{e}_1 + z\overline{k}_1)\overline{Q}_{12} + (\overline{e}_2 + z\overline{k}_2)\overline{Q}_{22}, \tag{11}$$

$$\overline{\sigma}_{x\theta} = (\overline{\gamma} + 2z\overline{\tau})\overline{Q}_{66}. \tag{12}$$

The force and moment resultants are defined by

$$\{N_x, N_\theta, N_{x\theta}\} = \int_{-h/2}^{h/2} \{\overline{\sigma}_x,\ \overline{\sigma}_\theta,\ \overline{\sigma}_{x\theta}\}\,\mathrm{d}z, \tag{13}$$

$$\{M_x, M_\theta, M_{x\theta}\} = \int_{-h/2}^{h/2} \{\overline{\sigma}_x,\ \overline{\sigma}_\theta,\ \overline{\sigma}_{x\theta}\}\,z\,\mathrm{d}z, \tag{14}$$

where $N_{x\theta}$ N and $N_{x\theta}$ are force components in axial, circumferential and shear directions, respectively and $M_{x\theta}$ M and $M_{x\theta}$ are moment components in axial, circumferential and shear directions, respectively.

Eqs. (7), (13) and (14) are combined as

$$\boldsymbol{N} = \boldsymbol{L}\overline{\boldsymbol{e}}, \tag{15}$$

where $\boldsymbol{L}$ and $\overline{\boldsymbol{e}}$ are expressed as follows:

$$\boldsymbol{N}^{\mathrm{T}} = \{N_x, N_\theta, N_{x\theta}, M_x, M_\theta, M_{x\theta}\}, \tag{16}$$

$$\overline{\boldsymbol{e}}^{\mathrm{T}} = \{\overline{e}_1, \overline{e}_2, \overline{\gamma}, \overline{k}_1, \overline{k}_2, 2\overline{\tau}\}, \tag{17}$$

and $\boldsymbol{L}$ for multiple layered cylindrical shell is defined as

$$\boldsymbol{L} = \begin{pmatrix} \boldsymbol{X} & \boldsymbol{Y} \\ \boldsymbol{Y} & \boldsymbol{Z} \end{pmatrix}, \tag{18}$$

where $\boldsymbol{X}$, $\boldsymbol{Y}$, $\boldsymbol{Z}$ are the extensional, coupling, bending stiffness, expressed as follows:

$$\boldsymbol{X} = \begin{pmatrix} X_{11} & X_{12} & 0 \\ X_{12} & X_{22} & 0 \\ 0 & 0 & X_{66} \end{pmatrix}, \tag{19}$$

$$\boldsymbol{Y} = \begin{pmatrix} Y_{11} & Y_{12} & 0 \\ Y_{12} & Y_{22} & 0 \\ 0 & 0 & Y_{66} \end{pmatrix}, \tag{20}$$

$$\boldsymbol{Z} = \begin{pmatrix} Z_{11} & Z_{12} & 0 \\ Z_{12} & Z_{22} & 0 \\ 0 & 0 & Z_{66} \end{pmatrix}. \tag{21}$$

The matrix $\boldsymbol{L}$ in terms of X_{ij}, Y_{ij} and Z_{ij} can be written as

$$\boldsymbol{L} = \begin{pmatrix} X_{11} & X_{12} & 0 & Y_{11} & Y_{12} & 0 \\ Y_{12} & Y_{22} & 0 & Y_{12} & Y_{22} & 0 \\ 0 & 0 & X_{66} & 0 & 0 & Y_{66} \\ Y_{11} & Y_{12} & 0 & Z_{11} & Z_{12} & 0 \\ Y_{12} & Y_{21} & 0 & Z_{12} & Z_{22} & 0 \\ 0 & 0 & X_{66} & 0 & 0 & Z_{66} \end{pmatrix}. \tag{22}$$

For a multiple layered shell, the stiffness are given by LAM and LOY[4]:

$$x_{ij} = \sum_{k=1}^{T} Q_{ij}^{\ k}(t_k - t_{k-1}), \tag{23}$$

$$Y_{ij} = \frac{1}{2}\sum_{k=1}^{T} Q_{ij}^{\ k}({t_k}^2 - {t_{k-1}}^2), \tag{24}$$

$$Z_{ij} = \frac{1}{3}\sum_{k=1}^{T} Q_{ij}^{\ k}({t_k}^3 - {t_{k-1}}^3). \tag{25}$$

where t_k and t_{k-1} are the distances from the mid-surface of the multiple cylindrical shell, T is number of layers and $Q_{ij}^{\ k}$ is the reduced stiffness for the k-th layer as defined in Eq. (6). By substituting Eqs. (16)–(21) into Eq. (15) for a multiple layered cylindrical shell, thus

$$\begin{Bmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{Bmatrix} = \begin{pmatrix} X_{11} & X_{12} & 0 & Y_{11} & Y_{12} & 0 \\ Y_{12} & Y_{22} & 0 & Y_{12} & Y_{22} & 0 \\ 0 & 0 & X_{66} & 0 & 0 & Y_{66} \\ Y_{11} & Y_{12} & 0 & Z_{11} & Z_{12} & 0 \\ Y_{12} & Y_{21} & 0 & Z_{12} & Z_{22} & 0 \\ 0 & 0 & X_{66} & 0 & 0 & Z_{66} \end{pmatrix} \begin{Bmatrix} \overline{e}_1 \\ \overline{e}_2 \\ \overline{\gamma} \\ \overline{k}_1 \\ \overline{k}_2 \\ 2\overline{\tau} \end{Bmatrix}. \tag{26}$$

3 Energy Equations

To investigate the relationship between lateral pressure with strain energy and kinetic energy, consider a thin-walled multiple layered cylindrical shell under uniform external lateral pressure as shown in Fig 2.

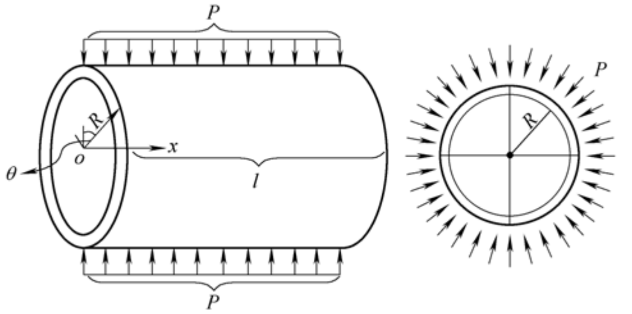

Fig. 2. Geometry of a multiple layered cylindrical shell subjected to uniform external lateral pressure

The potential energy of the external lateral pressure for multiple layered cylindrical shell F, is

$$F=\int_0^L\int_0^{2\pi}\frac{P}{2}\left[\left(\frac{\partial^2 w}{\partial\theta^2}+w\right)w\right]\mathrm{d}\theta\,\mathrm{d}x. \tag{27}$$

The strain energy of thin-walled multiple layered cylindrical shell U is

$$U=\frac{1}{2}\int_0^L\int_0^{2\pi}\bar{\boldsymbol{e}}^{\mathrm{T}}\boldsymbol{L}\,\bar{\boldsymbol{e}}R\mathrm{d}\theta\,\mathrm{d}x. \tag{28}$$

Substitution of $\boldsymbol{L}$ and $\bar{\boldsymbol{e}}$ into the strain energy for multiple layered cylindrical shell, thus

$$\begin{aligned}U=&\frac{1}{2}\int_0^L\int_0^{2\pi}\{\bar{e}_1^2X_{11}+\bar{e}_1\bar{e}_2X_{12}+\bar{k}_1\bar{e}_1Y_{11}+\bar{k}_2Y_{12}\bar{e}_1+\\&\bar{e}_1\bar{e}_2X_{12}+\bar{e}_2^2X_{22}+\bar{k}_1\bar{e}_2Y_{12}+\bar{k}_2\bar{e}_2Y_{22}+\bar{\gamma}^2X_{66}+2\bar{\tau}\,\bar{\gamma}\,Y_{66}+\\&\bar{k}_1\bar{e}_1Y_{11}+\bar{k}_1\bar{e}_2Y_{12}+\bar{k}_1^2Z_{11}+\bar{k}_1\bar{k}_2Z_{12}+\bar{k}_2\bar{e}_1Y_{12}+\bar{k}_2Y_{22}\bar{e}_2+\\&\bar{k}_1\bar{k}_2Z_{12}+\bar{k}_2^2Z_{22}+2\bar{\tau}\,\bar{\gamma}\,Y_{66}+4\bar{\tau}^2Z_{66}\}\,R\,\mathrm{d}\theta\,\mathrm{d}x.\end{aligned} \tag{29}$$

The total potential energy for vibration of multiple layered cylindrical shell under lateral pressure is

$$U_{\text{total}}=U+F. \tag{30}$$

The kinetic energy for vibration of thin-walled multiple layered cylindrical shell under uniform external lateral pressure, is given by

$$T=\frac{1}{2}\int_0^L\int_0^{2\pi}\rho_T\left[\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial v}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right]R\mathrm{d}\theta\,\mathrm{d}x. \tag{31}$$

where ρ_T is the mass density per unit surface area, defined for multiple layered cylindrical shell:

$$\rho_T=\sum_{k=1}^{T}\rho_k(t_k-t_{k-1}). \tag{32}$$

4 Displacement Field and Boundary Conditions

The displacement field for multiple layered cylindrical shell under lateral pressure with asymmetric boundary conditions can be expressed as follows:

$$\begin{cases}u=E_1\dfrac{\partial\Omega(x)}{\partial x}\cos(n\theta)\cos(\omega t),\\ v=E_2\Omega(x)\sin(n\theta)\cos(\omega t),\\ w=E_3\Omega(x)\cos(n\theta)\cos(\omega t),\end{cases} \tag{33}$$

where E_1, E_2 and E_3 are constants denoting vibrations in the axial u, circumferential v and radial w directions. The axial modal function is denoted by $\Omega(x)$, n denotes the number of circumferential waves in the mode shape and ω is the natural frequency of the vibration.

The axial modal function $\Omega(x)$ in Eq. (33) is chosen to satisfy condition at both ends of the multiple layered cylindrical shell. The beam modal function has been chosen as the axial modal function and expressed by MOON and SHAW[33].

$$\begin{aligned}\Omega(x)=&\Psi_1\cosh\left(\frac{\Phi_{\mathrm{m}}x}{L}\right)+\Psi_2\cos\left(\frac{\Phi_{\mathrm{m}}x}{L}\right)-\\&\mu_{\mathrm{m}}\left[\Psi_3\sinh\left(\frac{\Phi_{\mathrm{m}}x}{L}\right)+\Psi_4\sin\left(\frac{\Phi_{\mathrm{m}}x}{L}\right)\right].\end{aligned} \tag{34}$$

where, the values of $\Psi_i\,(i=1,\cdots,4)$, Φ_{m} and μ_{m} for multiple layered cylindrical shell under lateral pressure with asymmetric boundary conditions considered are given in Table 1.

Table 1. Values of boundary conditions

Boundary condition	$\Psi_i\,(i=1,\cdots,4)$	Φ_{m}	μ_{m}
SS-C	$\Psi_1=1,\ \Psi_2=-1,$ $\Psi_3=1,\ \Psi_4=-1.$	$\tan\Phi_{\mathrm{m}}=\tanh\Phi_{\mathrm{m}}$	$\dfrac{\cosh\Phi_{\mathrm{m}}-\cos\Phi_{\mathrm{m}}}{\sinh\Phi_{\mathrm{m}}-\sin\Phi_{\mathrm{m}}}$
C-F	$\Psi_1=1,\ \Psi_2=-1,$ $\Psi_3=1,\ \Psi_4=-1.$	$\cos\Phi_{\mathrm{m}}\cosh\Phi_{\mathrm{m}}=-1$	$\dfrac{\sinh\Phi_{\mathrm{m}}-\sin\Phi_{\mathrm{m}}}{\cosh\Phi_{\mathrm{m}}+\cos\Phi_{\mathrm{m}}}$
F-SS	$\Psi_1=1,\ \Psi_2=-1,$ $\Psi_3=1,\ \Psi_4=1.$	$\tan\Phi_{\mathrm{m}}=\tanh\Phi_{\mathrm{m}}$	$\dfrac{\cosh\Phi_{\mathrm{m}}-\cos\Phi_{\mathrm{m}}}{\sinh\Phi_{\mathrm{m}}-\sin\Phi_{\mathrm{m}}}$

The geometric boundary conditions for simply supported, clamped and free that satisfy the applied boundary conditions at the ends of multiple layered cylindrical shell, $x=0$ and $x=L$ can be expressed in terms of $\Omega(x)$ as follows.

Simply supported boundary condition:

$$\Omega(0)=\frac{\partial^2\Omega(L)}{\partial x^2}=0. \tag{35}$$

Free boundary condition:

$$\frac{\partial^2\Omega(0)}{\partial x^2}=\frac{\partial^3\Omega(L)}{\partial x^3}=0. \tag{36}$$

Clamped boundary condition:

$$\Omega(0)=\frac{\partial\Omega(L)}{\partial x}=0. \tag{37}$$

5 Ritz Method

Ritz method is commonly used as an approximation method for a numerical solution of various boundary value problems in mechanics. To determine the natural frequency of vibration for multiple layered cylindrical shells under lateral pressure, the Ritz method is used. The energy functional, $\varPi$, defined by the Lagrangian function for vibration of multiple layered cylindrical shells under external lateral pressure is

$$\varPi = T_{\max} - U_{\text{total max}}. \tag{38}$$

Substituting Eq. (33) into Eqs. (27), (29) and (31) and applying Ritz method with minimizing the energy functional $\varPi$ with respect to the unknown coefficients as follows:

$$\frac{\partial \varPi}{\partial E_1} = \frac{\partial \varPi}{\partial E_2} = \frac{\partial \varPi}{\partial E_3} = 0. \tag{39}$$

This equation includes three equations of motion for multiple layered cylindrical shell under lateral pressure is obtained.

The three governing eigenvalues of the equations of motion can be expressed in matrix from as

$$\begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{pmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}. \tag{40}$$

The solution is obtained by setting the determinant of matrix $\boldsymbol{C}$ equals to zero, i.e.,

$$|\boldsymbol{C}| = 0. \tag{41}$$

The solution of Eq. (41) is the characteristic of the multiple layered cylindrical shell under lateral pressure expressed in the power of ω as

$$\beta_0\omega^6 + \beta_1\omega^4 + \beta_2\omega^2 + \beta_3 = 0, \tag{42}$$

where $\beta_i (i = 0,1,2,3)$ are some constants. Eq. (42) can be solved using Newton-Raphson procedure where three positive and three negative roots are obtained. The three positive roots obtained are the natural frequencies of multiple layered cylindrical shell with asymmetric boundary conditions under lateral pressure. The smallest of the three positive roots is the natural frequency of interest in the present study.

6 Numerical Results

To validate the accuracy of the model developed, the results for multiple layered cylindrical shell without lateral pressure are compared with those in the literature. The material properties of the three multiple layered cylindrical shell are given in Table 2.

Table 2. Material properties of the multiple layered cylindrical shell

Layers status	Type of materials	Young's coefficient $E/(\mathrm{GN\cdot m^{-2}})$	Poisson ratio ν	Density $\rho/(\mathrm{kg\cdot m^{-3}})$
Outer layer	Stainless steel	210	0.28	7.8×10^3
Middle layer	Aluminum	70	0.35	2.7×10^3
Inner layer	Stainless steel	210	0.28	7.8×10^3

Table 3 shows the comparison of the natural frequency parameter $\varGamma = \omega R\sqrt{(1-\nu^2)\rho / E}$ with those in LOY, et al[34] and ZHANG, et al[35] for multiple layered cylindrical shell without lateral pressure with simply support- clamped boundary conditions. Table 4 shows the comparison of the natural frequency (Hz) with those in SHARMA[36] for multiple layered cylindrical shell without lateral pressure with clamped-free boundary conditions. From the comparisons presented in Tables 3 and 4, it can be seen that the results agreed well with those in the literature. The comparisons with cylindrical shells subjected to lateral pressure are not presented as the results for a multiple layered cylindrical shell are not found in the literature. The finding could be validated experimentally which is out of the scope of this study. The experiment could be designed based on the study of buckling under pressure to create the effect of lateral pressure[37].

Table 3. Comparison of values of the natural frequency parameter $\varGamma = \omega R\sqrt{(1-\nu^2)\rho / E}$ for a multiple layered cylindrical shell without lateral pressures with SS-C boundary condition (m = 1, R/L = 0.005, R/h = 100)

n	LOY, et al[34]	ZHANG, et al[35]	Present
1	0.032 8	0.034 8	0.031 1
2	0.013 9	0.014 0	0.012 2
3	0.022 6	0.022 7	0.021 4
4	0.042 2	0.042 2	0.043 7
5	0.068 0	0.068 1	0.069 3
6	0.099 7	0.099 8	0.098 4
7	0.137 2	0.137 3	0.138 6
8	0.180 8	0.180 6	0.182 1
9	0.229 5	0.229 6	0.229 9
10	0.284 4	0.284 5	0.285 0

Table 4. Natural frequency of multiple layered cylindrical shell without lateral pressure with C-F boundary condition. (L = 502 mm, R = 63.5 mm, h = 1.63 mm)

m	n	SHARMA[36]	Present
1	2	319.5	318.462
1	3	769.9	768.323
1	4	1 465.8	1 467.78
1	5	2 367.1	2 366. 02
1	6	3 470.3	3 468.89

7 Results and Discussion

In this paper, a multiple layered cylindrical shell is subjected to lateral pressure is analysed. The analyses are conducted by assuming lateral pressures equal 50 kPa and 100 kPa. Altogether, three boundary conditions are discussed in this paper. The effects of the asymmetric boundary conditions on the natural frequency for multiple layered cylindrical shell subjected to lateral pressure as the function of circumferential wave numbers (n) is studied.

Figs. 3–5 show the variation of the natural frequency of a multiple layered cylindrical shell for different circumferential wave numbers (n) with and without lateral pressure for the asymmetric boundary conditions. All of these graphs show the vibration characteristics of the multiple layered cylindrical shell under the effects of lateral pressure. For all the three boundary conditions when the lateral pressure is zero, the natural frequency initially decreases and then increases.

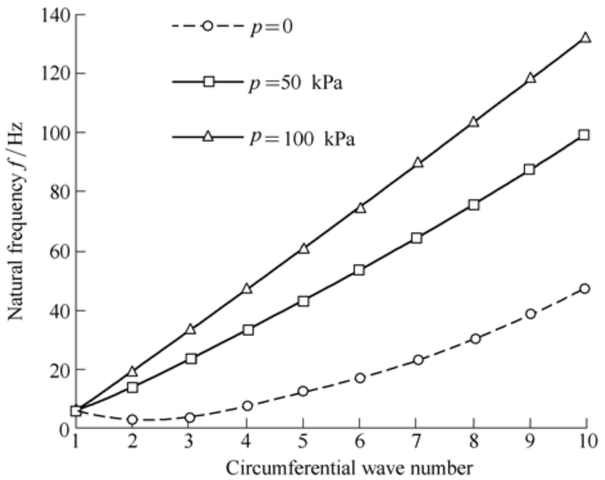

Fig. 3. Variation of the natural frequency for different lateral pressure under clamped-free (C-F) boundary condition ($h/R = 0.002$, $L/R = 20$, $R = 1$, $m = 1$)

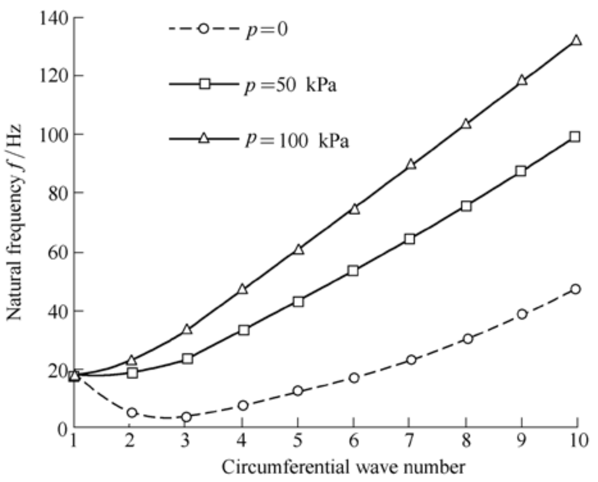

Fig. 4. Variation of the natural frequency for different lateral pressure under clamped-simply supported (C-SS) boundary condition ($h/R = 0.002$, $L/R = 20$, $R = 1$, $m = 1$)

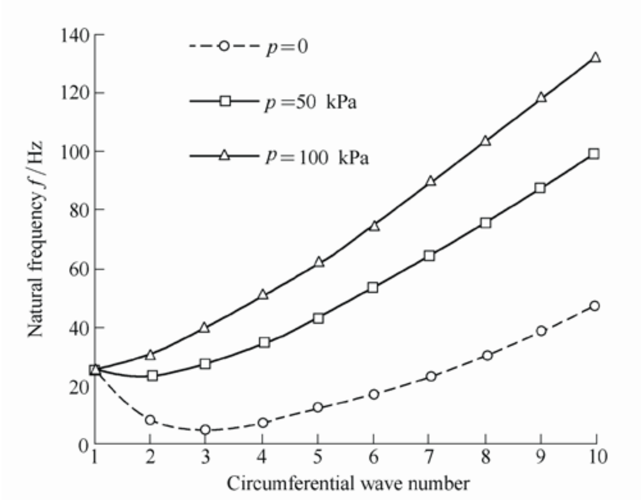

Fig. 5. Variation of the natural frequency for different external lateral pressure under free-simply supported (F-SS) boundary condition ($h/R = 0.002$, $L/R = 20$, $R = 1$, $m = 1$)

When multiple layered cylindrical shell is subjected to lateral pressure, for all asymmetric boundary conditions the natural frequencies increase as the circumferential wave number n is increased. The results show the effect of lateral pressure on the natural frequency of a multiple layered cylindrical shell which causes the natural frequency to increase. When the value of the lateral pressure is large, the natural frequency is higher. The results obtained also show the natural frequency characteristics of a multiple layered cylindrical shell with and without lateral pressure are different for different boundary conditions. It should be noted that the natural frequencies of multiple layered cylindrical shells with and without lateral pressure for all the graphs are calculated for $m = 1$.

Figs. 6–8 show the variation of the natural frequency of a multiple layered cylindrical shell subjected to lateral pressure for different thickness to radius ratio (h/R) with asymmetric boundary conditions. The analysis is conducted by assuming lateral pressure equal to 50 kPa. All of these graphs show the vibration behaviour of the multiple layered cylindrical shell under the effects of lateral pressure.

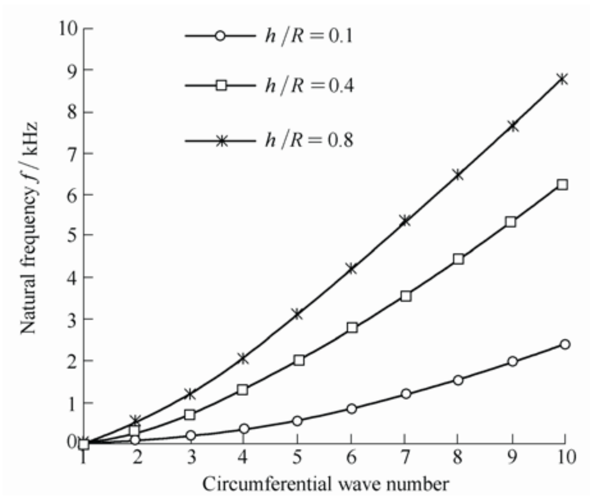

Fig. 6. Variation of natural frequency for various h/R ratios with C-F boundary condition ($L/R = 20$, $R = 1$, $m = 1$, $P = 50$ kPa)

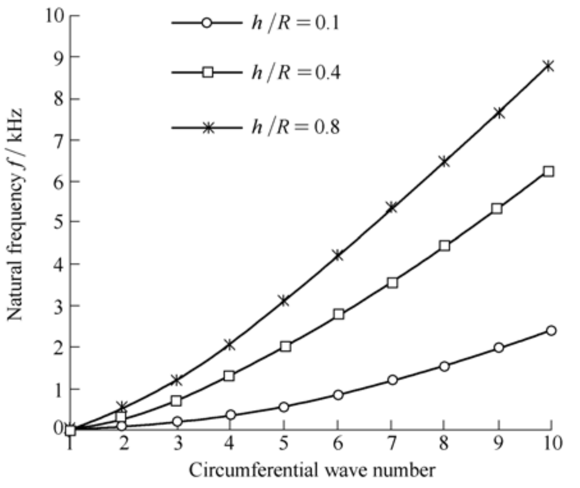

Fig. 7. Variation of natural frequency for various h/R ratios with C-SS boundary condition ($L/R = 20$, R=1, $m = 1$, $P = 50$ kPa)

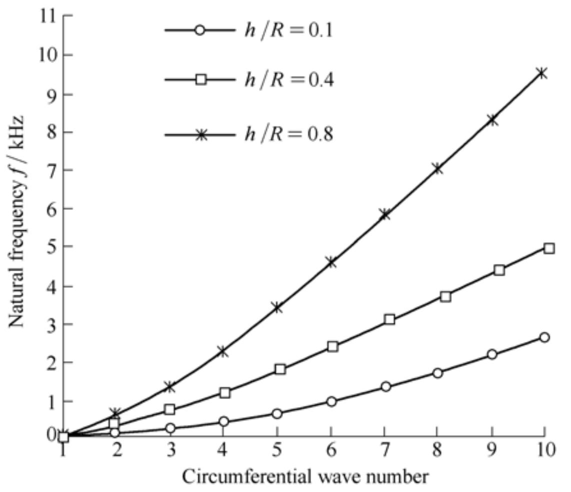

Fig. 8. Variation of natural frequency for various h/R ratios with F-SS boundary condition ($L/R = 20$, $R = 1$, $m = 1$, $P = 50$ kPa)

The results show that the natural frequency is higher for larger thickness to radius ratio (h/R). Thus lateral pressure, different boundary conditions and different thickness to radius ratios have effect on the natural frequency of a multiple layered cylindrical shell.

Tables 5–7 show the variation of the natural frequency against circumferential wave number for different L/R ratios under three boundary conditions for multiple layered cylindrical shell with lateral pressure. In these tables the effect of lateral pressure and asymmetric boundary conditions on the natural frequency are illustrated. For all asymmetric conditions, the natural frequency is found to decrease as the length to radius ratios (L/R) is increased. This frequency behavior shows, the effect of lateral pressure on natural frequency characteristics is significant at different L/R ratios. These tables show that asymmetric boundary conditions have effect on the natural frequency characteristics of multiple layered cylindrical shell. For example in Table 5 with clamped-free boundary conditions, the natural frequency difference between n=1 and 5 at L/R=20 is about 86% and at $L/R = 70$ is about 80% while in Table 6 with clamped-simply supported boundary condition, the natural frequency difference between n=1 and 5 at L/R=20 is 59.5% and at L/R=70 is 96.4%.

Table 5. Variation of the natural frequency for various L/R ratios with C-F boundary condition ($h/R = 0.002$, $R = 1$, $m = 1$, $P = 50$ kPa)

n	L/R=20	L/R=30	L/R= 40	L/R=70
1	6.136	2.755	1.555	0.509
2	13.84	13.53	13.01	12.70
3	23.93	23.38	22.87	22.16
4	33.84	33.23	32.72	32.10
5	43.84	43.22	42.61	42.11

Table 6. Variation of the natural frequency for various L/R ratios with C-SS boundary condition ($h/R = 0.002$, $R = 1$, $m = 1$, $P = 50$ kPa)

n	L/R=20	L/R=30	L/R=40	L/R=70
1	17.730	8.206	4.687	1.559
2	14.926	13.35	12.68	11.91
3	24.082	23.65	22.12	21.81
4	33.883	33.25	32.84	32.22
5	43.853	43.14	42.73	42.22

Table 7. Variation of the natural frequency for various L/R ratios with F-SS boundary condition ($h/R = 0.002$, $R = 1$, $m = 1$, $P = 50$ kPa)

n	L/R=20	L/R=30	L/R=40	L/R=70
1	25.317	11.540	6.554	2.165
2	16.005	14.187	13.85	12.72
3	24.233	23.979	23.43	22.82
4	33.920	33.559	32.83	32.21
5	43.869	43.346	42.84	42.13

8 Conclusions

(1) The vibration of thin-walled multiple layered cylindrical shells under lateral pressure for three asymmetrical boundary conditions are investigated.

(2) The type of material for multiple layered cylindrical shell are stainless steel and aluminum where the outer and inner layers are stainless steel, while the middle layer is assumed to be of aluminum.

(3) The models are formulated based on Love's shell theory with beam functions to describe the vibration problem. The governing equations are derived using energy functional with Ritz method. The boundary conditions considered are simply supported-clamped (SS-C), free-clamped (F-C) and simply supported-free (SS-F).

(4) For this multiple layered cylindrical shell the frequency characteristics, the influence of lateral pressure on the natural frequency and the effect of different boundary conditions on the natural frequency are

investigated. The lateral pressure has effect on the natural frequency of a multiple layered cylindrical shell and increase the natural frequency.

(5) The results obtained also show that the natural frequency characteristics for a multiple layered cylindrical shell with lateral pressure are different for different boundary conditions.

(6) The results are very useful for engineering application when studying vibration of shells with lateral pressure and this study can be used to validate numerical methods.

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