DOI: 10.3901/CJME.2014.02.358, available online at www.springerlink.com; www.cjmenet.com; www.cjmenet.com.cn

Bifurcation and Chaos Analysis of Nonlinear Rotor System with Axial-grooved Gas-lubricated Journal Bearing Support

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Received January 31, 2013; revised August 13, 2013; accepted December 31, 2013

Abstract: Axial-grooved gas-lubricated journal bearings have been widely applied to precision instrument due to their high accuracy, low friction, low noise and high stability. The rotor system with axial-grooved gas-lubricated journal bearing support is a typical nonlinear dynamic system. The nonlinear analysis measures have to be adopted to analyze the behaviors of the axial-grooved gas-lubricated journal bearing-rotor nonlinear system as the linear analysis measures fail. The bifurcation and chaos of nonlinear rotor system with three axial-grooved gas-lubricated journal bearing support are investigated by nonlinear dynamics theory. A time-dependent mathematical model is established to describe the pressure distribution in the axial-grooved compressible gas-lubricated journal bearing. The time-dependent compressible gas-lubricated Reynolds equation is solved by the differential transformation method. The gyroscopic effect of the rotor supported by gas-lubricated journal bearing with three axial grooves is taken into consideration in the model of the system, and the dynamic equation of motion is calculated by the modified Wilson-θ-based method. To analyze the unbalanced responses of the rotor system supported by finite length gas-lubricated journal bearings, such as bifurcation and chaos, the bifurcation diagram, the orbit diagram, the Poincaré map, the time series and the frequency spectrum are employed. The numerical results reveal that the nonlinear gas film forces have a significant influence on the stability of rotor system and there are the rich nonlinear phenomena, such as the periodic, period-doubling, quasi-periodic, period-4 and chaotic motion, and so on. The proposed models and numerical results can provide a theoretical direction to the design of axial-grooved gas-lubricated journal bearing-rotor system.

Keywords: axial-grooved gas journal bearing, differential transformation method, nonlinear, bifurcation, chaos

1 Introduction

Many researches have been done on the nonlinear characteristics of the rotor system with oil film journal bearing support[1–5]. Due to low frictional losses, cleanliness, little heat generation, and easy availability of gas as lubricant, the gas-lubricated bearings have been successfully used in engineering such as high speed machine tools, computer drive disks, dental drills, precision instruments, micro gas turbines and turbochargers etc. The characteristics of hydrostatic gas journal bearings[6] and flexure pivot hydrostatic pad gas bearing[7] were investigated both theoretically and experimentally. These works provided better understanding and desirable application of hydrostatic gas bearing in engineering. Zhou, et al[8], analyzed the static characteristics of rigid surface and compliant surface aerodynamic gas thrust bearing by the finite difference method(FDM). The analysis results indicated that the maximum load-carrying capacity and smaller friction moment could be obtained with suitable values of bearing structure parameters. The dynamic coefficients and stability of self-acting tilting-pad journal bearing were studied under the assumption of linear field in references[9–12]. Due to nonlinearity of gas-lubricated bearing in nature, the linear analysis measure fails to provide insights into nonlinear behaviors of gas-lubricated bearing to some extent. BOU-SAÏD, et al[13], made a comparison between linear and nonlinear analysis approaches. Their work showed that nonlinear analysis measure has to be adopted as the linear approach may lead to incorrect result.

Refs. [14–15] investigate nonlinear dynamic behaviors of a flexible rotor supported by relative short gas journal bearing and relative short herringbone-grooved gas journal

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This project is supported by National Natural Science Foundation of China(Grant No. 51075327), National Key Basic Research and Development Program of China(973 Program, Grant No. 2013CB035705), Shaanxi Provincial Natural Science Foundation of China(Grant No. 2013JQ7008), Open Project of State Key Laboratory of Mechanical Transmission of China(Grant No. SKLMT-KFKT-201011), Tribology Science Fund of State Key Laboratory of Tribology of China(Grant No. SKLTKF11A02), and Scientific Research Program of Shaanxi Provincial Education Department of China(Grant Nos. 12JK0661, 12JK0680)

bearing. In both works, the finite difference method with successive over relaxation method was employed to solve the governing equation that is obtained from Reynolds equation in gas lubrication by neglecting the circumferential pressure distribution. In later studies, a hybrid method combining the finite difference method and differential transformation method(DTM) was used to implement nonlinear analysis of a rigid[16] and a flexible[17] rotor with spherical gas-lubricated journal bearing support. The evolution in the dynamic behaviors of the bearing systems with respect to the rotor mass and bearing number was systematically examined. YANG, et al[18], obtained the nonlinear gas film force of cylindrical gas-lubricated bearing by the finite difference method. By taking rotor mass, periodic external exciting force and aspect ratio as system parameters for stability analysis, two unstable threshold values of the system parameters were obtained. A Newmark-β-based local iteration method with Floquet theory was proposed to calculate the nonlinear responses of Jeffcott rotor system supported in cylindrical gas journal bearings and analyze the stability of periodic solution and its bifurcation by ZHANG, et al[19]. In Refs. [20–22], by considering noncircular gas-lubricated journal bearings with lobe as the supports of rigid rotor system and using finite element method and Runge-kutta method, the nonlinear behaviors of rigid rotor system with non-circular gas-lubricated journal bearings are investigated. The authors studied bifurcation of nonlinear responses of rigid rotor system supported by two-lobe and three-lobe and four-lobe noncircular gas-lubricated journal bearing, also provided insights into the effect of preload on nonlinear behaviors of rigid rotor system with three-lobe and four-lobe support. WANG, et al[23], investigated the nonlinear responses and bifurcation of flexible rotor supported by self-acting gas-lubricated bearing with two axial grooves by using finite difference method and Euler integral method. This work indicated that stability of the rotor system can be enhanced by imbalance mass to some extent. The bifurcation and nonlinear dynamics of united gas-lubricated bearing system were investigated using a hybrid numerical method combining DTM and FDM by WANG[24]. In his work, the changes of the dynamic behaviors of a single mass rotor system with two degrees-of-freedom with respect to the rotor mass and bearing number were analyzed. ERTAS, et al[25], experimentally demonstrated the ability of the damped gas bearing to safely withstand rotor vibration levels while subjected to severe imbalance loading. ZHANG[26] studied the static performance of micro gas-lubricated journal bearing by spectra collocation method. In his work, by considering the effects of length-to-diameter ratio and slip, the corresponding stability boundaries were analyzed.

In the works above, nonlinear behaviors of symmetrical rigid or flexible rotor system without consideration of the moments of the inertia of the rotor were reported. Due to low stability of cylindrical and noncircular lobe self-acting gas journal bearing, the easiest way to overcome the shortcoming is that axial grooves are designed in the inner surface of the bearing along the axial direction. Because axial-grooved gas-lubricated journal bearing can prevent the pressure perturbation that may spread along the axial direction, this is extremely important to self-acting gas-lubricated bearing that clearance-to- diameter ratio of the bearing is usually very small.

This paper focuses on the gas-lubricated journal bearings with three axial grooves and the rotor with the moments of the inertia of the rotor. As the gas film forces are nonlinear, it is difficult and time consuming to solve Reynolds equation in gas lubrication. In order to save computational cost and improve the calculating accuracy, the differential transformation method is employed to obtain nonlinear gas film pressure distribution due to its rapid convergence rate and minimal calculation error. For a more precise description of the actual rotors, the effect of gyroscopic moment is taken into consideration in the model of the rotor system. The dynamic equation of an unsymmetrical rotor system supported by three axial-grooved gas-lubricated bearing is solved by using the modified Wilson-θ-based method[3], and the nonlinear dynamic behaviors, bifurcation and chaos of the bearing-rotor system are investigated. The numerical methods and results can provide the reference for the nonlinear dynamics design of gas-lubricated bearing-rotor systems.

2 Equation of System

The schematic diagram of an unsymmetrical rotor supported by gas-lubricated journal bearing is shown in Fig. 1.

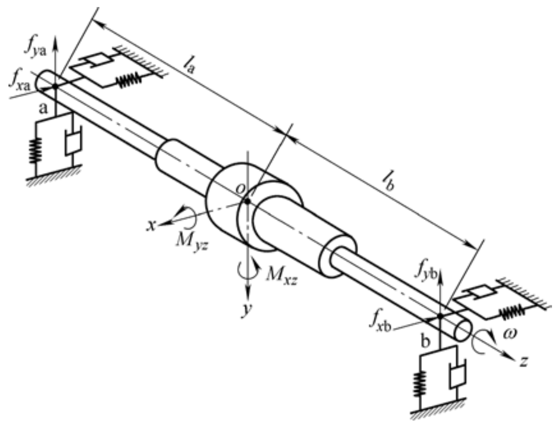


Fig. 1. Schematic diagram of an unsymmetrical rotor with gas-lubricated bearing support

The equation of motion for an unsymmetrical rotor with the axial-grooved gas-lubricated journal bearing can be written as follows：

$$\boldsymbol{m}\ddot{\boldsymbol{x}} + \boldsymbol{g}\dot{\boldsymbol{x}} = \boldsymbol{w} + \boldsymbol{q} + \boldsymbol{f}, \tag{1}$$

where $\boldsymbol{m}$ is the mass matrix, $\boldsymbol{g}$ is the gyroscopic matrix, $\boldsymbol{x}$ is the displacement vector, $\boldsymbol{w}$ is the weight vector, $\boldsymbol{q}$ is the external excitation force vector, $\boldsymbol{f}$ is the nonlinear gas film force vector, those are

$$\boldsymbol{x}=\begin{pmatrix}x_{\mathrm{a}} & y_{\mathrm{a}} & x_{\mathrm{b}} & y_{\mathrm{b}}\end{pmatrix}^{\mathrm{T}},$$

$$\boldsymbol{m}=\begin{pmatrix}\frac{l_{\mathrm{b}}}{l}m & 0 & \frac{l_{\mathrm{a}}}{l}m & 0\\ 0 & \frac{l_{\mathrm{b}}}{l}m & 0 & \frac{l_{\mathrm{a}}}{l}m\\ -\frac{J_{oy}}{l} & 0 & \frac{J_{oy}}{l} & 0\\ 0 & -\frac{J_{ox}}{l} & 0 & \frac{J_{ox}}{l}\end{pmatrix},$$

$$\boldsymbol{g}=\frac{J_{oz}\omega}{l}\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 1 & 0 & -1\\ -1 & 0 & 1 & 0\end{pmatrix},$$

$$\boldsymbol{w}=\begin{pmatrix}0 & mg & 0 & 0\end{pmatrix}^{\mathrm{T}},$$

$$\boldsymbol{q}=\begin{pmatrix}me_x\omega^2\cos(\omega t)+me_y\omega^2\sin(\omega t)\\ me_y\omega^2\cos(\omega t)-me_x\omega^2\sin(\omega t)\\ 0\\ 0\end{pmatrix},$$

$$\boldsymbol{f}=\begin{pmatrix}-f_{x\mathrm{a}}-f_{x\mathrm{b}}\\ -f_{y\mathrm{a}}-f_{y\mathrm{b}}\\ f_{x\mathrm{a}}l_{\mathrm{a}}-f_{x\mathrm{b}}l_{\mathrm{b}}\\ f_{y\mathrm{a}}l_{\mathrm{a}}-f_{y\mathrm{b}}l_{\mathrm{b}}\end{pmatrix}.$$

where m is the mass of the rotor, x_j and y_j (j=a, b) are the displacements of the journal centers in the x and y directions at bearing “a” and “b” stations, J_{ox} and J_{oy} are the equatorial moments of the inertia of the rotor, J_{oz} is the polar moment of the inertia of the rotor, f_{xj} and f_{yj} (j=a, b) are the nonlinear gas film forces acting on the rotor in the x and y directions at bearing “a” and “b” stations. l_{a} is the distance between the center of the left bearing (bearing “a”) and the center of mass, l_{b} is the distance between the center of the right bearing (bearing “b”) and the center of mass, l is the distance between the centers of two bearings, g is the acceleration of gravity, e_x and e_y are the mass eccentricities of the rotor in the x and y directions, ω is the rotating speed of the rotor.

The following dimensionless variables are defined:

$$X=\frac{x}{c},\ Y=\frac{y}{c},\ X'=\frac{\dot{x}}{\omega c},\ Y'=\frac{\dot{y}}{\omega c},\ X''=\frac{\ddot{x}}{\omega^2 c},$$
$$Y''=\frac{\ddot{y}}{\omega^2 c},\ \tau=\omega t,\ \bar{g}=\frac{g}{\omega^2 c},\ E_x=\frac{e_x}{c},\ E_y=\frac{e_y}{c},$$

$$\bar{J}_{ox}=\frac{J_{ox}}{mc^2},\ \bar{J}_{oy}=\frac{J_{oy}}{mc^2},\ \bar{J}_{oz}=\frac{J_{oz}}{mc^2},\tag{2}$$

where c is the radial clearance.

$$F_x=\frac{f_x}{p_{\mathrm{a}}R^2},\ F_y=\frac{f_y}{p_{\mathrm{a}}R^2},\ \bar{M}=\frac{mc\omega^2}{p_{\mathrm{a}}R^2},\tag{3}$$

where p_{a} is the ambient pressure, R is the radius of the journal.

Substitution of Eqs. (2), (3) into Eq. (1) gives the dimensionless dynamic equation as follows:

$$\boldsymbol{MX}''+\boldsymbol{GX}'=\boldsymbol{W}+\boldsymbol{Q}+\boldsymbol{F},\tag{4}$$

Eq. (4) can be rewritten in the following form

$$\begin{gathered}\begin{pmatrix}\frac{l_{\mathrm{b}}}{l}\bar{M} & 0 & \frac{l_{\mathrm{a}}}{l}\bar{M} & 0\\ 0 & \frac{l_{\mathrm{b}}}{l}\bar{M} & 0 & \frac{l_{\mathrm{a}}}{l}\bar{M}\\ -\bar{J}_{oy}\bar{M} & 0 & \bar{J}_{oy}\bar{M} & 0\\ 0 & -\bar{J}_{ox}\bar{M} & 0 & \bar{J}_{ox}\bar{M}\end{pmatrix}\begin{pmatrix}X''_{\mathrm{a}}\\ Y''_{\mathrm{a}}\\ X''_{\mathrm{b}}\\ Y''_{\mathrm{b}}\end{pmatrix}+\\ \begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & \bar{J}_{oz}\bar{M} & 0 & -\bar{J}_{oz}\bar{M}\\ -\bar{J}_{oz}\bar{M} & 0 & \bar{J}_{oz}\bar{M} & 0\end{pmatrix}\begin{pmatrix}X'_{\mathrm{a}}\\ Y'_{\mathrm{a}}\\ X'_{\mathrm{b}}\\ Y'_{\mathrm{b}}\end{pmatrix}=\\ \begin{pmatrix}0\\ \bar{M}\bar{g}\\ 0\\ 0\end{pmatrix}+\begin{pmatrix}\bar{M}E_x\cos(\tau)+\bar{M}E_y\sin(\tau)\\ \bar{M}E_y\cos(\tau)-\bar{M}E_x\sin(\tau)\\ 0\\ 0\end{pmatrix}+\\ \begin{pmatrix}-F_{x\mathrm{a}}-F_{x\mathrm{b}}\\ -F_{y\mathrm{a}}-F_{y\mathrm{b}}\\ F_{x\mathrm{a}}\left(\frac{l_{\mathrm{a}}l}{c^2}\right)-F_{x\mathrm{b}}\left(\frac{l_{\mathrm{b}}l}{c^2}\right)\\ F_{y\mathrm{a}}\left(\frac{l_{\mathrm{a}}l}{c^2}\right)-F_{y\mathrm{b}}\left(\frac{l_{\mathrm{b}}l}{c^2}\right)\end{pmatrix}.\end{gathered}\tag{5}$$

3 Reynolds Equation and Nonlinear Gas Film Forces

3.1 Dimensionless Reynolds equation

Under the isothermal operating condition, the gas pressure governing equation is modeled by Reynolds equation as follows:

$$\frac{\partial}{\partial\varphi}\left(\bar{P}\bar{H}^3\frac{\partial\bar{P}}{\partial\varphi}\right)+\frac{\partial}{\partial\lambda}\left(\bar{P}\bar{H}^3\frac{\partial\bar{P}}{\partial\lambda}\right)=\varLambda\frac{\partial(\bar{P}\bar{H})}{\partial\varphi}+2\varLambda\frac{\partial(\bar{P}\bar{H})}{\partial\tau},\tag{6}$$

where $\bar{P}=p/p_{\mathrm{a}}$ is the dimensionless pressure distribution, $\bar{H}=1+\varepsilon\cos(\phi-\theta)$ is the dimensionless gas gap between journal and bushing, $\varepsilon=\sqrt{e_x^2+e_y^2}\big/c$ is the dimensionless eccentricity, ϕ is the angle which is calculated from negative y axis to gas film location(radian), θ is the deviation angle (radian), $\Lambda=(6\mu\omega/p_{\mathrm{a}})\times(R/c)^2$ is the bearing number, μ is the gas dynamic viscosity, $\tau=\omega t$ is the dimensionless time, φ is the dimensionless circumferential coordinate which is calculated from deviation line to gas location (as shown in Fig. 2) (radian), λ is the axial coordinate.

The pressure distribution boundary conditions of the axial-grooved gas-lubricated journal bearing are stated as follows.

(1) Gas pressure on both ends of the bushing is equal to ambient pressure p_{a}, i.e., $\overline{P}_i(\varphi,\pm B/2R)=1,\ i=1,2,3\cdots$. $\overline{P}_i$ is the pressure distribution of the ith pad, B is the width of the axial-grooved gas-lubricated journal bearing, $B/2R$ is the width-to-diameter ratio.

(2) Gas pressure distribution $\overline{P}$ is an even function for λ, i.e., $\overline{P}(\varphi,\lambda)=\overline{P}(\varphi,-\lambda)$.

(3) Gas Pressure $\overline{P}$ is continuous at λ=0, i.e.,

$$\left.\frac{\partial\overline{P}}{\partial\lambda}\right|_{\lambda=0}=0.$$

(4) Gas Pressure $\overline{P}_i$ obeys $\left.\overline{P}_i\right|_{\varphi=\beta+(i-1)(\alpha+\xi)-\theta}=\left.\overline{P}_i\right|_{\varphi=\beta+i\alpha+(i-1)\xi-\theta}=1$, i.e., $\overline{P}_i=1$ at the leading and trailing edge of the ith pad, respectively. β is the location angle of the pad which is calculated from negative y axis to the leading edge of the 1st pad, α is the arc bushing angle of the axial-grooved gas-lubricated journal bearing, ξ is the groove width angle of the axial-grooved gas-lubricated journal bearing(as shown in Fig. 2).

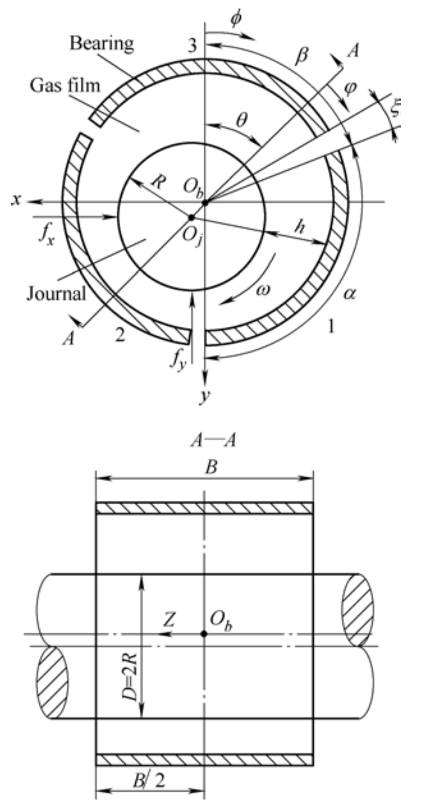


Fig. 2. Calculation coordinates for a three axial grooves gas-lubricated journal bearing

In Fig. 2, $h=c\overline{H}$ is the gas film thickness, O_{b} is the center of the axial-grooved gas-lubricated bearing, O_{j} is the journal center of the rotor.

3.2 Solution of the dimensionless Reynolds equation

The differential transformation method is presented[27] based on Taylor expansion and first applied to engineering domain. Differential transformation is usually employed to solve differential equations because of its rapid convergence rate and minimal calculation error[28–30].

By applying differential transformation with respect to time into Eq. (6), the following equation can be obtained:

$$\begin{gathered}3Z\otimes\frac{\partial H}{\partial n}\otimes\frac{\partial\Theta}{\partial n}+\varXi\otimes\frac{\partial^2\Theta}{\partial n^2}+3Z\otimes\frac{\partial H}{\partial\lambda}\otimes\frac{\partial\Theta}{\partial\lambda}+\\ \varXi\otimes\frac{\partial^2\Theta}{\partial\lambda^2}=2\Lambda\frac{\partial H}{\partial n}\otimes P+2\Lambda\frac{\partial P}{\partial n}\otimes H+\\ 4\Lambda\frac{\partial H}{\partial\tau}\otimes P+4\Lambda\frac{\partial P}{\partial\tau}\otimes H,\end{gathered}\tag{7}$$

where operator $\otimes$ represents convolution, n is the transformation order, and

$$Z(n)=H\otimes H=\sum_{m=0}^{n}H(n-m)H(m),$$

$$\Theta(n)=P\otimes P=\sum_{m=0}^{n}P(n-m)P(m),$$

$$\varXi(n)=H\otimes H\otimes H=\sum_{m=0}^{n}H(n-m)\sum_{l=0}^{m}H(m-l)H(l).$$

By using the first-order and second-order central-difference method to discretize Eq. (7) with respect to φ and λ, Eq. (7) can be expressed as

$$\begin{gathered}3\sum_{m=0}^{n}Z_{i,j}(n-m)\sum_{l=0}^{m}\left[\left(\frac{H_{i+1,j}(m-l)-H_{i-1,j}(m-l)}{2\Delta\varphi}\right)\times\right.\\ \left.\left(\frac{\Theta_{i+1,j}(l)-\Theta_{i-1,j}(l)}{2\Delta\varphi}\right)\right]+\sum_{m=0}^{n}\left[\varXi_{i,j}(n-m)\times\right.\\ \left.\left(\frac{\Theta_{i+1,j}(m)-2\Theta_{i,j}(m)+\Theta_{i-1,j}(m)}{(\Delta\varphi)^2}\right)\right]+3\sum_{m=0}^{n}Z_{i,j}(n-m)\times\\ \sum_{l=0}^{m}\left[\left(\frac{H_{i,j+1}(m-l)-H_{i,j-1}(m-l)}{2\Delta\lambda}\right)\times\left(\frac{\Theta_{i,j+1}(l)-\Theta_{i,j-1}(l)}{2\Delta\lambda}\right)\right]+\\ \sum_{m=0}^{n}\left[\varXi_{i,j}(n-m)\left(\frac{\Theta_{i,j+1}(m)-2\Theta_{i,j}(m)+\Theta_{i,j-1}(m)}{(\Delta\lambda)^2}\right)\right]=\\ 2\Lambda\sum_{m=0}^{n}\left[\left(\frac{H_{i+1,j}(n-m)-H_{i-1,j}(n-m)}{2\Delta\varphi}\right)P_{i,j}(m)\right]+\\ 2\Lambda\sum_{m=0}^{n}\left[\left(\frac{P_{i+1,j}(n-m)-P_{i-1,j}(n-m)}{2\Delta\varphi}\right)H_{i,j}(m)\right]+\\ 4\Lambda\sum_{m=0}^{n}\left[\left(\frac{m+1}{\Delta\tau}\right)H_{i,j}(n-m)P_{i,j}(m+1)\right]+\end{gathered}$$

$$4\Lambda\sum_{m=0}^{n}\left[\left(\frac{m+1}{\Delta\tau}\right)P_{i,j}(n-m)H_{i,j}(m+1)\right], \tag{8}$$

where *i*, *j* are the numbers of the nodes in the circumferential and axial directions of the gas field, *l*, *m* and *n* are the orders of differential transformations ($n \geqslant m \geqslant l$), $\Delta\varphi$ is the step in circumferential direction, $\Delta\lambda$ is the step in axial direction, $\Delta\tau$ is the dimensionless time step.

If n=0, then m=l=0, Eq. (8) is written in the following form:

$$3Z_{i,j}(0)\left[\frac{H_{i+1,j}(0)-H_{i-1,j}(0)}{2\Delta\varphi}\times\frac{\Theta_{i+1,j}(0)-\Theta_{i-1,j}(0)}{2\Delta\varphi}\right]+$$
$$\Xi_{i,j}(0)\left(\frac{\Theta_{i+1,j}(0)-2\Theta_{i,j}(0)+\Theta_{i-1,j}(0)}{(\Delta\varphi)^2}\right)+$$
$$\Xi_{i,j}(0)\left(\frac{\Theta_{i,j+1}(0)-2\Theta_{i,j}(0)+\Theta_{i,j-1}(0)}{(\Delta\lambda)^2}\right)=$$
$$2\Lambda\left(\frac{H_{i+1,j}(0)-H_{i-1,j}(0)}{2\Delta\varphi}\right)P_{i,j}(0)+$$
$$2\Lambda\left(\frac{P_{i+1,j}(0)-P_{i-1,j}(0)}{2\Delta\varphi}\right)H_{i,j}(0)+$$
$$4\Lambda\left(\frac{1}{\Delta\tau}\right)H_{i,j}(0)P_{i,j}(1)+4\Lambda\left(\frac{1}{\Delta\tau}\right)P_{i,j}(0)H_{i,j}(1). \tag{9}$$

If $n=0$, the following equations can be obtained:

$$\Theta_{i,j}(0)=P_{i,j}(0)P_{i,j}(0), \tag{10}$$

$$Z_{i,j}(0)=H_{i,j}(0)H_{i,j}(0), \tag{11}$$

$$\Xi_{i,j}(0)=H_{i,j}(0)H_{i,j}(0)H_{i,j}(0). \tag{12}$$

Substitution of Eqs. (10) to (12) into Eq. (9) gives the following equation:

$$\frac{3H_{i,j}^2(0)\big(H_{i+1,j}(0)-H_{i-1,j}(0)\big)+4H_{i,j}^3(0)}{4(\Delta\varphi)^2}\Theta_{i+1,j}(0)-$$
$$\frac{3H_{i,j}^2(0)\big(H_{i+1,j}(0)-H_{i-1,j}(0)\big)-4H_{i,j}^3(0)}{4(\Delta\varphi)^2}\Theta_{i-1,j}(0)+$$
$$\left(\frac{H_{i,j}^3(0)}{(\Delta\lambda)^2}\right)\Theta_{i,j+1}(0)+\left(\frac{H_{i,j}^3(0)}{(\Delta\lambda)^2}\right)\Theta_{i,j-1}(0)-$$
$$\left(\frac{2H_{i,j}^3(0)}{(\Delta\varphi)^2}+\frac{2H_{i,j}^3(0)}{(\Delta\lambda)^2}\right)\Theta_{i,j}(0)=$$
$$\Lambda\left(\frac{H_{i,j}(0)}{\Delta\varphi}\right)P_{i+1,j}(0)-\Lambda\left(\frac{H_{i,j}(0)}{\Delta\varphi}\right)P_{i-1,j}(0)+$$
$$\Lambda\left(\frac{H_{i+1,j}(0)-H_{i-1,j}(0)}{\Delta\varphi}\right)P_{i,j}(0)+$$
$$4\Lambda\left(\frac{1}{\Delta\tau}\right)P_{i,j}(0)H_{i,j}(1)+4\Lambda\left(\frac{1}{\Delta\tau}\right)H_{i,j}(0)P_{i,j}(1), \tag{13}$$

where $H_{i,j}(0)$ is the thickness of gas film at previous one step, $H_{i,j}(1)$ is the associated variable which can be obtained by the known variables, $P_{i,j}(0)$ is the pressure of gas film at previous one step, $P_{i,j}(1)$ is the associated variable which can be obtained by Eq. (13).

Since transformation function of the dimensionless thickness of gas film, i.e., $H=1+\varepsilon\cos\varphi$ is not function of the axial coordinate λ, one obtains $\partial H/\partial\lambda=0$. If n=1, then m=l=0 and 1, Eq. (8) is written in the following form:

$$3Z_{i,j}(1)\left(\frac{H_{i+1,j}(0)-H_{i-1,j}(0)}{2\Delta\varphi}\frac{\Theta_{i+1,j}(0)-\Theta_{i-1,j}(0)}{2\Delta\varphi}\right)+$$
$$3Z_{i,j}(0)\left(\frac{H_{i+1,j}(1)-H_{i-1,j}(1)}{2\Delta\varphi}\frac{\Theta_{i+1,j}(0)-\Theta_{i-1,j}(0)}{2\Delta\varphi}\right)+$$
$$3Z_{i,j}(0)\left(\frac{H_{i+1,j}(0)-H_{i-1,j}(0)}{2\Delta\varphi}\frac{\Theta_{i+1,j}(1)-\Theta_{i-1,j}(1)}{2\Delta\varphi}\right)+$$
$$\Xi_{i,j}(1)\left(\frac{\Theta_{i+1,j}(0)-2\Theta_{i,j}(0)+\Theta_{i-1,j}(0)}{(\Delta\varphi)^2}\right)+$$
$$\Xi_{i,j}(0)\left(\frac{\Theta_{i+1,j}(1)-2\Theta_{i,j}(1)+\Theta_{i-1,j}(1)}{(\Delta\varphi)^2}\right)+$$
$$\Xi_{i,j}(1)\left(\frac{\Theta_{i,j+1}(0)-2\Theta_{i,j}(0)+\Theta_{i,j-1}(0)}{(\Delta\lambda)^2}\right)+$$
$$\Xi_{i,j}(0)\left(\frac{\Theta_{i,j+1}(1)-2\Theta_{i,j}(1)+\Theta_{i,j-1}(1)}{(\Delta\lambda)^2}\right)=$$
$$2\Lambda\left(\frac{H_{i+1,j}(0)-H_{i-1,j}(0)}{2\Delta\varphi}\right)P_{i,j}(1)+$$
$$2\Lambda\left(\frac{H_{i+1}(1)-H_{i-1}(1)}{2\Delta\varphi}\right)P_{i,j}(0)+$$
$$2\Lambda\left(\frac{P_{i+1,j}(0)-P_{i-1,j}(0)}{2\Delta\varphi}\right)H_{i,j}(1)+$$
$$2\Lambda\left(\frac{P_{i+1,j}(1)-P_{i-1,j}(1)}{2\Delta\varphi}\right)H_{i,j}(0)+$$
$$4\Lambda\left(\frac{1}{\Delta\tau}\right)P_{i,j}(1)H_{i,j}(1)+4\Lambda\left(\frac{2}{\Delta\tau}\right)P_{i,j}(0)H_{i,j}(2)+$$
$$4\Lambda\left(\frac{1}{\Delta\tau}\right)H_{i,j}(1)P_{i,j}(1)+4\Lambda\left(\frac{2}{\Delta\tau}\right)H_{i,j}(0)P_{i,j}(2). \tag{14}$$

If n=1, the following equations can be obtained:

$$\Theta_{i,j}(1)=2P_{i,j}(0)P_{i,j}(1), \tag{15}$$

$$Z_{i,j}(1) = 2H_{i,j}(0)H_{i,j}(1), \tag{16}$$

$$\Xi_{i,j}(1) = 3H_{i,j}(0)H_{i,j}(0)H_{i,j}(1). \tag{17}$$

Substituting Eqs. (10)–(12) and Eqs. (15)–(17) into Eq. (14), the following form can be obtained:

$$\begin{aligned}
&[6H_{i,j}(0)H_{i,j}(1)(H_{i+1,j}(0)-H_{i-1,j}(0))+3H_{i,j}^2(0)(H_{i+1,j}(1)-\\
&H_{i-1,j}(1))+12H_{i,j}^2(0)H_{i,j}(1)]/4(\Delta\varphi)^2\times\Theta_{i+1,j}(0)-\\
&[6H_{i,j}(0)H_{i,j}(1)(H_{i+1,j}(0)-H_{i-1,j}(0))+3H_{i,j}^2(0)\times\\
&(H_{i+1,j}(1)-H_{i-1,j}(1))-12H_{i,j}^2(0)H_{i,j}(1)]/4(\Delta\phi)^2\times\\
&\Theta_{i-1,j}(0)+\frac{3H_{i,j}^2(0)H_{i,j}(1)}{(\Delta\lambda)^2}\Theta_{i,j+1}(0)+\\
&\frac{3H_{i,j}^2(0)H_{i,j}(1)}{(\Delta\lambda)^2}\Theta_{i,j-1}(0)-\\
&\left(\frac{6H_{i,j}^2(0)H_{i,j}(1)}{(\Delta\varphi)^2}+\frac{6H_{i,j}^2(0)H_{i,j}(1)}{(\Delta\lambda)^2}\right)\Theta_{i,j}(0)+\\
&\frac{3H_{i,j}^2(0)\left(H_{i+1,j}(0)-H_{i-1,j}(0)\right)+4H_{i,j}^3(0)}{4(\Delta\varphi)^2}\Theta_{i+1,j}(1)-\\
&\frac{3H_{i,j}^2(0)\left(H_{i+1,j}(0)-H_{i-1,j}(0)\right)-4H_{i,j}^3(0)}{4(\Delta\varphi)^2}\Theta_{i-1,j}(1)+\\
&\frac{H_{i,j}^3(0)}{(\Delta\lambda)^2}\Theta_{i,j+1}(1)+\frac{H_{i,j}^3(0)}{(\Delta\lambda)^2}\Theta_{i,j-1}(1)-\\
&\left(\frac{2H_{i,j}^3(0)}{(\Delta\varphi)^2}+\frac{2H_{i,j}^3(0)}{(\Delta\lambda)^2}\right)\Theta_{i,j}(1)=\\
&\left[\frac{\Lambda H_{i,j}(1)}{\Delta\varphi}P_{i+1,j}(0)-\frac{\Lambda H_{i,j}(1)}{\Delta\varphi}P_{i-1,j}(0)+\right.\\
&\left.\left(\frac{\Lambda\left(H_{i+1,j}(1)-H_{i-1,j}(1)\right)}{\Delta\varphi}+\frac{8\Lambda\left(H_{i,j}(2)\right)}{\Delta\tau}\right)P_{i,j}(0)\right]+\\
&\left[\frac{\Lambda H_{i,j}(0)}{\Delta\varphi}P_{i+1,j}(1)-\frac{\Lambda H_{i,j}(0)}{\Delta\varphi}P_{i-1,j}(1)+\right.\\
&\left.\left(\frac{\Lambda\left(H_{i+1,j}(0)-H_{i-1,j}(0)\right)}{\Delta\varphi}+\frac{8\Lambda\left(H_{i,j}(1)\right)}{\Delta\tau}\right)P_{i,j}(1)\right]+\\
&4\Lambda\left(\frac{2}{\Delta\tau}\right)H_{i,j}(0)P_{i,j}(2),
\end{aligned} \tag{18}$$

where $H_{i,j}(2)$ is the associated variable which can be obtained by the known variables, $P_{i,j}(2)$ is the associated variable which can be obtained from Eq. (18).

Substituting $H_i(0)$, $P_{i,j}(0)$ and $H_{i,j}(1)$ into Eq. (13), one can obtain $P_{i,j}(1)$, then $P_{i,j}(2)$ can be obtained from Eq. (18).

The transformation function $P_{i,j}$ of dimensionless pressure distribution $\overline{P}_{i,j}$ can be obtained in the following form:

$$P_{i,j} = P_{i,j}(0) + P_{i,j}(1) + P_{i,j}(2). \tag{19}$$

The dimensionless gas film forces of the ith pad in the x and y directions can be obtained by integrating pressure distribution in its gas-lubricated field:

$$\begin{cases} F_{xi} = -\int_{-\frac{B}{2R}}^{\frac{B}{2R}}\int_{\beta+(i-1)(\alpha+\xi)-\theta}^{\beta+i\alpha+(i-1)\xi-\theta}(\overline{P}_i-1.0)\sin(\varphi+\theta)\mathrm{d}\varphi\mathrm{d}\lambda, \\ F_{yi} = -\int_{-\frac{B}{2R}}^{\frac{B}{2R}}\int_{\beta+(i-1)(\alpha+\xi)-\theta}^{\beta+i\alpha+(i-1)\xi-\theta}(\overline{P}_i-1.0)\cos(\varphi+\theta)\mathrm{d}\varphi\mathrm{d}\lambda. \end{cases} \tag{20}$$

The dimensionless gas film resultant forces of the three axial grooves gas-lubricated journal bearing are the sum of each pad:

$$\begin{cases} F_x = F_{x1} + F_{x2} + F_{x3}, \\ F_y = F_{y1} + F_{y2} + F_{y3}. \end{cases} \tag{21}$$

4 Numerical Examples and Results

This paper presents two different cases. The detailed parameters of three axial grooves gas-lubricated journal bearings are B=8.25 mm, R=5 mm, α=115º, ξ=5º, c=0.005 mm, μ=18 μPa • s, respectively. The parameters of the rotor are l_1=60 mm (length of the 1st segment), d_1=10 mm(diameter of the 1st segment), l_2=30 mm, d_2=14 mm, l_3=20 mm, d_3=20 mm, l_4=30 mm, d_4 =15 mm, l_5=60 mm, d_5=10 mm, respectively.

Case 1: The rotor has the mass eccentricities of e_x=0.015 mm and e_y=0.015 mm. The calculated results are shown in dimensionless form. The rotating velocity of the rotor ω (rad/s) is chosen as the bifurcation parameter. With the increase of ω, the unbalanced periodic response of the system loses its stability and turns into quasi-periodic motion. As the rotating velocity of the rotor ω increases continually, quasi-periodic motion turns into subharmonic motion. This phenomenon, in which the ratio of the forced frequency and the response frequency becomes rational, is called phase locking or mode locking. A continuous increase of ω to 1640 rad/s results in a chaotic motion of the system. Fig. 3 represents the periodic-quasi periodic-mode locking-chaotic routine. The periodic solution is stable before quasi-periodic bifurcation appears. With the increase of the rotating velocity of the rotor ω, the periodic motion of the system bifurcates to quasi-periodic motion.

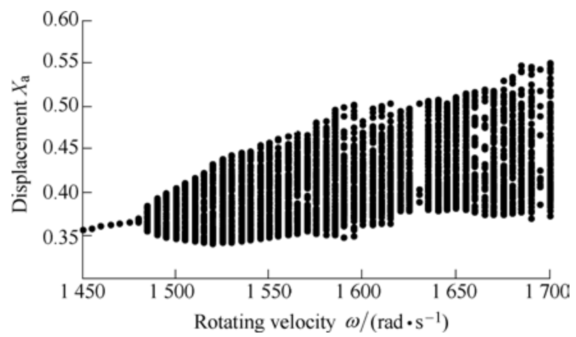


Fig. 3. Bifurcation diagram of X_a versus ω at bearing "a" station

The unbalanced response of the system, i.e., gas film whirl(gas film whirl cause nonlinear characteristics of the system) is the stable periodic motion at ω=1450 rad/s, which is shown in Fig. 4. The quasi-periodic orbit of the journal center, Poincaré map, time series and FFT spectrum diagram for ω=1500 rad/s are shown in Fig. 5. The Poincaré map in Poincaré section is the tours attractors(Fig. 5(b)), the FFT spectrum is discrete and the ratio of frequencies is irrational(Fig. 5(d)). The unbalanced subharmonic response of the rotor is the period-4 motion at ω=1630 rad/s, which is shown in Fig. 6. In fact, subharmonic motion appears in very tiny rotating velocity intervals. Comparing to subharmonic motion, quasi-periodic motion is more frequently occurred. As rotating velocity of the rotor reaches to ω=1640 rad/s, the motion of the journal center becomes chaotic motion. The characteristics of chaotic motion are low power and wide band. The time series of chaotic motion is irregular, and the Poincaré map on the selected Poincaré plane presents a typical characteristic of strange attractors. Chaotic orbit of the journal center, projection of the Poincaré map of the journal center on X-Y plane, time series of Y_b at bearing "b" station for ω=1700 rad/s are shown in Fig. 7.

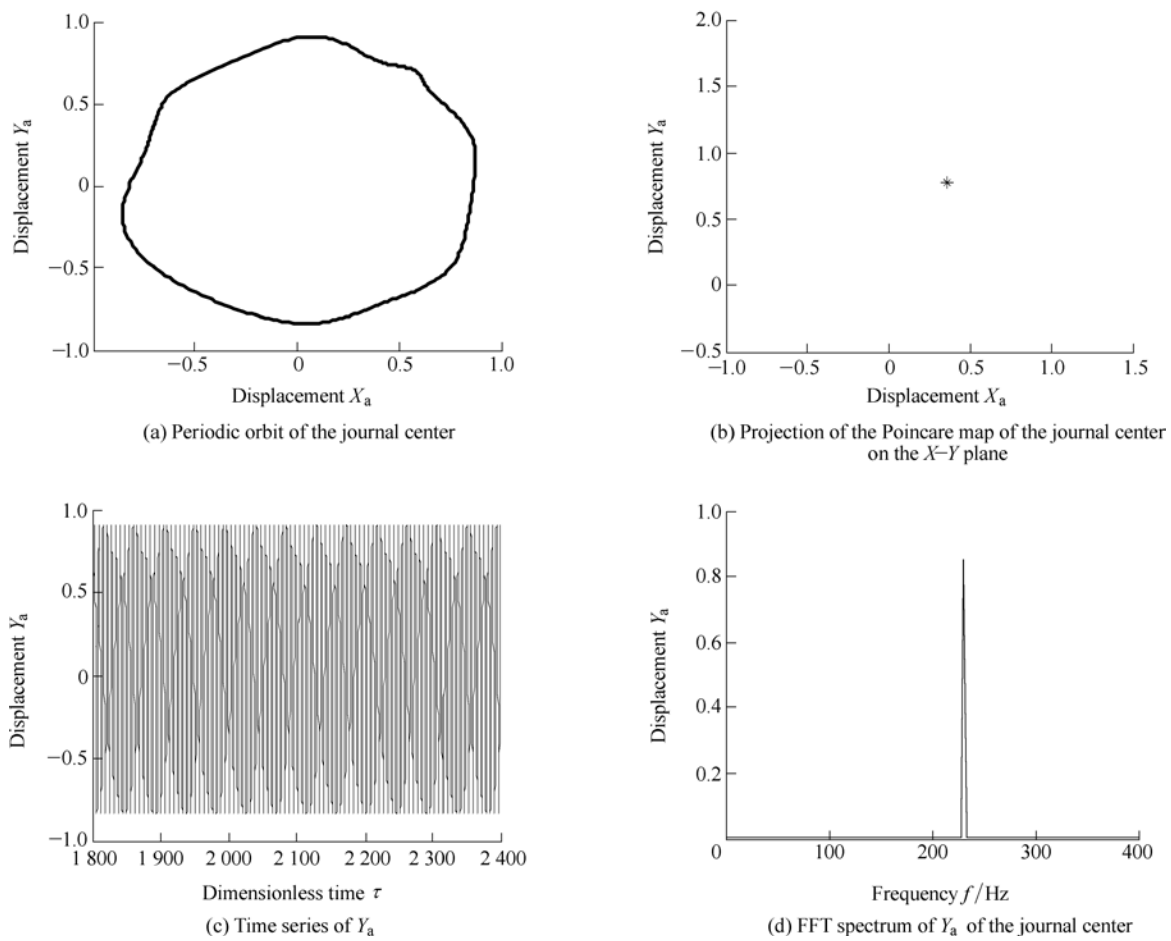


Fig. 4. Periodic orbit of the journal center, projection of the Poincaré map of the journal center on X-Y plane, time series of Y_a and spectrum of Y_a at bearing "a" station for ω=1450 rad/s

Case 2: The rotor has the mass eccentricities of e_x=0.001 mm and e_y=0.001 mm. The unbalanced response of the system is the stable periodic motion when the rotating velocity is low. With the increase of the rotating velocity of the rotor ω, the periodic motion bifurcates to period-doubling motion, i.e., the half-speed whirl of the rotor, which is considered as an obstacle in the application and development of gas bearing with higher speed. Period-doubling orbit of the journal center, projection of the Poincaré map of the journal center on the X-Y plane, time series of Y_b and FFT spectrum of Y_b at bearing "b" station for ω=2 200 rad/s are shown in Fig. 8.

If the rotating velocity is higher, the system motion will become unstable.

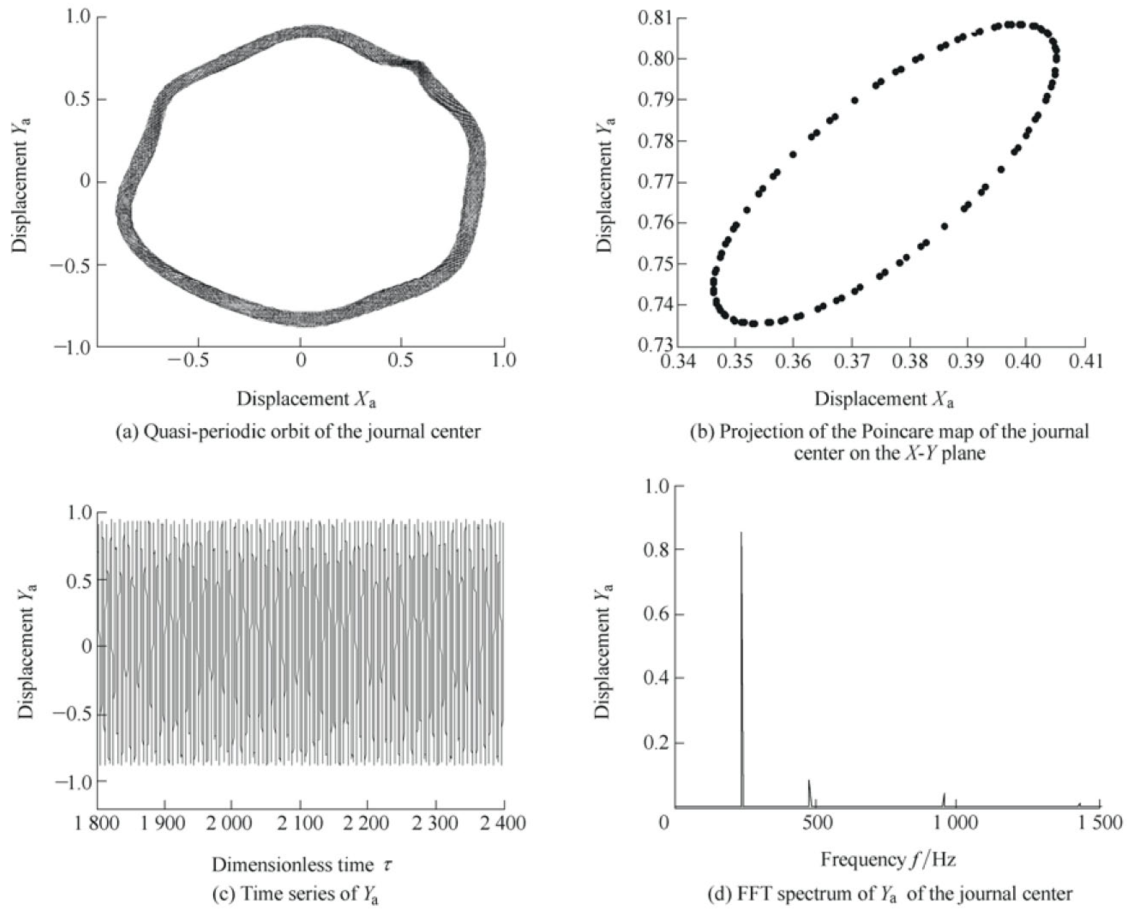


Fig. 5. Quasi-periodic orbit of the journal center, projection of Poincaré map of the journal center on X-Y plane, time series of Y_a and spectrum of Y_a at bearing "a" station for ω=1500 rad/s

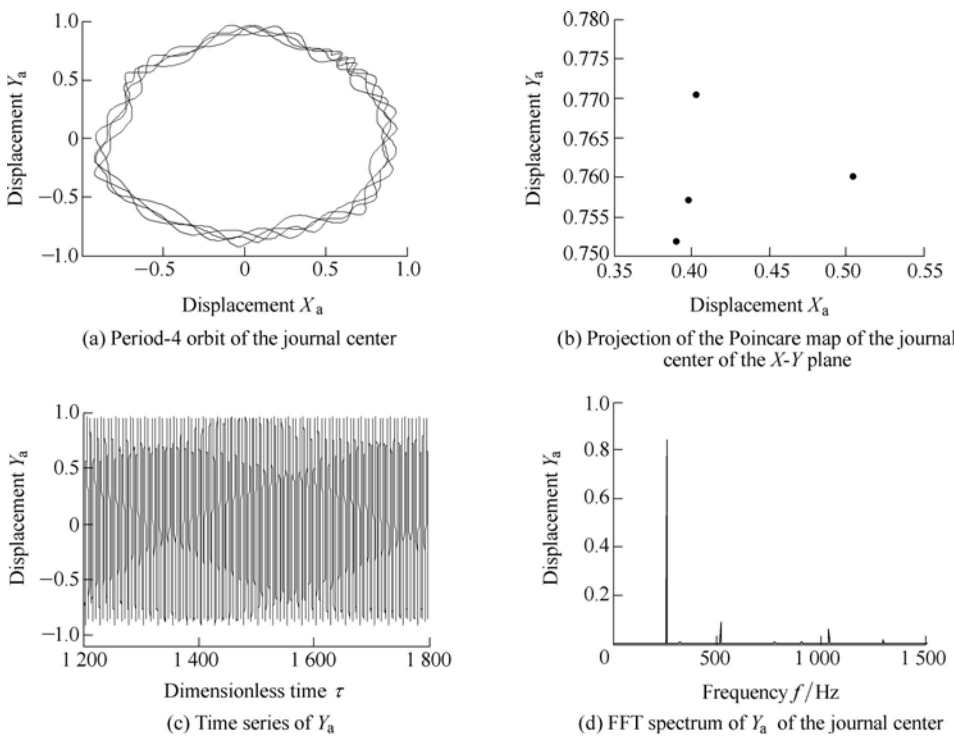
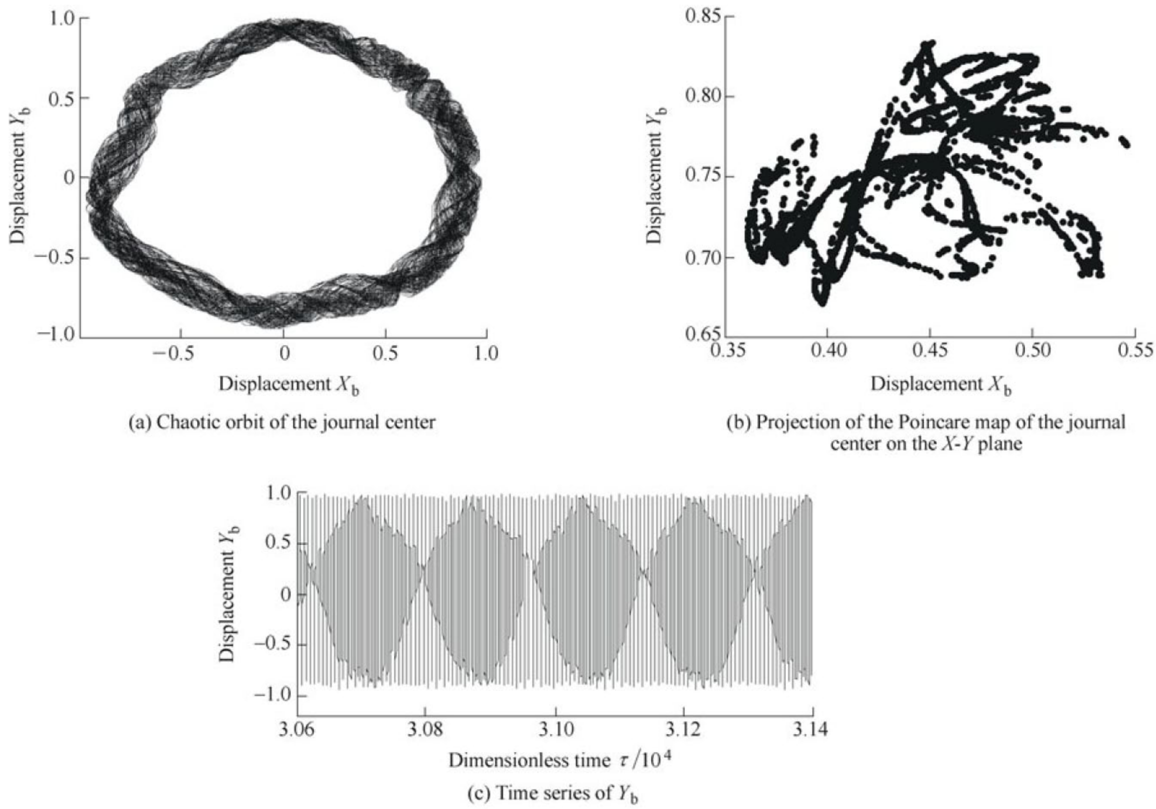


Fig. 6. Subharmonic Period-4 orbit of the journal center, projection of the Poincaré map of the journal center on the X-Y plane, time series of Y_a and FFT spectrum of Y_a at bearing "a" station for ω=1630 rad/s

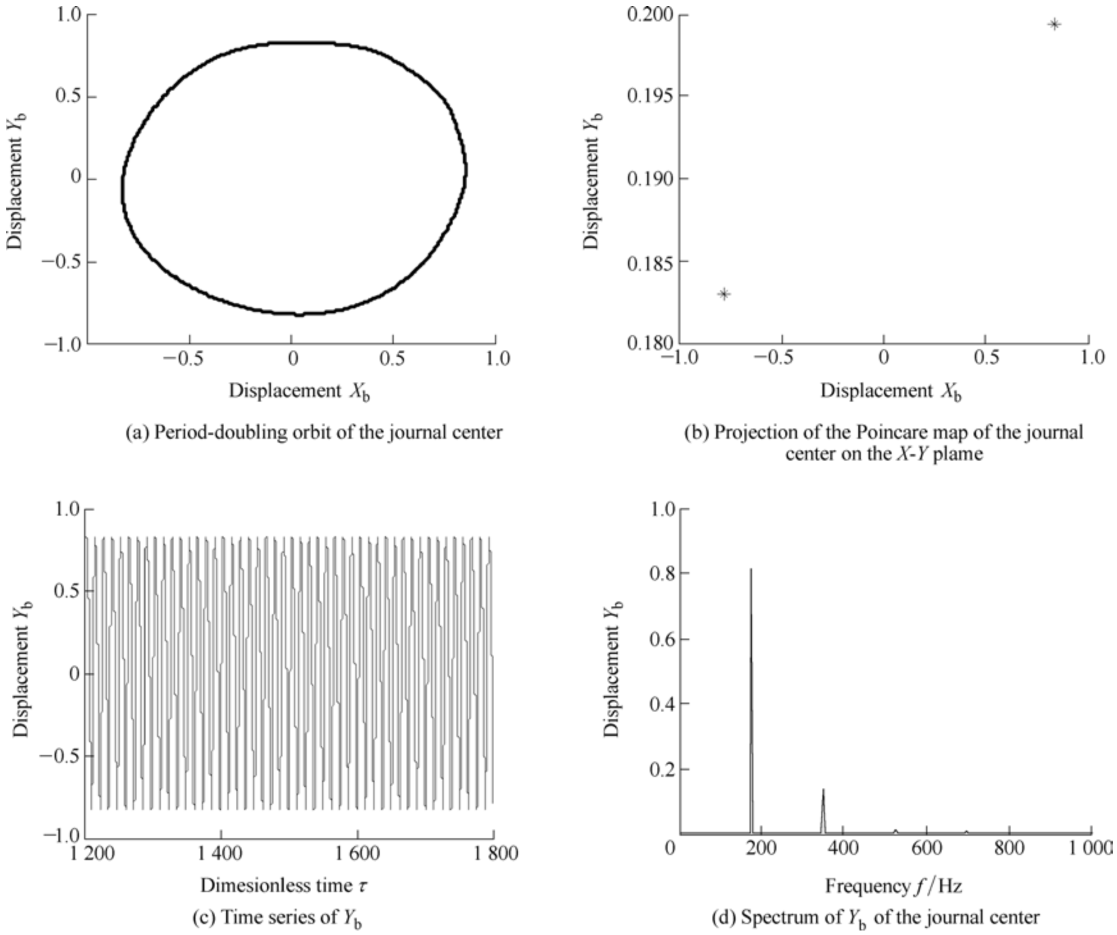


(a) Chaotic orbit of the journal center

(b) Projection of the Poincare map of the journal center on the X-Y plane

(c) Time series of Y_b

Fig. 7. Chaotic orbit of the journal center, projection of the Poincaré map of the journal center on X-Y plane, time series of Y_b at bearing "b" station for ω=1700 rad/s



(a) Period-doubling orbit of the journal center

(b) Projection of the Poincare map of the journal center on the X-Y plame

(c) Time series of Y_b

(d) Spectrum of Y_b of the journal center

Fig. 8. Period-doubling orbit of the journal center, projection of the Poincaré map of the journal center on X-Y plane, time series of Y_b and FFT spectrum of Y_b at bearing "b" station for ω=2 200 rad/s

5 Conclusions

A three axial-grooved gas-lubricated journal bearing-unsymmetrical rotor system with gyroscopic effect is modeled. In order to reduce the computational cost and minimize the computational error, the gas film pressure distribution is solved by using the differential transformation method. The dynamic equation of motion of three axial-grooved gas-lubricated journal bearing-unsymmetrical rotor system is solved by using the modified Wilson-θ-based method. The unbalanced dynamic responses of the system are investigated via bifurcation diagram, orbit diagram, Poincaré map, time series and frequency spectrum diagram. There exists the phenomenon of stable periodic motion, critical quasi-periodic motion and instable chaotic motion in nonlinear rotor system with axial-grooved gas-lubricated journal bearing support. According to the proposed models and numerical results, the following conclusions can be drawn.

(1) The strong nonlinearity of the gas film forces has significant influence on the stability of gas-lubricated journal bearing-rotor system.

(2) In case 1, with the increase of ω, the unbalanced periodic response of the system loses its stability and turns into quasi-periodic motion. As ω increases continually, quasi-periodic motion turns into subharmonic motion. When ω reaches to ω=1640 rad/s, the motion of the journal center becomes chaotic motion. The periodic-quasi periodic-mode locking-chaotic routine is shown in Fig. 3.

(3) In case 2, with the increase of ω, the periodic motion bifurcates to period-doubling motion, i.e., the half-speed whirl of the rotor, which is considered as an obstacle in the application and development of gas bearing with higher speed. If the rotating velocity is higher, the system motion will become unstable.

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