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Strength-Toughening Model of Eutectic Ceramic Composite with Inherent Defects

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Abstract: Strengthening and toughening mechanisms in composite ceramics is complex. A change in a single parameter induces multiple property variations. The multiple changes in properties are often incompletely represented in theoretical models. This incompleteness in the parameter chosen fails to explain the mechanism of failure in composite ceramics. The exponential toughness function is used to represent the pull-out toughening mechanism, which dominates the crack growth resistance curve(*R*-curve). The strengthening-toughening model is established based on the Mori-Tanaka method(M-T method). The influence of inherent defects on toughness function and strength is analyzed by using this model. The theoretical result is compared with the experiment data. This model exactly reflects the change in strength. The theoretical result indicates that defects change the toughness function. Moreover, micro-cracks increase toughness size a_c, and the strength of crack instable extensions acutely decreases as defect content increases. This presented model establishes the relationship among the important mechanical parameters of defect, strength, elastic modulus, and the *R*-curve.

Keywords: inherent defects, pull-out toughening, toughness function, strength

1 Introduction

The inherent brittleness of ceramic materials make them extremely sensitive to micropores, microcracks, and other defects. To improve the inherent defect tolerance of ceramic materials, modern ceramic composites greatly promote toughness by introducing fibers[1–2], toughness particles[3–5], and a transformation phase[6–7] into the ceramic matrix. Relative theoritical models have been established to explain their mechanisms. However, the mechanisms of composite ceramic fracture and toughening have become more complex because of the introduction of other phases. Explaining these complex mechanisms by accurately using theoretical models is difficult.

Strengthening and toughening mechanisms are usually analyzed separately in most theoretical research of composites. When the strengthening and toughening effects of particles, fibers and eutectic rods are studied theoretically, the focus are usually on additives and the defects are usually ingnored[8–9]. However, toughness is treated as a constant in crack propagation analysis when analyzing the influence of defects [10–11]. The fact that the value of composite material toughness is a function of crack propagation length and its interaction with inherent defects are not considered. These theoretical models cannot completely reflect the fracture mechanism in composite ceramics. Toughness is undeniably a constant if crack length is sufficiently large[11]. However, crack growth resistance behavior is obvious when the crack length is small. Establishing a strength model controlled by the crack growth resistance curve(*R*-curve) of composite materials, in which the effect of defects is taken into consideration, is necessary.

Two common mathematical models consider changes in crack growth resistance with crack propagation length[12]. COOK and CLARKE[13] proposed a crack growth resistance curve in the form of the following power function:

$$K = K_0(a/d)^n \quad (a \geqslant d). \tag{1}$$

This formula is widely used in characterizating the *R*-curve of metal materials. This power function derived from the toughness function is only an empirical formula without physical basis and cannot entirely reflect the crack propagation law. Moreover, its extrapolation based on the formula is unreliable[14].

RAMACHANDRAN, et al[15], proposed an exponential function *R*-curve:

$$K = K_\infty - (K_\infty - K_0)\exp(-\Delta a/\lambda), \tag{2}$$

where a is the crack length, Δa is crack extension length, λ is crack normalized size, and K_∞ and K_0 are respectively the

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intrinsic toughness of matrix material and critical fracture toughness value of composite in these two functions. The meaning of λ in the exponential functionis not very clear. Moreover, the corresponding values for this type of material is obtained through experiment fitting, and the formula is not explained theoretically. In addition, these two models do not consider the influence of inherent defects. However, with experimental and theoretical result verification, the exponential function is effective in explaining the bridging toughness mechanism[16–17], and it has been widely used in materials that experience bridging, toughening, and pull-out toughening such as Si_3N_4 fiber reinforced composites[18–19] and the Al_2O_3/Al composite[20–21]. The crack growth resistance curve conforms greatly with the exponential toughening function. Thereby, using the exponential toughening function to characterizate the R-curve in pull-out and bridging toughening composites is reasonable. However, some characteristic parameters must be defined to better explain the decrease in strength as a result of inherent defects.

Unlike short fiber reinforced compsite material, defects are broadly disrtibuted in eutectic ceramic composites. Such defects decrease composite strength, fracture toughness, elastic modulus, and other properties. Thus, an exact mechanical model should reflect the comprehensive effect of defects. To study the influence of toughness on composite fracture, exponential function R-curve token by $T(a)$ is cited. Some characteristic parameters derived theoretically are based on the pull-out toughening mechanism. A semi-empirical strength-toughness model is established based on the Mori-Tanaka method(M-T method).

2 Toughness Function

In eutectic composites, multi-toughening mechanisms work in turn with crack propagation. Cracks mostly originated in the matrix of ceramic composites. The transformation toughening mechanism initially takes effect in the matrix during crack extension. Once the crack tip reaches the eutectic rod, the bridging and pull-outing of the eutectic rod strongly reduced the stress intensity factor of the applied loads. Fig. 1 shows the toughening process and sequence.

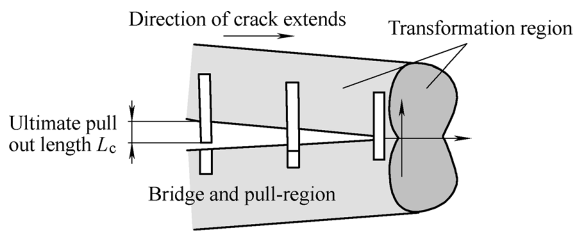

Fig. 1. Sketch of multi-toughening mechanism

2.1 Characteristic parameters in toughness function

The toughness function of eutectic composite ceramics is an exponential function of crack length:

$$T(a)=\begin{cases} T_0, \ a<a_1, \\ T_\infty-(T_\infty-T_0)\exp\left(\dfrac{-k(a-a_1)}{a_c}\right), \ a\geqslant a_1, \end{cases} \tag{3}$$

where a_1 represents the average length of crack extensions in the matrix(for ceramic composites with a eutectic-rod, this variable indicates the average distance between the surface of the eutectic-rod); k is the slope of the toughness function, which can obtained by experimentation for a specific composite; T_0 is the matrix intrinsic toughness; and T_∞ is the critical fracture toughness when the crack has extended adequately and can be obtained through the specific toughening mechanisms or experimental data[22–23]. Pull-out toughening is the predominant toughening mechanism in ceramic composites with eutectic rod[24]. NI[9] provided the pull-out toughening value expression of ceramic composite with a random distributed rod-like eutectic as

$$\Delta K=\frac{\sigma_{fu}}{4}\sqrt{\frac{fER}{\mu_s\sigma_{22}^{r}}}, \tag{4}$$

where f is the volume fraction of the rod-like eutectic, σ_{22}^{r} is the thermal residual stress perpendicular to the eutectic rod axes(which can be obtained using the interact direct derivative(IDD) estimate)[25]; E is the effective elastic modulus of ceramic composite; R is the average radius of the eutectic rod (the value is approximately 10 to 20 μm for the Al_2O_3-ZrO_2 eutectic); μ_s is the friction coefficient between the eutectic and the surrounding medium(which usually composes 0.2 of this composite); and σ_{fu} is the strength of the eutectic(which can be computed using dislocation pile-up theory[9] and the value of which ranges from 2 GPa to 3 GPa). Other toughening mechanisms devoted to critical fracture toughness is limited, and the value is approximately 4 MPa•$m^{1/2}$ [9]. The, critical fracture toughness can be shown as

$$T_c=4+\Delta K. \tag{5}$$

a_c is the token of the toughening size in Eq. (3), defined as the minimum crack length when toughness values no longer increase with crack length. Thus, a eutectic rod is pulled out completely when crack length reaches a_c. When $a>a_c$, the rod would not consume any energy as a crack extension, and it can be computed using the formula in the following section.

Assuming that the penny crack in a homogeneous material is subjected to simple tension σ perpendicular to the crack plane, then crack opening displacement b satisfies the following formula[26]:

$$b(r)=\frac{8(1-\upsilon^2)\sqrt{(a^2-r^2)}}{\pi E}\sigma, \tag{6}$$

where a is the crack radius and r is the distance from an arbitrary point on the crack surface to the crack center.

If crack radius a extends to a_c such that $\sigma=\sigma_c$ as crack opening displacement exactly equals the ultimate pull-out length of eutectic, then Eq. (6) becomes

$$L_c=\frac{8(1-\upsilon^2)a_c\sigma_c}{\pi E}, \tag{7}$$

$$\sigma_c=T_c/\sqrt{\pi a_c}. \tag{8}$$

The eutectic rod in crack regions will break when the crack surface displacement reaches the ultimate pull out length L_c, that is, when the total force on the end and the cylindrical surface of the eutectic rod reaches the tensile strength. If the contact area of the contact surface is proportional to the inherent defect content, and the interface strength between the eutectic and the surrounding medium is weakened by defects, the critical condition is

$$2\pi RL_c\tau_{mc}(1-f_2)+\pi R^2\sigma_{mc}(1-f_2)=\pi R^2\sigma_{fu}, \tag{9}$$

where f_2 is the inherent defect volume fraction; σ_{fu} is the fracture strength of eutectic rod; τ_{mc} and σ_{mc} are the average interfacial shear strength and the average interface tensile strength with surrounding matrix medium. By combining Eqs. (5), (7), (8), and (9), toughening size a_c is obtained as the following formula:

$$a_c=\pi^3\left[\frac{ER[\sigma_{fu}-\sigma_{mc}(1-f_2)]}{16(1-\upsilon^2)\tau_{mc}(1-f_2)\left(4+\frac{\sigma_{fu}}{4}\sqrt{\frac{fER}{\mu_s\sigma_{22}^{r}}}\right)}\right]^2 \tag{10}$$

2.2 Influence of defect volume friction to toughness function

The matrix thermal expansion coefficient for Al_2O_3-ZrO_2 ceramic composites becomes lower than the eutectic. Residual stress in the eutectic is tensile and concentrated on the eutectic end. The equation $\tau_{mc}=\sigma_{mc}=m\cdot\sigma_m$ simplifies the calculation because of the weak interface contact between eutectic and around media. Due to the lack of experiment data for micro-strength, alumina fiber tensile strength is taken as the matrix strength and σ_m=2 GPa, (for alumina fiber, tensile strength approximately 1.4–2.6GPa[27]). Considering that the microstructure of the alumina matrix is more complex with a weak interface, m=0.95 is adopted experientially as a conservative computing. According to Eq. (8), toughening size a_c is related to the volume detects volume, as shown in Fig. 2.

Fig. 2 shows that toughness size a_c gradually increases as defect fraction increases. Fig. 3 shows that critical fracture toughness is related to the volume fraction of defects.

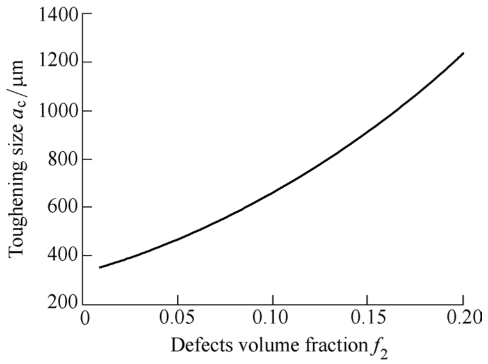

Fig. 2. Toughening size to defects volume fraction

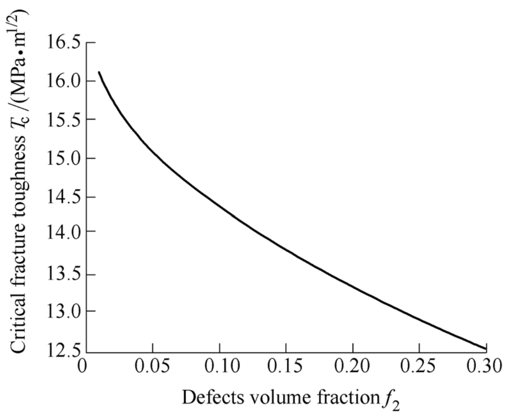

Fig. 3. Critical fracture toughness to defects volume fraction

Fig. 3 shows the relationship between critical fracture toughness and the defects volume fraction. The critical fracture toughness of ceramic composite decreases as the defects volume fraction increases. When the fraction is low, its decrease is relatively significant. The toughness function changes significantly under the combined effects of these two effects. Toughness size a_c has a significant influence. Fig. 4 shows that the increasing interval of toughness with many defects is greater than that with few defects, and toughness function increases more acutely in ceramic composites with few defects.

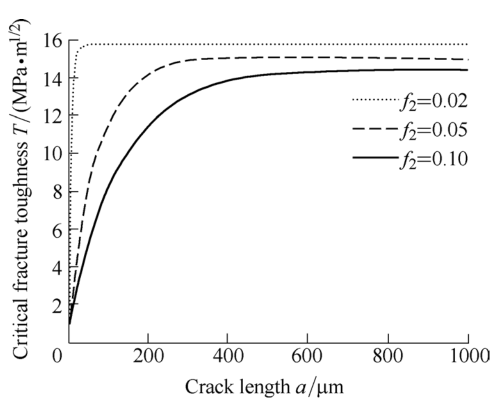

Fig. 4. Influence of defects parameter to toughness function

3 Strength of Composite

Eutectic rod random distribution forms an effective matrix of composites. Microcracks and some inhomogametic inclusions are randomly embedded in the matrix. The cracks are assumed to have the same size. Fig. 5 shows the distribution of micro cells. The coordinate system is shown in Fig. 6. The coordinate transformation is as follows:

$$\boldsymbol{x}' = \boldsymbol{P}\boldsymbol{X}, \tag{11}$$

$$\boldsymbol{P} = \begin{pmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{pmatrix}, \tag{12}$$

where

$$l_1 = \cos\theta\cos\varphi, l_2 = \cos\theta\sin\varphi, l_3 = -\sin\theta,$$
$$m_1 = -\sin\varphi, m_2 = \cos\varphi, m_3 = 0,$$
$$n_1 = \sin\theta\cos\varphi, n_2 = \sin\theta\sin\varphi, n_3 = \cos\theta.$$

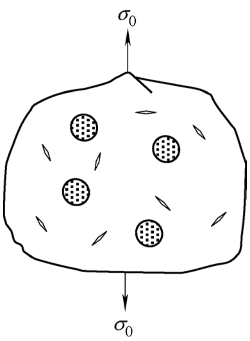

Fig. 5. Inclusions and crack distribution model

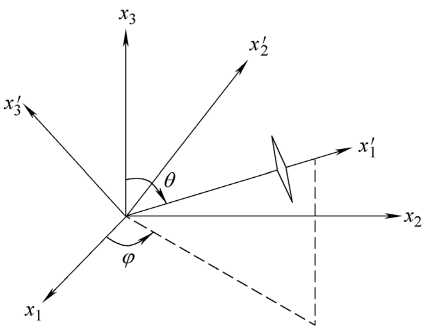

Fig. 6. Microcrack and its local coordinate system

Based on the M-T method[28], the effective eigen-strain ε^{1*} in inclusions under stress $\boldsymbol{\sigma}_0$ under global system is

$$\boldsymbol{\varepsilon}^{1*} = -\left(\Delta\boldsymbol{C}\boldsymbol{T}_{\mathrm{s}}^{-1}\boldsymbol{M}_1'\boldsymbol{T}_{\mathrm{s}} + \boldsymbol{C}_0\right)^{-1}[\Delta\boldsymbol{C}\left(\boldsymbol{\varepsilon}^0 + \tilde{\boldsymbol{\varepsilon}}\right) + \boldsymbol{C}_1\boldsymbol{\varepsilon}^{\mathrm{t}}], \tag{13}$$

where

$$\boldsymbol{T}_{\mathrm{s}} = \begin{pmatrix} l_1^2 & l_2^2 & l_3^2 & l_2l_3 & l_3l_1 & l_1l_2 \\ m_1^2 & m_2^2 & m_3^2 & m_2m_3 & m_1m_3 & m_2m_1 \\ n_1^2 & n_2^2 & n_3^2 & n_2n_3 & n_1n_3 & n_2n_1 \\ 2m_1n_1 & 2m_2n_2 & 2m_3n_3 & m_2n_3+m_3n_2 & n_3m_1+n_1m_3 & m_1n_2+m_2n_1 \\ 2n_1l_1 & 2n_2l_2 & 2n_3l_3 & l_2n_3+l_3n_2 & n_3l_1+n_1l_3 & l_1n_2+l_2n_1 \\ 2l_1m_1 & 2l_2m_2 & 2l_3m_3 & l_2m_3+l_3m_2 & m_3l_1+m_1l_3 & l_1m_2+l_2m_1 \end{pmatrix},$$

and $\boldsymbol{M}_1'$ is the Eshelby tensor under local coordinate system from Ref. [29], $\Delta\boldsymbol{C}=\boldsymbol{C}_1-\boldsymbol{C}_0$, $\boldsymbol{C}_0$ and $\boldsymbol{C}_1$ is stiffness of matrix and inclusions, respectively; $\boldsymbol{\varepsilon}^{\mathrm{t}}$ is thermal mismatch strain between particles and matrix.

On account that stress in microcrack is zero, the eigen-strain of microcrack under global system is

$$\boldsymbol{\varepsilon}^{2*} = -\left(\boldsymbol{T}_{\mathrm{s}}^{-1}\boldsymbol{M}_2'\boldsymbol{T}_{\mathrm{s}} - \boldsymbol{I}\right)^{-1}\left(\boldsymbol{\varepsilon}^0 + \tilde{\boldsymbol{\varepsilon}}\right). \tag{14}$$

Free energy variation caused by microcracks when the composite is under the applied $\boldsymbol{\sigma}_0$ is

$$\Delta W = -\frac{1}{2}\int_{V_2}\boldsymbol{\sigma}_0\boldsymbol{\varepsilon}^{2*}\mathrm{d}V. \tag{15}$$

The free energy variation for each microcrack is

$$E_{\mathrm{int}}^2 = -\frac{\Omega_2}{2a_2}\int_{\phi_1}^{\phi_2}\int_{\theta_1}^{\theta_2}\boldsymbol{\sigma}_0\boldsymbol{\varepsilon}^{2*}g_2(\theta,\phi)\sin\theta\mathrm{d}\theta\mathrm{d}\phi, \tag{16}$$

where $\Omega_2=4\pi a^2t/3$ is the volume of a single penny microcrack that the penny crack is treated as an extremely oblate ellipsoid; a and t are the radius and thickness of the microcrack, respectively; and $t \ll a$. $g_2(\theta,\phi)$ is the probability distribution function. According to the expression of $\boldsymbol{M}_2'$, if $t/a\to 0$, then Eq. (12) can be shown as

$$E_{\mathrm{int}}^2 = -\frac{8(1-\upsilon)^2a^3}{(1-2\upsilon)(2-\upsilon)}\boldsymbol{\sigma}_0\boldsymbol{Q}\left(\boldsymbol{\varepsilon}^0 + \tilde{\boldsymbol{\varepsilon}}\right), \tag{17}$$

where

$$\boldsymbol{Q} = \int_{\phi_1}^{\phi_2}\int_{\theta_1}^{\theta_2}\boldsymbol{T}_{\mathrm{s}}^{-1}\boldsymbol{T}_1\boldsymbol{T}_{\mathrm{s}}g_2(\theta,\phi)\sin\theta\mathrm{d}\theta\mathrm{d}\phi, \tag{18}$$

and

$$\boldsymbol{T}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{\upsilon}{1-\upsilon} & 0 & 0 & 0 & 0 & 0 \\ \frac{\upsilon}{1-\upsilon} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\upsilon}{1-\upsilon} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\upsilon}{1-\upsilon} \end{pmatrix}. \tag{19}$$

The energy release rate of each microcrack is

$$G = \left|\frac{E_{\mathrm{int}}^2}{\Delta A}\right| = \left|\frac{\partial E_{\mathrm{int}}^2}{\partial(\pi a)^2}\right|. \tag{20}$$

Substituting Eq. (17) into Eq. (20),

$$G = \frac{8(1-\upsilon)^2 a}{\pi(1-2\upsilon)(2-\upsilon)} \boldsymbol{\sigma}_0 \boldsymbol{Q}\left(\boldsymbol{\varepsilon}^0 + \tilde{\boldsymbol{\varepsilon}}\right). \tag{21}$$

Critical energy release rate $G_c(a)$ can be obtained from the toughness of the composite using the following formula:

$$G_c(a) = (1-\upsilon^2)T(a)^2 / E, \tag{22}$$

where E and υ are the effective elastic modulus and poisson's ratio of composite material. The compute method can be found in Ref. [29]. If $G=G_c(a)$, then crack initiation stress is satisfied with

$$\boldsymbol{\sigma}_0 \boldsymbol{Q}\left(\boldsymbol{\varepsilon}^0 + \tilde{\boldsymbol{\varepsilon}}\right) = \frac{\pi(1-2\upsilon)(2-\upsilon)}{8(1-\upsilon)^2 a} G_c(a), \tag{23}$$

and the crack unstable propagation condition is

$$\begin{cases} G(a) = G_c(a), \\ \mathrm{d}G(a)/\mathrm{d}a = \mathrm{d}G_c(a)/\mathrm{d}a. \end{cases} \tag{24}$$

4 Numerical Example

Assuming that Al_2O_3-ZrO_2 ceramic composite is subjected to simple tensile loads, wherein the particle volume fraction is f_1=5% and the defects volume fraction is f_2=3%, then the resulting crack initiation stress to microcrack length is that shown in Fig. 7.

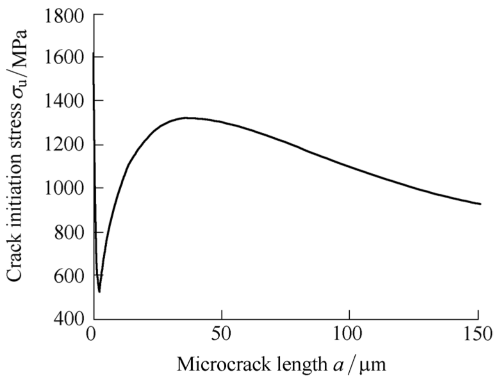

Fig. 7. Crack initiation stress in different crack length

Fig. 7 shows that crack initiation stress is no longer a simple linear relationship with the square root of the half-length for the changes of toughness value with crack length. As the length of crack propagation increases, the crack initiation stress is reduced before the crack radius reaches a_1. However, when crack radius is greater than a_1, the interval of crack initiation stress increases because of the sharp increasing interval of the toughness function. This increasing process greatly increases the tolerance of defects. Thus, ceramic composite material is relatively insensitive to defect size.

According to the crack instability condition in Eq. (24), the stress of unstable crack propagation in simple tension for Al_2O_3-ZrO_2 ceramic composite is 1.08 GPa, and the corresponding crack length is 37.5 μm with a defect volume fraction of f_1=5%. Thus, the strength of ceramic composite is 1.08 GPa. When the applied stress is smaller than this value, Al_2O_3-ZrO_2 ceramic composite with defect volume fraction of f_1=5% can withstand the stress of 1.08 GPa without instability propagation. This result corresponds with the data tested by ZHAO, et al[30].

The volume fraction of defects significantly influences crack initiation stress. This relationship is shown in Fig. 8.

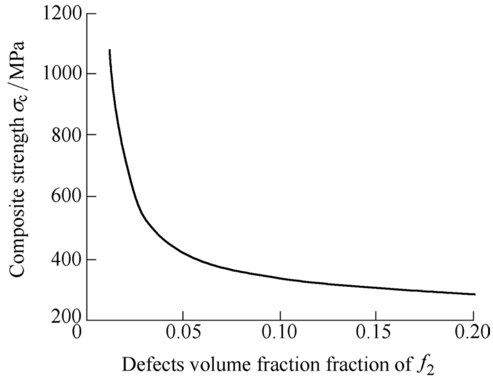

Fig. 8. Strength in different defects volume fraction

Fig. 8 shows that the strength of tensile instability is reduced sharply as the defect volume fraction increases if the crack length is the same. Strength is extremely sensitive to defect volume fraction when the fraction is low. Reducing defects as much as possible is significant when improving the strength of ceramics.

Particles also influence crack initiation strength. Thermal residual stress causes stress concentration and even causes interface debonding at low applied stress because of the difference in the thermal expansion coefficient between particles and the matrix. For example, if the crack radius is a=50 μm and defect volume fraction is f_2=3%, then the influence of Al_2O_3 particles is similar to that shown in Fig. 9.

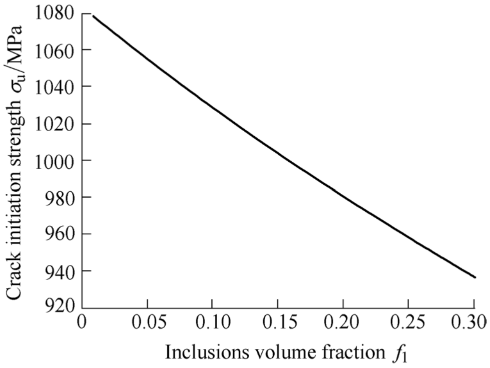

Fig. 9. Crack initiation stress to inclusion volume fraction

The theoretical calculations and experimental results of fracture strengths are shown in Table 1.

Table 1. Comparison of theoretical and experimental result

Material number	1[31]	2[31]	3[31]	4[30]	5[32]	6[33]
Relative density ρ / %	95.5	96.3	97.2	98.8	99.2	99.8
Elastic modulus E / GPa	–	–	–	410	–	422
Fracture toughness T_c / (MPa • m$^{0.5}$)	7.1	8.8	12	12.6	13.7	14.8
Fracture strength(experiment) σ_c / MPa	620	860	1080	1168	1278	1568
Fracture strength(calculated) σ_c / MPa	815	858	928	1062	1143	1283
Error of fracture strength e / %	31.4	–0.2	–14.1	–9.1	–10.5	–18.2

"–" means there's no experiment data in related reference

In Table 1, critical fracture toughness and toughening size is obtained according to defect fraction based on Eqs. (5) and (10), respectively. Subsequently, material strength calculated using Eq. (24). The elastic modulus of the unknown is calculated using the theory in Ref. [29]. Table 1 shows that the theoretical results are always smaller than the experimental results except for material 1. The results can be explained with the following reasons. First, the defects, including micropores and microcracks, are treated as microcracks in theoretical calculations, which exaggerate the influence of defects. Second, the superposition of phase transformation toughening and bridging toughening increases the slope of the toughness function at the beginning of the segment. However, this effect is not sufficiently taken into consideration. Toughening size is smaller than that in pull-out toughening, which decreases the strength of the theoretical. Third, the defect fraction often changes with sintering technology and procedures. Different sintering processes may produce ceramic composite with similar defect fraction values, but with enormous variance in microscopic structure and toughening mechanism. The theory does not take these factors into consideration. For instance, material 1 contains large amounts of ZrO_2 phase, in which large numbers of spherules was observed and the toughening mechanism changed significantly[33]. In addition, the small relative density indicates that the presence of many macro pores in this composite, which caused composite fracture at low loads. Moreover, in material 7, the eutectic in the ceramic mostly formed triangular symmetric colonies with excellent mechanical properties. Such colonies formed the actual matrix of the composite. The pull-out length of the eutectic was limited and size of defects was very small. The fracture process may not reach the stable state of eutectic-rod pulling-out. Therefore, taking full account of the scale of the microstructure size and the micro-toughening mechanism is necessary for explaining the strength of composites.

5 Conclusions

A theoretical method is improved to predict the strength of eutectic ceramic composites based on the M-T method according to the pull-out toughening mechanism of eutectic composite ceramics. This model presents a reasonable explanation for the decrease in strength of toughened ceramic. Taking full account of the scale of the microstructure size and the micro-toughening mechanism is necessary. This paper makes the following conclusions:

(1) Defects in ceramic composite have little effect on the value of critical fracture toughness. However, toughness size a_c is significantly increased and toughness function is obviously decreased. The material is also susceptible to unstable propagation. In addition, material strength is reduced as the number of defects increase.

(2) The pull-out mechanism is prevalent in ceramic composites. The toughening effect is present until the length of the crack is larger than a_1.

(3) The volume fraction of the microcracks and particles in composites has a significant effect on the strength of the unstable propagation of cracks. It is also rapidly reduced as the volume fraction of the microcracks increases, especially when the fraction is low. Strength is extremely sensitive to defects volume fraction. Particles cause strength to decrease significantly.

(4) All defects are treated as microcracks in calculations. Thus, theoretical results are smaller than the experimental results.

References

[1] MARSHALL D B, COX B N, EVANS A G. The mechanics of matrix cracking in brittle-matrix fiber composites[J]. *Acta Metall*, 1985, 33(11): 2013–2021.

[2] JAE-SEOL L, TOYOHIKO Y. Fabrication of short-fiber- reinforced SiC composites by polycarbosilane infiltration[J]. *Journal of the European Ceramic Society*, 2004, 24(1): 25–31.

[3] KRSTIC V V, NICHOISON P S, HOAGLAND R G. Toughening of glasses by metallic particles[J]. *J. Am. Ceram. Soc.*, 1981, 64(9): 499–504.

[4] YEOMANS J A. Ductile particle ceramic matrix composites-scientific curiosities or engineering materials?[J]. *Journal of the European Ceramic Society*, 2008, 28(7): 1543–1550.

[5] LIU Y G, ZHOU J Q, SHEN T D. Effect of nano-metal particles on the fracture toughness of metal-ceramic composite[J]. *Materials and Design*, 2013, 45: 67–71.

[6] EVANS A G. Perspective on the development of high-toughness ceramics[J]. *J. Am. Ceram. Soc.*,1990, 73: 187–195.

[7] KELLY P M, FRANCIS ROSE L R. The martensitic transformation in ceramics-its role in transformation toughening [J]. *Prog Mater Sci*, 2002, 47(5): 463–557.

[8] QIAO Y, KONG X G. Unstable crack advance across a regular array of short fibers in brittle matrix[J]. *Composites Science and Technology*, 2004, 64(5): 711–717.

[9] NI X H. *Research on micromechanical model and macro mechanical properties for composite ceramics with interphase*[D]. Shijiazhuang: Ordnance Engineering College, 2008. (in Chinese)

[10] LI W F, DU S Y. Stress and strength of elastic matrix with misoriented microcracks and prolate spheroids[J]. *Acta Mechanica Sinica,* 1994, 26(5): 541–550. (in Chinese)

[11] PALIWAL B, RAMESH K T. An interacting micro-crack damage model for failure of brittle materials under compression[J]. *Journal of the Mechanics and Physics of Solids*, 2008, 56(3): 896–923.

[12] SONG S X, QIU X N. Discussion on the expressions of *R*-curve behavior of ceramic materials[J]. *Journal of University of Jinan*, 2007, 21(3): 197–199. (in Chinese)

[13] COOK R F, CLARKE D R. Fracture stability R-curves strength variability[J]. *Act Metal*, 1988, 36(3): 555–562.

[14] CHEN J P, XI J, GUAN Z D. Study on the regressive parameters of *R*-curve behavior for ceramic composite[J]. *Journal of the Chinese Ceramic Society*, 1997, 25(1): 37–43. (in Chinese)

[15] RAMACHANDRAN N, SHETTY D K. Rising crack-growth-resistance(R-curve) behavior of toughened alumna and silicon nitride[J]. *J Am Ceram Soc*, 1991, 74(10): 2634–2641.

[16] DONG Q W, FENG P, SUN Y, et al. Influence of bridging characteristics on fracture resistance curves[J]. *Journal of Harbin Institute of Technology*, 2002, 34(3): 348–351. (in Chinese)

[17] RADDATZ O, SCHNEIDER G A, CLAUSSEN N. Modelling of *R*-curve behavior in ceramic/metal composites[J]. *Acta Mater*., 1998, 46(18): 6381–6395.

[18] FÜNFSCHILLING S, FETT T, HOFFMANN M J, et al. Mechanisms of toughening in silicon nitrides: The roles of crack bridging and microstructure[J]. *Acta Materialia*, 2011, 59(10): 3978–3989.

[19] ZOU L H, HUANG Y, PARK D S, et al. *R*-curve behavior of Si_3N_4 whisker-reinforced Si_3N_4 matrix composites[J]. *Ceramics International*, 2004, 31(1): 197–204.

[20] RADDATZ O, SCHNEIDER G A, CLAUSSEN N. Modeling of R-curve behavior in ceramic/metal composites[J]. *Acta Mater*., 1998, 46(18): 6381–6395.

[21] RADDATZ O, SCHNEIDER G A, MACKENS W, et al. Bridging stresses and R-curves in ceramic/metal composites[J]. *Journal of the European Ceramic Society*, 2000, 20(13): 2261–2273.

[22] ZHAO Z M, ZHANG L, SONG Y L, et al. Composition microstructures and properties of $Al_2O_3/ZrO_2(4Y)$ prepared by combustion synthesis under high gravity[J]. *Special Casting & Nonferrous Alloys*, 2008, (S1): 297–302. (in Chinese)

[23] PAN C Z, ZHANG L, ZHAO Z M, et al. Growth mechanisms of $Al_2O_3/ZrO_2(Y_2O_3)$ eutectic composite ceramic[J]. *Materials Science and Engineering of Powder Metallurgy*, 2007, 12(5): 290–295. (in Chinese)

[24] YU G B, YAN W, WANG S H, et al. Toughening mechanism of lined Al_2O_3-ZrO_2 multiphase ceramics in SHS composite pipes[J]. *Journal of University of Science and Technology Beijing*, 2006, 13(2): 178–182.

[25] ZHENG Q S, DU D X. An explicit and universally applicable estimate for the effective properties of multiphase composites which accounts for inclusion distribution[J]. *J. Mech. Phys. Solids,* 2001, 49(11): 2765–2788.

[26] YU S W, FENG X Q. *Damage mechanics*[M]. Beijing: Tsinghua University Press, 1997. (in Chinese)

[27] WANG D G, ZHONG L L, GU L X. Preparation and application of alumina fibers[J]. *New Chemical Materials*, 2002, 30(4): 17–19. (in Chinese)

[28] MORI T, TANAKA K. Average stress in matrix and average energy of materials with misfitting inclusion[J]. *Act. Metal,* 1973, 21: 571–574.

[29] FU Y W, LIU X Q, NI X H, et al. Effective elastic property prediction of ceramic composite with inherent defect[J]. *Journal of Mechanical Engineering*, 2012, 48(16): 46–51. (in Chinese)

[30] ZHAO Z M, ZHANG L, BAI H B, et al. Microstructure and different-scale multistage toughening of self-toughening composite ceramics with in-situ growing nano-micron fibers[J]. *Rare Metal Materials and Engineering*, 2006, 35(S2): 91–95. (in Chinese)

[31] ZHAO Z M, ZHANG L, SONG Y L, et al. Influences of ZrO_2 on $Al_2O_3/ZrO_2(4Y)$ prepared by combustion synthesis under high gravity[J]. *Rare Metal Materials and Engineering,* 2009, 38(S2): 98–101. (in Chinese)

[32] ZHAO Z M, ZHANG L, ZHANG S Y, et al. Microstructures and mechanical properties of large-scale $Al_2O_3/ZrO_2(Y_2O_3)$ self-growing ceramic plates prepared by combustion synthesis under high gravity[C]//*International Conference on Smart Materials and Nanotechnology in Engineering*, Harbin, China, July 1–4, 2007: 64235B1–64235B8.

[33] ZHAO Z M, ZHANG L, SONG Y G, et al. $Al_2O_3/ZrO_2(Y_2O_3)$ self-growing composites prepared by combustion synthesis under high gravity[J]. *Scripta Materialia*, 2008, 58(3): 207–210.

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