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Turned Trochoidal Disturbance on a Liquid Jet Surface

SADIK Shalom[1, *], KIRZHNER Felix[2], and KRAMARENKO Denis[3]

1 *Department of Mechanical Engineering, ORT Braude College, Karmiel 21982, Israel*
2 *Division of Environmental, Water and Agricultural Engineering, Faculty of Civil and Environmental Engineering, Technion-IIT, Haifa 32000, Israel*
3 *Biotechnology Engineering, ORT Braude College, Karmiel 21982, Israel*

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Abstract: This paper shows that a turned trochoidal function disturbance may lead to peripheral drops production. The resulting model is used to describe that a turned trochoidal disturbance leads to peripheral drops production on the liquid jet surface without the necessity for superimposed disturbances. The trochoid is a non-unique parametric function. Only non-unique parametric functions disturbances may lead to peripheral drops production. The trochoidal function disturbance is decomposed to Fourier series. Every Fourier element receives an amplification factor in accordance to the Rayleigh inviscid jet model. Peripheral drops are received on the jet surface. The paper shows that all trochoidal disturbance functions, prolate cycloid, cycloid and curtate cycloid have a capability of peripheral drops producing. A limited capability of peripheral drops production is introduced for the trochoidal curtate cycloid. Produced drops size are reduced for increasing the jet velocity and wave number. Smaller drops are also received by transition from the prolate cycloid to curtate cycloid disturbance.

Keywords: drops, liquid jet, trochoidal function

1 Introduction

Mathematical models of spray atomization were mostly developed for serial drops formation. The RAYLEIGH linear model[1] was the first to show this development. Subsequent works used non-linear models which introduced satellite drops between the main drops but all those were serial. Spray model that defined peripheral drops production was proposed in Ref. [2] which was based on harmonic superimposed disturbances. The current work reveals peripheral drops creation without refined to superimposed disturbances.

The turned trochoid which is a parametric function leads to a disturbance function that is not unique. A non-unique disturbance may lead to peripheral drops production. The choice of examining a trochoidal disturbance was supported by lot of works about incompressible fluid waves.

The RAYLEIGH jet model[1, 3] leaded by normal modes disturbances to serial drops production only. These normal modes are unique functions. A unique function cannot leads to peripheral drops production. So, the RAYLEIGH model leaded only to serial drops production. RAYLEIGH demonstrated that a disturbance on the jet surface may be developed and grows. This disturbance leads to the jet breakup into serial drops. ZIMMELS, et al[2], proposed a model to show that a sequence of superimposed disturbances led to peripheral drops production. The superimposed disturbances are non-unique functions. So, the disturbances lead to peripheral drops production. Only non-unique function may lead to peripheral drops production. This paper focus on a trochoidal disturbance which leads to peripheral drops production on the jet surface without the necessity for building superimposed disturbances as was suggested by ZIMMELS, et al[2].

Turned trochoidal waves as a solution for an inviscid incompressible fluid were introduced by GERSTNER[4]. GERSTNER's analysis was performed by using a Lagrangian formulation of non-viscous fluid motion and mass conservation. Turned trochoidal-waves solution was also received by WEBER[5] for incompressible viscous fluid. WEBER's analysis was also performed by using a Lagrangian formulation but of viscous fluid motion. Those analyses were performed for two dimensional Cartesian coordinate system.

Turned trochoidal waves figure solution was received for cylindrical coordinates by INOGAMOV[6]. This theory was not developed for incompressible fluid jets. The theory describes a nonlinear wave on the surface of a cavity in a rotating fluid. CONSTANTIN, et al[7], introduced particles paths solution for turned trochoidal waves by the incompressible two dimensional Euler equations. The Euler equations usage means dealing with inviscid fluid. The

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* Corresponding author. E-mail: shaloms@braude.ac.il

solution was performed for Cartesian coordinate system by using the Euler two dimensional equations, mass conservation equation for incompressible fluid and boundary conditions: kinematic and dynamic boundary conditions for the free surface. Intersectional epitrochoidal and intersectional hypotrochoidal particle paths were received as a solution. The epitrochoidal path is parallel to the straight trochoid function while the hypotrochoidal path is parallel to the turned trochoid function: $0 < k \cdot A/2 < 1$.

DARLES, et al[8–9], used as a forward step the turned trochoidal wave as a stage of spray production. They showed as is introduced in Fig. 1[9] the trochoidal configuration possibilities. It is shown that the trochoidal may has a curtate cycloid shape while $0 < (k \cdot A/2) < 1$, cycloid shape while $(k \cdot A/2) = 1$ and prolate cycloid shape while $(k \cdot A/2) > 1$, where k is the wave number and A is the wave amplitude. DARLES, et al[9], claimed that the prolate cycloid shape which intersected itself did not relate to realistic water wave figure, so, the value of the item $(k \cdot A/2)$ cannot go through one.

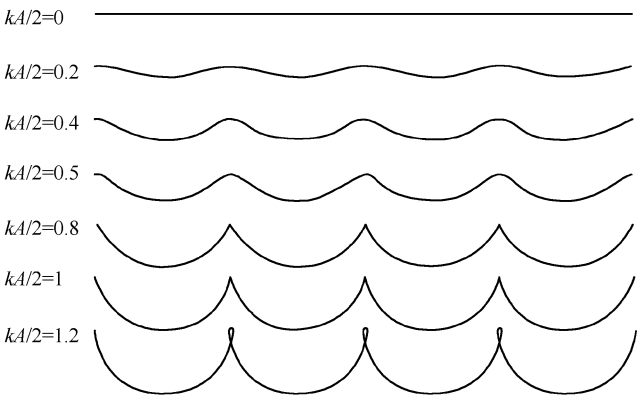


Fig. 1. Shapes of water trochoidal waves

WU, et al[10], examined experimentally the effects of initial flow conditions on the primary breakup of non-turbulent and turbulent round liquid jets in still gases. The turbulence degree was controlled by boundary controlled cutter and its length to diameter ratio. The transition from laminar to turbulent flow occurred by enlargement the Reynolds number or/and enlargement the cutter length to diameter ratio. WU, et al[10], introduced a scheme which explains the mechanism for non-turbulent primary breakup; this scheme is introduced in Fig. 2.

The drops production mechanism which is introduced by Fig. 2 begins from the boundary layer developed around the contracted injector passage. Vorticities are produced into the boundary layer. While the jet is going out the vorticities are going larger and spread out as drops.

Sharp ends or spikes which are elongated to ligaments along the jet surface are noted in literature as a preliminary stage for drops production. Spikes may be identified in Fig. 2. An exhaustive description of ligaments as a preliminary stage for drops creation was noted by POMEAU, et al[11].

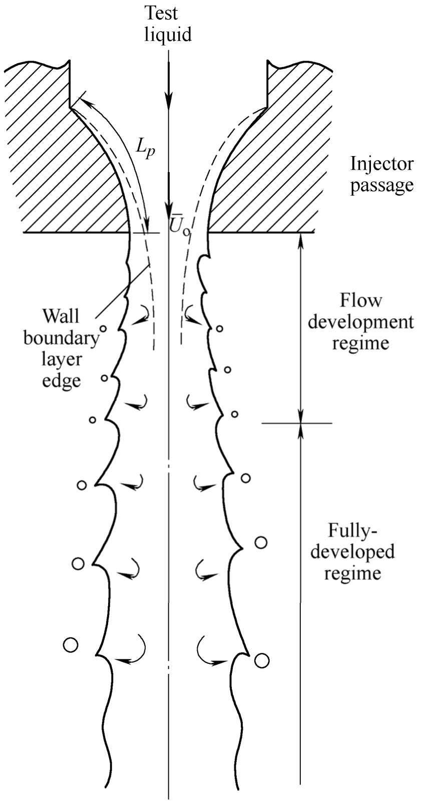


Fig. 2. Sketch of the injector passage vorticity mechanism for non-turbulent primary breakup

Trochoidal surface shape of water waves was also received by considering the effect of viscosity as was investigated theoretically on small amplitude Gerstner waves in deep water by WEBER[12]. A review by QIAN, et al[13] on atomization modeling introduced the peripheral drops atomization as the leading stage of the jet atomization and was indicated as the primary atomization. The peripheral atomization leads to a production of large unstable drops undergoes further disruption into smaller drops, this stage was designated as the secondary atomization. Primary and secondary atomization stages were introduced also by DESJARDINS, et al[14] onto a detailed numerical investigation of turbulent jets in quiescent air, with the focus on the processes leading to a liquid atomization. Focus on the primary atomization stage was introduced by SHINJO, et al[15] onto using the detailed numerical simulation data of primary atomization and characterization of the liquid jet surface instability development that leads to atomization. Focus on Primary atomization investigation was also done by OLIVIER, et al[16], the work simulate the breakup of turbulent jet under diesel conditions. HOEVE, et al[17], investigated the formation of micro droplets from micron-sized jets, as might be expected the atomization process leaded only to serial drops production. Capillary breakup of a liquid torus was introduced by MEHRABIAN, et al[18] onto computation of capillary instability of a Newtonian liquid torus suspended in an immiscible Newtonian medium. Analysis of Kelvin-Helmholtz instability of cylindrical interface with axial electric field was introduced by

AWASTHI, et al[19] for viscous potential flow. Viscosity enters through normal stress balance in viscous potential flow theory and tangential stresses are not considered. By obtained stability criterion it was shown that the axial electric field has stabilizing effect while relative velocity has destabilizing effect. The instability and subsequent atomization of viscous liquid jet emanated into a high-pressure gaseous surrounding was investigated both computationally and experimentally by IBRAHIM, et al[20]. The theoretical analysis was based on a simplified mathematical formulation of the continuity and momentum equations in their conservative form. A satisfactory agreement was received for the model prediction and experimental data for drop sizes at gas pressures of 150 and 300 psia for lower values of the investigated Weber number. VUORINEN, et al[21], investigated the droplet size effects on mixing, the study dealt with the specific interest on fuel injection and droplet atomization in low temperature combustion diesel engines, the primary result was that smallest droplets mix the best. YANG, et al[22], developed a theoretical model in order to investigate the instability of a viscoelastic liquid jet with axisymmetric and non-axisymmetric disturbances which is moving in a swirling air stream. Result showed that the viscoelastic liquid jet is more unstable than its Newtonian counterpart. RUO, et al[23], introduced a three –dimensional instability analysis of an electrified non-Newtonian liquid jet in order to examine the influence of viscoelastic stresses, electric force and surface tension. Results showed that the viscoelastic stresses play a stabilizing role while electrification destabilizes the disturbances with shorter wavelength and higher azimuthal wave numbers, surface tension enhances axisymmetric disturbances with wavelength comparable to the jet circumference while suppressing all the non-axisymmetric disturbances. Investigation of charged liquid jet was done by ZHAKIN, et al[24–26]. Results showed that in the long-wavelength region, the axisymmetric perturbations are always suppressed by the electric field while under specific value in the short-wave region the electric field destabilizes the axisymmetric perturbations. An increase in viscosity in the long-wavelength region can effectively stabilize the jet, while in the short-wave region this increase can lead to the development of instability. GRIGOR'EV, et al[27], derived a dispersion relation for capillary waves on the surface of a charged cylindrical jet of an ideal incompressible conducting liquid moving relative to an ideal dielectric medium. It was found that for increased jet velocity, axisymmetric waves leads to finer disintegration. RUO, et al[28], developed a theoretical model in order to investigate the effect of an axial magnetic field on the instability of a charged liquid jet. Results showed that the magnetic force induced by the motion of charged surface is insignificant in comparison with the electric force and does not have effect on the instability of a dielectric liquid jet. However, for liquid with high electrical conductivity, Lorenz force induced by a conducting current becomes significant, suppressing destabilizing mechanism.

This paper is devoted to reestimate theoretical peripheral drops production by non-superimposed surface trochoidal disturbance. The work phases were supported by previous paperworks about water waves and by a prior analysis about harmonic superimposed disturbances on liquid jet surface. Elementary liquid jet parameters on potential drops formation were principally studied.

2 Theoretical Approach

The theoretical approach of this work is the RAYLEIGH inviscid jet model. After linearization the Euler equations and the boundary conditions for a liquid rounded jet, RAYLEIGH demonstrated the disturbance development of normal modes. The normal mode disturbance function is $\hat{\xi}\exp(st+\mathrm{i}(kx+n\theta))$, where $\hat{\xi}$ is the disturbance amplitude which has length dimension, s is an amplification factor which has time^{-1} dimension, t is time (time dimension), k is the disturbance wave number (length dimension), x is a length dimension coordinate along the jet axis, n is a no dimensional parameter which actually introduce the peripheral wave number. For achieving peripheral amplification disturbance the peripheral wave length must be larger than the jet perimeter. The amplification factor was estimated as:

$$s=\left[\frac{\gamma}{a^3\rho}\frac{\alpha I_n{}'(\alpha)}{I_n(\alpha)}(1-\alpha^2-n^2)\right]^{1/2}, \tag{1}$$

where γ is the jet liquid surface tension, a is the jet radius, ρ is the jet liquid density, α is the no dimensional wave number where its value is $\alpha=ka$, $I_n(\alpha)$ is the modified Bessel function of the first kind of order n and $I_n{}'(\alpha)$ is the modified Bessel function derivative. The effect of these parameters on the amplification factor is shown clearly in Eq. (1). This work use this result as it is, so it is not dealt with these effects.

This work does not deal with peripheral disturbances, so $n=0$. The amplification factor will be:

$$s=\left[\frac{\gamma}{a^3\rho}\frac{\alpha I_1(\alpha)}{I_0(\alpha)}(1-\alpha^2)\right]^{1/2}, \tag{2}$$

while $I_0{}'(\alpha)=I_1(\alpha)$.

3 Turned Trochoidal Disturbance Function

The trochoidal function is received by these parametric equations:

$$x=a_{\mathrm{tr}}\phi-b_{\mathrm{tr}}\sin\phi, \tag{3}$$

$$y=a_{\mathrm{tr}}\phi-b_{\mathrm{tr}}\cos\phi, \tag{4}$$

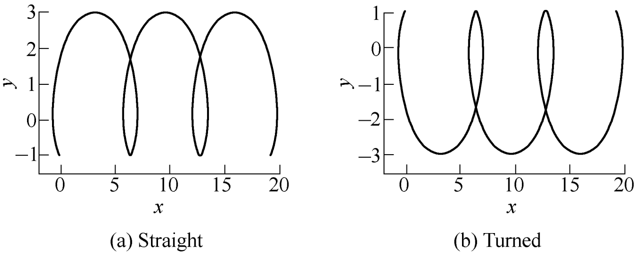
where ϕ in Eq. (3) and Eq. (4) is the no dimensional trochoidal parameter, a_{tr} and b_{tr} are dimensional parameters. The choice of the relation between a_{tr} and b_{tr} determine the trochoidal figure. There are three types of trochoidal functions; the curtate cycloid which is characterized by the relation $b_{\mathrm{tr}} < a_{\mathrm{tr}}$, the prolate cycloid which is characterized by the relation $b_{\mathrm{tr}} > a_{\mathrm{tr}}$ and the cycloid which is characterized by the relation $b_{\mathrm{tr}} = a_{\mathrm{tr}}$. It is believed that the relation $b_{\mathrm{tr}} > a_{\mathrm{tr}}$ (the prolate cycloid) is the appropriate relation for producing peripheral drops. This straight trochoid function is shown in Fig. 3(a).

The turned trochoidal parametric equations are as follows:

$$x = a_{\mathrm{tr}}\phi - b_{\mathrm{tr}}\sin\phi, \tag{5}$$

$$y = -a_{\mathrm{tr}} + b_{\mathrm{tr}}\sin\phi. \tag{6}$$

It is believed that the turned trochoidal disturbance function may lead to the peripheral drops production. The turned trochoid function is shown in Fig. 3(b).



(a) Straight (b) Turned

Fig. 3. Trochoid function for $b_{\mathrm{tr}} = 2a_{\mathrm{tr}}$

In the interval $0 < \phi < 2\pi$ the function ϕ is decomposed to Fourier series as:

$$\pi - 2\left(\frac{\sin\phi}{1} + \frac{\sin(2\phi)}{2} + \frac{\sin(3\phi)}{3} + \cdots\right). \tag{7}$$

The turned trochoid may be introduced as:

$$x = a_{\mathrm{tr}}\left(\pi - 2\sum_{n=1}^{n=\infty}\frac{\sin(n\phi)}{n}\right) - b_{\mathrm{tr}}\sin\phi, \tag{8}$$

$$y = -a_{\mathrm{tr}} + b_{\mathrm{tr}}\cos\phi, \tag{9}$$

or as a complex function:

$$x = \mathrm{Re}\left[a_{\mathrm{tr}}\left(\pi - 2\sum_{n=1}^{n=\infty}\frac{\exp(\mathrm{i}n\phi)}{n}\right) - b_{\mathrm{tr}}\exp(\mathrm{i}\phi)\right], \tag{10}$$

$$y = \mathrm{Re}\left[-a_{\mathrm{tr}} + b_{\mathrm{tr}}\exp(\mathrm{i}\phi)\right]. \tag{11}$$

The functions x and y in Eq. (10) and Eq. (11) will get appropriately the values y_1 and y_2. The function ϕ will get the value $\phi = kx$ while k is the disturbance wave number and x is the polar coordinate along the jet axis. So, for this work, Eq. (10) and Eq. (11) will be changed to:

$$y_1 = \mathrm{Re}\left[a_{\mathrm{tr}}\left(\pi - 2\sum_{n=1}^{n=\infty}\frac{\exp(\mathrm{i}nk\phi)}{n}\right) - b_{\mathrm{tr}}\exp(\mathrm{i}k\phi)\right], \tag{12}$$

$$y_2 = \mathrm{Re}\left[-a_{\mathrm{tr}} + b_{\mathrm{tr}}\exp(\mathrm{i}k\phi)\right]. \tag{13}$$

The time changing is $t = x/u$ where x as aforementioned is the polar coordinate along the jet axis and u is the jet velocity. So, the turned trochoidal disturbance functions will be written as follows:

$$y_{1D} = \mathrm{Re}\left[a_{\mathrm{tr}}\left(\pi - 2\sum_{n=1}^{n=\infty}\frac{\exp(s_1 x/u + \mathrm{i}nkx)}{n}\right) - b_{\mathrm{tr}}\exp(s_2 x/u + \mathrm{i}kx)\right], \tag{14}$$

$$y_{2D} = \mathrm{Re}\left[-a_{\mathrm{tr}} + b_{\mathrm{tr}}\exp(s_2 x/u + \mathrm{i}kx)\right], \tag{15}$$

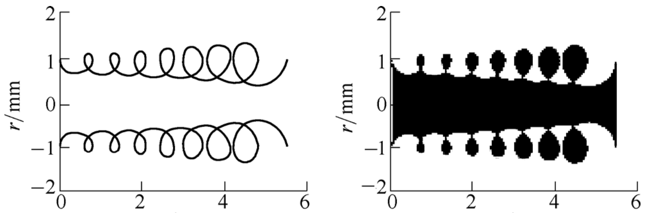
where
$$s_1 = s_1(n) = \left(\frac{\gamma\alpha_1 I_1(\alpha_1)(1-\alpha_1^2)}{a^3\rho I_0(\alpha_1)}\right)^{\frac{1}{2}},$$
$$k_1 = k_1(n) = nk, \quad \alpha_1 = \alpha_1(n) = k_1 a,$$
$$s_2 = \left(\frac{\gamma\alpha_2 I_1(\alpha_2)(1-\alpha_2^2)}{a^3\rho I_0(\alpha_2)}\right)^{\frac{1}{2}},$$
$$k_2 = k, \ \alpha_2 = ka.$$

The turned trochoidal disturbance function is received by plotting y_{2D} as a function of y_{1D}, $y_{2D} = y_{2D}(y_{1D})$.

4 Results

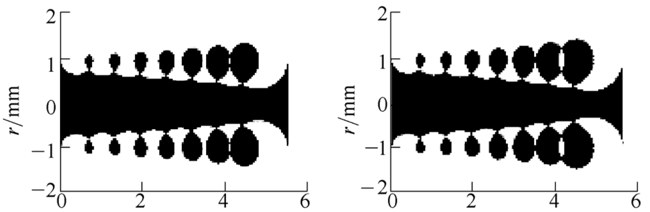
Despite this work deals with fundamental phenomenon, the results introduced are for water jet which its parameters are: water density (ρ=1000 kg/m^3), water surface tension (γ=72.8×10^{-3} kg/s^2) and the jet radius (a=1 mm=1×10^{-3} m). Fig. 4 are characterized by the wave number k=100 m^{-1}, a_{tr}=1×10^{-4} m and b_{tr}=2a_{tr}=2×10^{-4} m. Fig. 4(a) shows the basic trochoidal development for 10 m/s jet velocity (without coloring the jet and drops).

Fig. 4(b) shows the trochoidal development for 10 m/s jet velocity with coloring the jet and drops. Fig. 4(b) clarifies the data introduced in Fig. 4(a). While comparing the two figures it may be easily identify the jet and the drops produced in Fig. 4(a). Drops may be identified in the crests of the trochoidal surface. These drops are increased along the jet surface. While the jet velocity is reduced to 9 m/s as it is shown in Fig. 4(c) the drops intend merging beginning. As the jet velocity is reduced more, to 8 m/s as it is shown in Fig. 4(d), the far drops merging potential is going larger. Fig. 4(e), shows the trochoidal jet disturbance development while the jet velocity is increased over the Figs. 4(a), and 4(b) jet velocity. It is shown that the drops have inverse trend, they are produced as discrete drops.



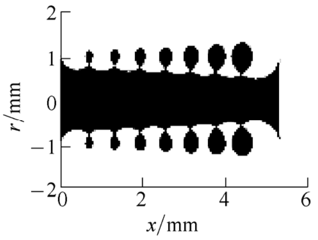
(a) 10 m/s, without coloring the jet and drops

(b) 10 m/s, with coloring the jet and drops



(c) 9 m/s

(d) 8 m/s

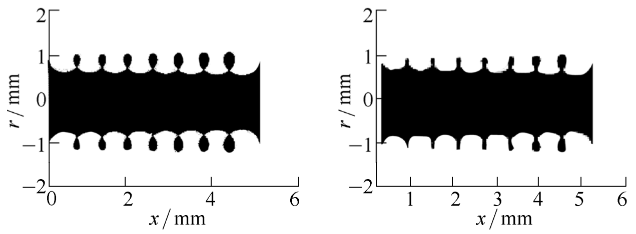


(e) 12 m/s

Fig. 4. Trochoidal disturbance development for the jet velocities

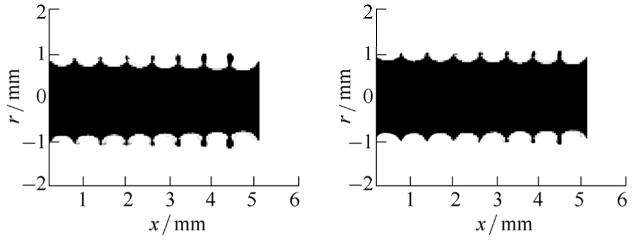
Fig. 5(a) is characterized by the same Fig. 4(e) parameters except that the wave number is $k = 900\ \mathrm{m}^{-1}$. It is shown that the larger wave number leads to smaller drops production. Figs. 5(b)–5(e) are characterized by the same Fig. 4(e) parameters except the relation between the trochoidal parameters a_{tr} and b_{tr}, for Fig. 5(b) $b_{\mathrm{tr}}/a_{\mathrm{tr}} = 1.2$, for Fig. 5(c) $b_{\mathrm{tr}}/a_{\mathrm{tr}} = 1$, for Fig. 5(d) $b_{\mathrm{tr}}/a_{\mathrm{tr}} = 0.8$ and for Fig. 5(e) $b_{\mathrm{tr}}/a_{\mathrm{tr}} = 0.5$. It is shown that the reduced $b_{\mathrm{tr}}/a_{\mathrm{tr}}$ relation leads to smaller drops production. It has to be emphasized here that it was expected that only the prolate cycloid disturbance $(b_{\mathrm{tr}} > a_{\mathrm{tr}})$ may lead to drop production but these figures show that drops are received also for the cycloid disturbance $(b_{\mathrm{tr}} = a_{\mathrm{tr}})$ and for the curtate cycloid disturbance $(b_{\mathrm{tr}} < a_{\mathrm{tr}})$. Indeed the curtate cycloid disturbance may lead to drops production as is shown in Fig. 5(d) $(b_{\mathrm{tr}} = 0.8a_{\mathrm{tr}})$ however its potential for drops production is limited as is shown in Fig. 5(e) $(b_{\mathrm{tr}} = 0.5a_{\mathrm{tr}})$, for this case it is shown that there is not drops production.

It may be noticed that despite the model complexity, the jet and drops mass tend to be conserved. All these figures show that the sum of the jet main body and the drops produced do not change, as the drops produced are larger, the jet main body is thinner or as the drops produced are smaller the jet main body is thicker.



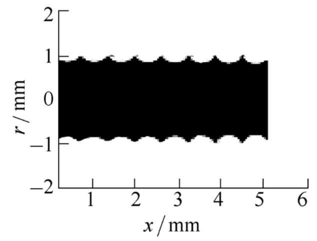
(a) $k = 900\ \mathrm{m}^{-1}$

(b) $b_{\mathrm{tr}} = 1.2\ a_{\mathrm{tr}}$



(c) $b_{\mathrm{tr}} = a_{\mathrm{tr}}$

(d) $b_{\mathrm{tr}} = 0.8\ a_{\mathrm{tr}}$



(e) $b_{\mathrm{tr}} = 0.5\ a_{\mathrm{tr}}$

Fig. 5. Trochoidal disturbance development for 12 m/s jet velocity

5 Discussion and Conclusions

Despite the literature content[4–5, 7] does not support specifically at turned trochoidal disturbances as characterize incompressible jets disturbances, it does so for planar incompressible fluid waves. This work examines an inverse trochoid disturbance because of its feasibility we believe for producing peripheral drops. As it is shown in Figs. 4(c) and 4(d), while the jet velocity is reduced, some drops in the jet front intend to be merged in order to produce larger drops. This result appropriate to our expectation that slower jet produce larger drops or the faster jet produce smaller drops. It is suitable to compare the set Figs. 4(c) and 4(d) to the set Figs. 4(b) and 4(e). It is shown that the set Figs. 4(b) and 4(e) introduce larger jet velocities compared to the set Figs. 4(c) and 4(d). Accordingly to our expectation it may be concluded that as the jet velocity increases, discrete and smaller drops are produced. Because the larger velocity jets produce smaller drops, it has to move along a long distance in order to produce large drops quantity. This conclusion might be considered as a disadvantage. The fact that the faster jet moving this distance in a shorter time might be considered as an advantage that covers the disadvantage conclusion. So, according to our expectations it may be concluded that in order to produce smaller drops, faster jets are preferred.

The influence of the wave number on the drops production may be concluded by comparing Fig. 4(e) and

Fig. 5(a). It is shown and may be concluded that as the wave number increases the drops size produced is reduced. Despite it may be indicated that smaller drops production is a meaningful result, the fact that the two jets (jets Fig. 4(e) and Fig. 5(a)) have the same velocity leads to the conclusion that in order to atomize the larger jet wave number, a longer time is needed which is performed along a longer distance.

The influence of the trochoidal parameters relation $b_{\mathrm{tr}}/a_{\mathrm{tr}}$ on the drops production may be concluded by comparing Fig. 4(e) and Figs. 5(b)–5(e). It is shown and may be concluded that as the trochoidal parameters relation $b_{\mathrm{tr}}/a_{\mathrm{tr}}$ is reduced, the drops produced are smaller. Because the basic trochoid function is doing loops only for $b_{\mathrm{tr}} > a_{\mathrm{tr}}$, see Figs. 3(a) and 3(b), it was expected that for trochoidal relations $b_{\mathrm{tr}} \leqslant a_{\mathrm{tr}}$ drops will not be produced. Figs. 5(c) and 5(d) show that it is not so, drops are produced for the relations $b_{\mathrm{tr}} = a_{\mathrm{tr}}$ and for the relation $b_{\mathrm{tr}} < a_{\mathrm{tr}}$. This result might be possible because the amplification factor exist into the trochoidal disturbance function and is not exist into the basic trochoid function. It may be summarized that all trochoidal disturbances functions have capability of producing drops. For the prolate cycloid only part of the domain relation $b_{\mathrm{tr}} < a_{\mathrm{tr}}$ may lead to peripheral drops production.

It has to explain the connection between the previous works[4–5, 7] and the current paper for liquid jet disturbances. These works which were noted in this paper deal with the same turned trochoidal function but for waves on plate surface. It may be expected that despite the difference between the plate surface and the rounded jet surface all the relevant works deal with similar surface disturbances. So, it may be supposed that those previous works support the current paper. Moreover, the turned trochoidal intersections as are introduced in Fig. 1 were rejected according to Ref. [9] as a realistic possibility of water wave configuration. Actually, the intersections which are introduced in this work and lead to drops production do not belong any more to the jet main body, so, the intersectional trochoid disturbance may be also seems as does not relate to the realistic liquid jet configuration according to DARLES claiming about trochoidal water waves.

Despite this paper as lot of others relates to discrete disturbances, it might be assumed that the jet exit from the nozzle is exposed to an assemblage of disturbances which may be differ from one to another by a number of physical parameters, in principal such parameters may be the wave number and the disturbance amplitude. The assemblage of these disturbances may lead to diverse of drop sizes. It has to be emphasized that assemblage of unique disturbances functions as the RAYLEIGH normal modes disturbances may lead only to serial drops and not to peripheral drops production. A diverse of peripheral drops sizes may be received only by an assemblage of non-unique parametric disturbances functions. Such assemblage of non-unique parametric disturbances functions were introduced by ZIMMELS and SADIK[2]. For the present work the physical parameters that may build the disturbances assemblage are the wave numbers (k) and the parametric relations ($b_{\mathrm{tr}}/a_{\mathrm{tr}}$). ZIMMELS and SADIK[2] dealt with superimposed disturbances sequence, so, the number of superimposed disturbances is actually another parameter that is relevant for building the disturbance assemblage. The present work does not deal with superimposed disturbances, so, it cannot be related to the number of superimposed disturbances as a relevant parameter for building a disturbance assemblage. Turned trochoidal superimposed disturbances are suggested for farther research.

It has to be emphasized that the peripheral drops received in the present work have actually a torus shape. It is assumed that the torus drops are not the ending of the atomization process. The torus drops continue to be factorized to smaller drops. The torus drops factorized is recommended for farther research. This farther research has to relate whether the torus drops atomization occurs serially after the torus drops are completely produced or whether the atomization process occurs simultaneously while disturbances are moving axially and peripherally on the jet surface.

This model is based on the RAYLEIGH model which was built by the Euler's equations that do not include the viscosity parameter which leads to vorticities production. Despite the vorticity mechanism was not a building block of this model the results received have very similar image to the scheme introduced by Fig. 2. The drops are going larger and are produced from spikes which are created along the jet surface. These spikes seem as short ligaments which are quoted in literature as a stage of drops production.

As a summation paragraph the liquid drops production may be divided into two channels; one channel is the liquid jet serial drops production and the second channel is the liquid jet peripheral drops production. The basic model for the serial drops production is the RAYLEIGH model while for the peripheral drops production a direction of superimposed disturbances was afforded by ZIMMELS and SADIK. This work shows that in order to produce peripheral drops another kind of disturbance may be offered-the trochoidal disturbance without the necessity for superimposed disturbances. It has to be emphasized that these peripheral drops production works use as a building block the RAYLEIGH jet serial drops production model.

Acknowledgements

We dedicate this paper to the late Prof. Yoram ZIMMELS, who left us and the academic world.

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Biographical notes

SADIK Shalom, born in 1952, is currently a staff member at *Department of Mechanical Engineering, Ort Braude College, Israel*. He teaches the courses Mechanics of Fluids and Heat Transfer. After long years working outside academy as leading new technologies projects he turned to the academic direction and completed his PhD thesis in the Technion at 2001. Thesis subject: Mechanisms of Nonaxial Disturbances in Evolution Processes of Jets into Sprays. Related to the thesis dissertation he published three quality papers. The first paper was published in *Applied Physics Letters*: Formation of Sprays from Liquid Jets by a Superimposed Sequence of Nonaxial Disturbances. His research interests include fluid mechanics and heat transfer.
Tel: 972-50-7776820; E-mail: shaloms@braude.ac.il

KIRZHNER Felix, born in 1949, is currently senior researcher at *Civil and Environmental Engineering Dept., in the Technion, Haifa, Israel*. He has published more than 70 papers and 3 books. His research interests include rook mechanics, particles behavior, waste water treatment technologies.
Tel: 972-4-8228898; E-mail: kfelix@techunix.technion.ac.il

KRAMARENKO Denis, born in 1986, is currently a research assistant at *MIT, HST-Harvard-MIT Division of Health Sciences and Technology*.
Tel: +1-(617)-803-5479; E-mail: kdenis@mit.edu