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Modeling and Calculation of Impact Friction Caused by Corner Contact in Gear Transmission

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Abstract: Corner contact in gear pair causes vibration and noise, which has attracted many attentions. However, teeth errors and deformation make it difficulty to determine the point situated at corner contact and study the mechanism of teeth impact friction in the current researches. Based on the mechanism of corner contact, the process of corner contact is divided into two stages of impact and scratch, and the calculation model including gear equivalent error－combined deformation is established along the line of action. According to the distributive law, gear equivalent error is synthesized by base pitch error, normal backlash and tooth profile modification on the line of action. The combined tooth compliance of the first point lying in corner contact before the normal path is inversed along the line of action, on basis of the theory of engagement and the curve of tooth synthetic compliance & load-history. Combined secondarily the equivalent error with the combined deflection, the position standard of the point situated at corner contact is probed. Then the impact positions and forces, from the beginning to the end during corner contact before the normal path, are calculated accurately. Due to the above results, the lash model during corner contact is founded, and the impact force and frictional coefficient are quantified. A numerical example is performed and the averaged impact friction coefficient based on the presented calculation method is validated. This research obtains the results which could be referenced to understand the complex mechanism of teeth impact friction and quantitative calculation of the friction force and coefficient, and to gear exact design for tribology.

Keywords: gear transmission, corner contact, oblique impact model, impact friction, frictional coefficient

1 Introduction

Corner contact, namely, contact outside the normal path of contact, can occur in gear transmission systems due to deformation of the teeth under working load or gear manufacturing error. Meshing outside the normal path of contact may lead to impact owing to sudden fluctuation of rotation velocity. In addition, meshing impact may induce dynamic load, vibration and noise[1].

Three kinds of corner contact (such as run-in corner contact, run-out corner contact and pitch corner contact) can appear in gear transmission system. The influence of pitch corner contact is least and the effect of run-out corner contact is less than run-in corner contact. The effects of corner contact on the transmission characteristics are analyzed based on transmission error models in Refs. [2–5]. Subsequently, partly gear profile error and teeth deformation are taken into account to analyze the corner contact in Refs. [6–7]. However, the corner contact position and impact force are depended on the gear teeth deformation mutually. The system error and gear teeth' comprehensive deformations are not combined in the corner contact analysis model, in Refs. [1–7], to investigate the corner contact position and impact force.

The impact force is derived from a finite element model in Ref. [8] and the results from basic experimental investigations with simple impact bodies are presented to validate the model. The impact motion is also analyzed in Refs. [9–10], but the meshing impact is different from the corner contact in present work. Besides, in Refs. [11–14], the dynamic load and contact fatigue are analyzed with multiple dynamic contact models under normal mesh condition, but the corner contact pattern and impact friction are not included.

The present work aims to develop calculation of the contact position, impact velocity, impact force and impact friction coefficient. The "gear equivalent error－combined deformation" model is constructed by combining with gear error, deformation and load effect. The initial contact position is searched based on the position criterion and deformation curve, the impact velocity and impact forces are obtained. Finally, the oblique impact friction model is constructed and the impact friction coefficient is deduced subsequently.

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2 Mechanism of Corner Contact and Numerical Model

2.1 Mechanism of corner contact

Theoretically, the normal pitch is equal to each other in normal mesh condition, namely, $p_{b1} = p_{b2}$. In practice, the gear manufacturing error and teeth deformation may lead to $p_{b1} \neq p_{b2}$. When $p_{b1} < p_{b2}$, one tooth may get into contact in advance and the corner contact occur as shown in Fig. 1. In Fig. 1, point E is theoretical initial meshing point without deformation and gear error. The gear pairs will rotation smoothly when the teeth get in to mesh at point E. However, the gear error or tooth deformation with change the meshing pattern and initial meshing point vary to D as shown in Fig. 1. Then the name basic radius becomes smaller than the theoretical basic radius and the line velocity along the line of action is different, which leads to mesh impact.

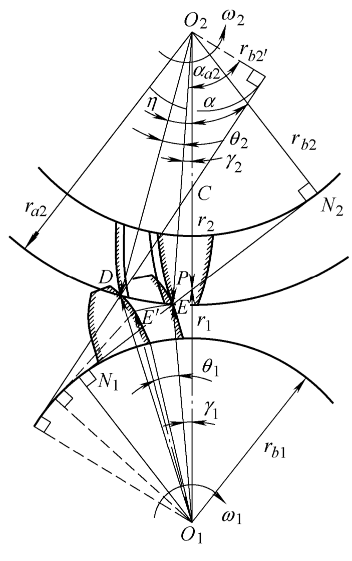

Fig. 1. Illustration of corner contact

Above all, the practice meshing process of corner contact mechanism is: the driven gear engages in advance from point D and the impact occurs. Then the driven gear scratches along the flank of driving gear to point E until the gear mesh along the line of action.

2.2 Calculation model

In gear transmission system, the main effect of factors of load sharing and dynamic load are normal pitch error f_{pb}, space error f_{pt} and normal backlash j_{bn}. According to the mechanism of corner contact, the main sources of the corner contact are the pitch error and normal backlash. Moreover, pitch error is origin from manufacturing and the normal backlash is mainly due to the assembling error. The effect of positive pitch error is larger than that of negative, so the positive pitch error is considered in the present work. In addition, the modification e_m may be considered[15]. Generally, the normal pitch error and normal backlash are random variables, and follow a standard normal distribution. The effects of normal pitch error, normal backlash and modification on the transmission precision and load sharing are independent[16]. These errors are integrated and projected on the line of action as gear equivalent error Δf_Σ

$$\Delta f_\Sigma = \frac{2 f_{pb} + e_m - j_{bn}}{\sqrt{2}}, \tag{1}$$

$$e_m = e_{m1} + e_{m2}, \tag{2}$$

where e_{m1} and e_{m2} are the modification of driving and driven gear respectively.

The tooth compliance combined with bending, shear, Hertz contact, root radius and foundation deformation are calculated by CORNELL, et al[17] and TAVAKOLI, et al[18]. LIN[19] improved the model by considering tooth compressive deformation. Based on these results, one can obtain the comprehensive deformation δ_Σ as

$$\delta_\Sigma = \delta_{\Sigma F'1} + \delta_{\Sigma F'2} + \delta_H, \tag{3}$$

$$\delta_{\Sigma F'i} = \delta_{Bi} + \delta_{Ni} + \delta_{Si} + \delta_{Gi}, i = 1, 2, \tag{4}$$

where $\delta_{\Sigma F'i}$ is the comprehensive deformation of gear i and δ_H is the Hertz deformation, δ_{Bi}, δ_{Ni}, δ_{Si} and δ_{Gi} are bending deformation, compressive deformation, shear deformation and foundation deformation respectively.

When the mesh point arrives at the mesh point F (for gear teeth 1–2) and another mesh point may appear at point E for teeth 3–4 as shown in Fig. 2 theoretically.

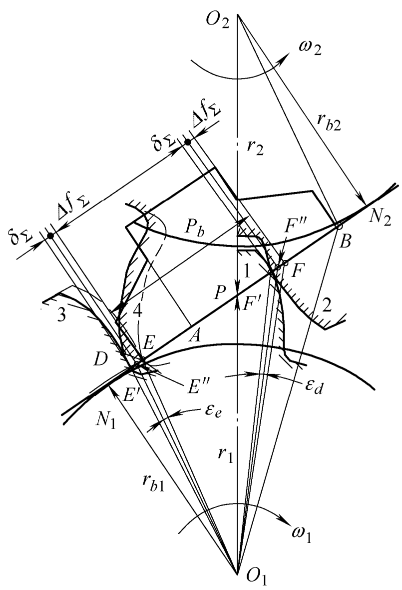

Fig. 2. Calculation model of equivalent error and comprehensive deformation

But when the system error and deformation are included, the gear mesh point D outside line of action will occur when gear teeth 1–2 arrive at F'. $\overline{F'F''}$ is the deformation along the line of action of gear teeth 1–2 and corresponding roll angle is ε_d. And $\overline{EE''}$ is the comprehensive system error for gear teeth 3–4 and roll angle is ε_e in driving gear frame. Then the gear equivalent error-combined deformation model is obtained as

$$\overline{F''F} = \Delta f_\Sigma + \delta_\Sigma. \tag{5}$$

The contact deformation with respect to roll angle is shown in Fig. 3[20], then the deformation when corner contact occurring can be calculated from Eq. (5) and substituting Eqs. (1)–(4) into Eq. (5), has

$$\overline{F'F} = \frac{2f_{pb} + \sum_{i=1}^{2} e_{mi} - j_{bn}}{\sqrt{2}} + \sum_{i=1}^{2} \delta_{F'i} + \delta_H. \tag{6}$$

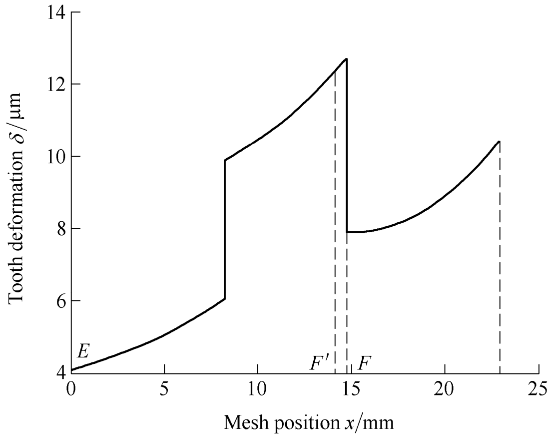

Fig. 3. Curve of tooth synthetic compliance and load-history

3 Initial Corner Contact Position and Impact Force

3.1 Initial corner contact position

To process the analysis, the initial corner contact point D, as shown in Fig. 1 must be determined. In the triangle $\triangle O_1DO_2$,

$$\overline{O_1D} = \left[r_{a2}^2 + (r_1 + r_2)^2 - 2r_{a2} \bullet (r_1 + r_2)\cos\eta\right]^{1/2}, \tag{7}$$

and in $\triangle PEO_2$,

$$\angle PEO_2 = \arcsin\frac{r_2 \sin(\pi/2 + \alpha)}{r_{a2}}, \tag{8}$$

$$\gamma_2 = \frac{\pi}{2} - \alpha - \angle PEO_2, \tag{9}$$

$$\eta = \gamma_2 + \theta_2, \tag{10}$$

$$\theta_2 = \frac{\theta_1}{i}, \tag{11}$$

$$\theta_1 = \varepsilon_d + \varepsilon_e. \tag{12}$$

In $\triangle O_1N_1E$ and $\triangle O_1N_1E''$, as shown in Fig. 2, one has

$$\angle N_1EO_1 = \arctan\frac{r_{b1}}{\overline{EN_1}}, \tag{13}$$

$$\angle N_1E''O_1 = \arctan\frac{r_{b1}}{\overline{EN_1} - \Delta f_\Sigma}, \tag{14}$$

$$\varepsilon_e = \angle N_1E''O_1 - \angle N_1EO_1, \tag{15}$$

$$\overline{N_1E} = (r_{a1}^2 - r_{b1}^2)^{1/2} - \varepsilon_\alpha p_b. \tag{16}$$

In $\triangle O_1N_1F'$ and $\triangle O_1N_1F''$,

$$\angle N_1F'O_1 = \arctan\frac{r_{b1}}{\overline{FN_1} - (\delta_\Sigma + \Delta f_\Sigma)}, \tag{17}$$

$$\angle N_1F''O_1 = \arctan\frac{r_{b1}}{\overline{FN_1} - \Delta f_\Sigma}, \tag{18}$$

$$\varepsilon_d = \angle N_1F'O_1 - \angle N_1F''O_1, \tag{19}$$

$$\overline{N_1F} = (r_{a1}^2 - r_{b1}^2)^{1/2} - (\varepsilon_\alpha - 1)p_b, \tag{20}$$

where r_1, r_2 are pitch radius, r_{a1}, r_{a2} are addendum radius, r_{b1} is the base cycle radius of driving gear, p_b is the normal pitch, ε_α is contact ratio, α is pressure angle, i is transmission ratio. Solving Eqs. (6)–(20), the position of corner contact point can be obtained.

3.2 Impact velocity

As shown in Fig. 4, the whole corner contact process $D \to E$, can be divided into two stages of impact $D \to D'$ and scratch $D' \to E$. Due to the corner contact, normal run-in impact velocity ΔV_{Dn} and relative velocity $\Delta V_{D\tau}$ along common tangent direction are as follows:

$$\Delta V_{Dn} = V_{D1}\cos\beta_1 - V_{D2}\cos\beta_2, \tag{21}$$

$$\Delta V_{D\tau} = V_{D2}\sin\beta_2 - V_{D1}\sin\beta_1, \tag{22}$$

where

$$V_{D1} = \overline{O_1D} \bullet \omega_1,\ V_{D2} = r_{a2} \bullet \omega_2, \tag{23}$$

$$\beta_1 = \angle DO_1N_{1'},\ \beta_2 = \angle O_2DN_{1'} - \frac{\pi}{2}, \tag{24}$$

$$\begin{cases} \angle DO_1N_{1'} = \arccos\dfrac{r_{b1}}{\overline{O_1D}}, \\ \angle O_2DN_{1'} = \arccos\dfrac{r_{a2}^2 + \overline{DN_{1'}}^2 - \overline{O_2N_{1'}}^2}{2r_{a2} \bullet \overline{DN_{1'}}}, \end{cases} \tag{25}$$

$$\overline{DN_{1'}} = \overline{O_1D}\sin\angle DO_1N_{1'}, \tag{26}$$

$$\overline{O_2N_{1'}} = \left[(r_1+r_2)^2 + r_{b1}^2 - 2(r_1+r_2)r_{b1}\right]^{1/2}\times [\cos(\angle DO_1N_{1'} + \angle DO_1P)]^{1/2}, \tag{27}$$

$$\angle DO_1P = \arccos\frac{(r_1+r_2)^2 + \overline{O_1D}^2 - r_{a2}^2}{2(r_1+r_2)\bullet\overline{O_1D}}, \tag{28}$$

where β, β_1, β_2 are the angles between ΔV, V_{D1}, V_{D2} and $\overline{N_{1'}N_{2'}}$ respectively, ω_1, ω_2 are the angle velocity of driving and driven gear, r_{b2} is the base cycle radius of driven gear.

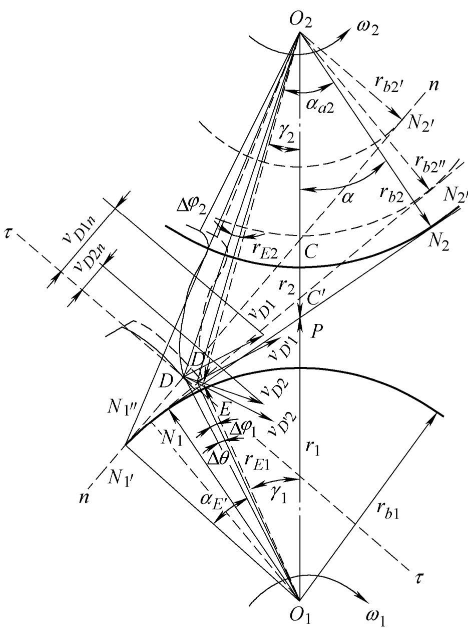

Fig. 4. Illustration of impact velocity

Similarly, for end corner contact point D', we have

$$\Delta V_{D'n} = V_{D'1}\cos\beta_1' - V_{D'2}\cos\beta_2', \tag{29}$$

$$\Delta V_{D'\tau} = V_{D'2}\sin\beta_2' - V_{D'1}\sin\beta_1', \tag{30}$$

$$V_{D'1} = \overline{O_1D'}\bullet\omega_1,\ V_{D'2} = V_{D2} + \frac{F_S T_S}{m_{\mathrm{red}}}, \tag{31}$$

$$T_S = \left[\frac{m_{\mathrm{red}}^2}{\pi^2 \Delta V_{Dn}\rho_{\Sigma}E'^2}\right]^{1/5}, \tag{32}$$

where T_S is impact duration, m_{red} is equivalent mass, ρ_1, ρ_2 are the curve radius of gear face 1–2. Equivalent radius

$$\rho_{\Sigma} = \frac{\rho_1\rho_2}{\rho_1+\rho_2},\quad \frac{1}{E'} = \frac{1-\upsilon_1^2}{E_1} + \frac{1-\upsilon_2^2}{E_2}.$$

And

$$\frac{1}{m_{\mathrm{red}}} = \frac{1}{m_{\mathrm{red1}}} + \frac{1}{m_{\mathrm{red2}}}, \tag{33}$$

$$\overline{O_1D'} = \overline{O_1D}\cos(\Delta\theta) - [\overline{O_1D}^2\cos^2(\Delta\theta) - \overline{O_1D}^2 + \overline{DD'}^2]^{1/2}, \tag{34}$$

$$\Delta\theta = \omega_1 \bullet T_S, \tag{35}$$

$$\overline{DD'} = r_{a2}\bullet\angle D'O_2D, \tag{36}$$

$$\angle D'O_2D = \frac{\Delta\theta}{i}, \tag{37}$$

$$\beta_1' = \angle D'O_1N_{1''},\ \beta_2' = \angle O_2D'N_{1''} - \frac{\pi}{2}, \tag{38}$$

$$\angle D'O_1N_{1''} = \arccos\frac{r_{b1}}{\overline{O_1D'}}, \tag{39}$$

$$\angle O_2D'N_{1''} = \arccos\frac{r_{a2}^2 + \overline{D'N_{1''}}^2 - \overline{O_2N_{1''}}^2}{2r_{a2}\bullet\overline{D'N_{1''}}}, \tag{40}$$

$$\overline{D'N_{1''}} = \overline{O_1D'}\sin\angle D'O_1N_{1''}, \tag{41}$$

$$\overline{O_2N_{1''}} = \left[(r_1+r_2)^2 + r_{b1}^2 - 2(r_1+r_2)r_{b1}\right]^{1/2}\times [\cos(\angle D'O_1N_{1''} + \angle D'O_1P)]^{1/2}, \tag{42}$$

$$\angle D'O_1P = \arccos\frac{(r_1+r_2)^2 + \overline{O_1D'}^2 - r_{a2}^2}{2(r_1+r_2)\bullet\overline{O_1D'}}, \tag{43}$$

$$\Delta V_n = \left|V_{D_n'} - V_{D_n}\right|, \tag{44}$$

$$\Delta V_\tau = \left|V_{D_\tau'} - V_{D_\tau}\right|, \tag{45}$$

where β_1', β_2' are the angle between V_{D_1}', V_{D_2}' and $\overline{N_1''N_2''}$ respectively.

3.3 Impact force

The impact force is closely related to impact velocity, impact duration, mesh stiffness and load. The impact forces corresponding to normal and tangential velocity are F_{In} and $F_{I\tau}$ as shown in Fig. 5, which transfer to impact kinetic energy and friction dissipation energy. The induced mass of gear pair are

$$m_{\mathrm{red}i} = \frac{J_i}{br_{bi}^2},\ i = 1, 2, \tag{46}$$

and inertia moment J_i is

$$J_1 = \frac{\pi\rho b}{2g}(r_{b1}^4 - r_h^4),\ J_2 = \frac{\pi\rho b}{2g}(r_{b2'}^4 - r_h^4), \tag{47}$$

$$\angle O_2N_{1'}D = \arccos\frac{\overline{O_2N_{1'}}^2 + \overline{DN_{1'}}^2 - r_{a2}^2}{2\overline{O_2N_{1'}} \bullet \overline{DN_{1'}}}, \tag{48}$$

$$r_{b2'} = \overline{O_2N_{1'}}\sin\angle O_2N_{1'}D, \tag{49}$$

where ρ is material density, b is face width, r_h is the radius of internal cycle of gear.

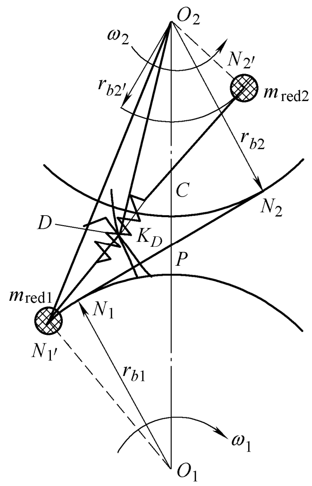

Fig. 5. Impact dynamics model

Then the normal kinetic energy is

$$E_k = \frac{1}{2}\bullet\frac{J_1J_2}{(J_1r_{b2'}^2 + J_2r_{b1}^2)b}\Delta V_n^2, \tag{50}$$

and dissipation energy is

$$W_f = \frac{1}{2}\bullet\frac{J_1J_2}{(J_1r_{b2'}^2 + J_2r_{b1}^2)b}\Delta V_\tau^2. \tag{51}$$

According to impact mechanics theory, the maximum deformation δ_s and maximum impact force F_s with relative to energy as

$$E_k = \frac{1}{2}\bullet\frac{J_1J_2}{(J_1r_{b2'}^2 + J_2r_{b1}^2)b}\Delta {V_n}^2 = \frac{1}{2q_s}\delta_s^2, \tag{52}$$

$$\delta_s = \frac{F_s}{bq_s}, \tag{53}$$

$$q_s(r) = q_1(r) + q_2(r) + q_{\mathrm{H}}(r), \tag{54}$$

$$q_{\mathrm{H}}(r) = \frac{2(1-\nu^2)}{\pi E}\left(1.27 + 0.781\ln\frac{m}{a}\right), \tag{55}$$

$$\frac{1}{q_i(r)} = (A_0 + A_1X_i) + (A_2 + A_3X_i)\frac{r - R_i}{(1+X_i)m}, \tag{56}$$

where q_s is the comprehensive compliance of the initial corner contact point D, q_1, q_2 are the compliance of gear teeth 1–2 (not including Hertz contact), and q_{H} is the contact compliance of teeth 1–2. The detailed descriptions of Eqs. (55) and (56) can be seen in Refs. [4] and [21], respectively. The impact force is

$$F_s = \Delta V_n\sqrt{\frac{bJ_1J_2}{(J_1r_{b2'}^2 + J_2r_{b1}^2)q_s}}. \tag{57}$$

4 Geometry Analysis of Corner Contact

As shown in Fig. 6, the triangle $\angle O_2DE$ can be discretized to n parts. For arbitrary point J, one has

$$\angle JO_2E = \frac{j}{n}\angle O_2DE, \tag{58}$$

$$\angle JO_2O_1 = \gamma_2 + \Delta\phi_{2j}, \tag{59}$$

$$O_1J = [r_{a2}^2 + (r_1 + r_2)^2 - 2(r_1 + r_2)\bullet r_{a2}]^{1/2}\times [\cos(\angle JO_2O_1)]^{1/2}, \tag{60}$$

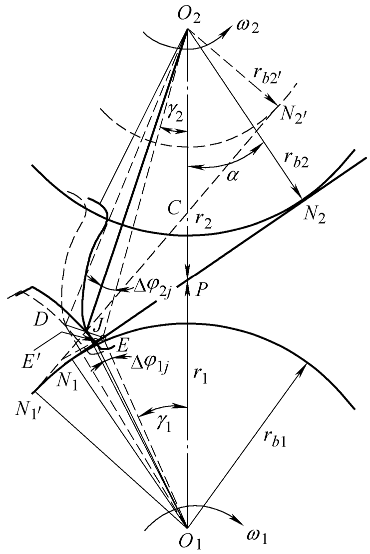

Fig. 6. Mesh position of corner contact

So the position and impact force at point D are determined as previous section, then the mesh point between points D and E can be calculated similarly. Note that, the gear error and tooth deformation enlarge the contact ratio, and the tooth root stress and the elastic deformation of the tooth become more complex.

5 Friction Coefficient of Corner Contact

In this section, friction coefficient of corner contact is built based on oblique impact friction theory and impact mechanics[22–23], as shown in Fig. 7. We assume that, (1) dry friction, (2) the duration between $D \to D'$ is considered and impact fluctuation is less important in the friction coefficient[11].

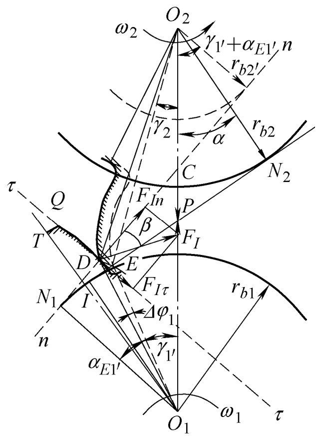

Fig. 7. Illustration of impact model

The averaged impact friction coefficient is defined as ratio of tangential impulse and normal impulse as

$$\overline{f}=\frac{\int_0^{T_S} F_{I\tau}\mathrm{d}t}{\int_0^{T_s} F_{In}\mathrm{d}t}, \tag{61}$$

$$m_{\mathrm{red}}\Delta V_\tau=\int_0^{T_S} F_{I\tau}\mathrm{d}t, \tag{62}$$

$$m_{red}\Delta V_n=\int_0^{T_S} F_{In}\mathrm{d}t, \tag{63}$$

then,

$$\overline{f}=\frac{\Delta V_\tau}{\Delta V_n}. \tag{64}$$

When the duration tends to 0, the averaged impact friction coefficient is equal to transit friction coefficient.

6 Numerical Simulation and Discussion

The gear parameters and material parameters used in the present paper are listed in Table 1. The modification is not considered (namely, $e_m=0$) and the gear and pinion are installed right (namely, $j_{bn}=0$). Moreover, the deformations of shaft, bearing and gearbox are not included.

Table 1. Gear parameters

Parameter		Value
Tooth number	z_1	20
	z_2	20
Pressure angle $\alpha/(°)$		20
Module m/mm		5
Contact ratio ε		1.556 8
Internal radius d_0/mm		20
Power P/kW		15
Speed $n_1/(\mathrm{r}\cdot\mathrm{min}^{-1})$		2000
Young's modulus $E_{1,2}$/GPa		205
Poisson's ratio $\nu_{1,2}$		0.3
Shear modulus G/GPa		78.8
Material density $\rho/(\mathrm{kg}\cdot\mathrm{m}^{-3})$		7800

For gear with 7 grade, the normal pitch error is $\pm f_{pb}=\pm 16$ μm, then the equivalent error $\Delta f_\Sigma=22.6$ μm. According to Ref. [20], the deformation δ_Σ is 24.7 μm. and $\overline{F''F}=47.3$ μm (obtained from Fig. 2). Substituting the parameters in the previous Eqs. (6)–(61), we obtain the averaged impact force $F_S=2186.4$ N, which is bigger than the theoretical meshing force $F_0=1260.8$ N, namely, the dynamic load factor is 1.734, closed to the result, 1.637 in Ref. [5]. The impact duration is $T_S=27$ μs and the averaged friction coefficient $\overline{f}=0.1811$. The friction coefficient is approximate to the experimental result $\overline{f}=0.179$ given in Ref. [24] and is less than the maximum value advised in Ref. [25], which varying in the range 0.1–0.3.

7 Conclusions

(1) The gear equivalent error—combined deformation model is proposed to analyze the corner contact. The gear equivalent error is synthesized by base pitch error, normal backlash and tooth profile modification on the line of action.

The bending, shear, Hertz, root radius and foundation deformation are calculated to obtain the initial position of corner contact point.

(2) The impact velocity, impact force from the beginning to the end during corner contact before the normal path, are calculated accurately.

(3) The averaged impact friction coefficient, defined as ratio of tangential impulse and normal impulse, is obtained and approved with the presented reference.

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