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Kinetostatics Analysis of a Novel 6-DOF Parallel Manipulator with Three Planar Limbs and Equipped with Three Fingers

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Abstract: It is significant to develop a robot hand with high rigidity by a 6-DOF parallel manipulator(PM). However, the existing 6-DOF PMs include spherical joint which has less capability of pulling force bearing, less rotation range and lower precision under alternately heavy loads. A novel 6-DOF PM with three planar limbs and equipped with three fingers is proposed and its kinematics and statics are analyzed systematically. A 3-dimension simulation mechanism of the proposed manipulator is constructed and its structure characteristics is analyzed. The kinematics formulae for solving the displacement, velocity, acceleration of the platform, the active legs and the fingers are established. The statics formulae are derived for solving the active forces of PM and the finger mechanisms. An analytic example is given for solving the kinematics and statics of proposed manipulator and the analytic solved results are verified by the simulation mechanism. It is proved from the error analysis of analytic solutions and simulation solutions that the derived analytic formulae are correct and provide the theoretical and technical foundations for its manufacturing, control and application.

Keywords: parallel manipulator, planar limbs, finger, kinematics, statics

1 Introduction

The 6-DOF parallel manipulators(PMs) are excellent candidates for advanced robotic applications by virtue of their high stiffness, high dexterity, compact size, and high power to weight ratio[1–3]. In a complex industrial assembly situation, the goal cannot be achieved in a single grasp. When multi-fingers are installed on the moving platform, the PM must be play more important roles in application of manufacturing and fixture of parallel machine tool, forging operator, assembly cells of automobile manufacturing or airplane manufacturing and flexible manufacturing system missions[4–10]. In this aspect, BANDE, et al[4], studied Gough-Stewart platform, and proposed assuming gripping as an end application. ZHANG, et al[5], designed a biologically inspired PM for the dexterous head section of a quadrupedal. MCCOLL, et al[6], proposed a generalized form of the antipodal method from multi-finger grasping and implemented for investigating the workspace of a wide range of planar wire-actuated PMs. LIU, et al[7], proposed a hybrid robot system for CT-guided surgery. WANG, et al[8], designed a reconfigurable robot for search/rescue. The fingers are key tools of assembly robots, forging operator, and mediation robots. MIR-NASIRI, et al[9], designed a four-axis parallel robotic arm for assembly operations. YAN, et al[10], established a coordinated kinematic modeling of gripper manipulator for heavy-duty manipulators. YI, et al[11], designed a parallel-type gripper mechanism with parallelogramic platform. VAHEDI, et al[12], studied three-fingers caging grasps of a given polygonal object with n edges. TERASAKI, et al[13], proposed an intelligent manipulator equipped with a parallel two-fingered gripper. In various PMs, the 6-DOF PMs with composite spherical joint S_c should be selected as the PM with multi-fingers because the moving platform can provide more room for arranging more fingers without interference among active limbs and finger mechanisms, and has more DOFs and higher stiffness[14–20]. In the aspects of PMs with S_c, PATRICIA, et al[14], ZHANG[15], HUANG, et al[16], JOKIN[17], ZHAO, et al[18], JIANG, et al[19], TONG, et al[20], synthesized various PMs with different limbs and different S_c. WU, et al[21–22], studied the performance and dynamics of a planar 3-DOF PM with actuation redundancy and a PRRRP PM. LI, et al[23], studied the performance of a 3DOF PM. LU, et al[24], studied kinematics and statics of a 5-DOF 4SPS+SPR PM with $2S_c$. In fact, it is difficulty to manufacture Sc, therefore, a novel spatial 6-DOF PM with 3 planar limbs and equipped with 3 fingers is proposed because the spatial 6-DOF PM with 3 planar limbs has several merits(see

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section 2) and its moving platform can provide more room for arranging more fingers. Since the 6-DOF PM with 3 planar limbs and 3 fingers are designed for high-speed and heavy-load coordinative operation, it becomes significant to establish its coordinative kinetostatics modeling and determine its characteristics for its structure optimization, control, manufacturing and applications. Therefore, this paper focuses on a novel 6-DOF PM with 3 planar limbs and 3 finger mechanisms and establishing its mathematic model of kinetostatics in the light of its application.

2 Structural Characteristics of 6-DOF PM with 3 Planar Limbs and 3 Fingers

A 3D prototype of 6-DOF PM with 3 planar limbs Q_i and equipped with 3 finger mechanisms is constructed, see Fig. 1. It includes a 6-DOF PM with 3Q_i and 3 one-DOF finger mechanisms. The 6-DOF PM with 3Q_i includes a moving platform m, a fixed base B, 3 identical planar limbs Q_i(i=1, 2, 3). Here, m is a regular triangle with 3 vertices b_i, 3 sides $l_{si}=l_s$ and a central point o; B is a regular triangle with 3 vertices B_i, 3 sides $L_{si}=L_s$ and a central point O. Each of Q_i includes a vertical rod r_{vi}, an upper beam g_i, a lower beam G_i and 2 translational active rods r_{ij}. Each of r_{ij} is composed a translational actuator, a cylinder and a piston rod. In each of Q_i, the middle of G_i connects with B by a vertical revolute joint R_{i1} at B_i; the one end of r_{vi} connects with m by a vertical revolute joint R_{i4} at b_i, the other end of r_{vi} connects with the middle of g_i by a horizontal revolute joint R_{i5}; the two ends of r_{ij} connect with the two ends of g_i and G_i by revolute joints; g_i, G_i and r_{ij} form a closed planar mechanism P_i with 2 translational actuators.

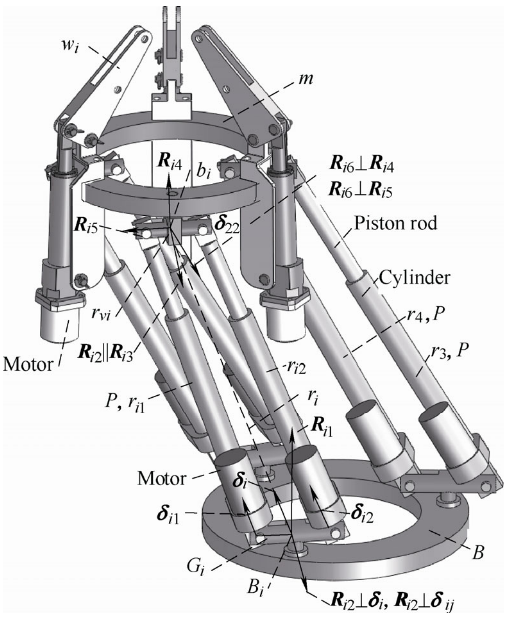


Fig. 1. 3D prototype of 6-DOF PM with 3 planar limbs and 3 one-DOF finger mechanisms

Let ⊥, ||, | be perpendicular, parallel, collinear constraints, respectively. Let (s_i), (m) and (B) be the coordinate systems on finger at o_i, on m at o and on B at O, respectively. This PM includes the following geometric conditions: $z\perp m$, $Z\perp B$, $R_{i1}\|Z$, $R_{i2}\perp\boldsymbol{\delta}_i$, $R_{i2}\perp\boldsymbol{\delta}_{ij}$, $R_{i2}\|R_{i3}$, $R_{i4}\|z$, $g_i\|m$, $R_{i2}\|B$, $G_i\|B$, (g_i, G_i, r_i, r_{ij}) being in Q_i, $b_1b_3\|x$, $ob_2|y$, $b_{i1}b_{i2}=g$, $B_{i1}B_{i2}=G$, $ob_i=e$, $ob_2|y$, $R_{i6}\perp R_{i4}$, $R_{i6}\perp R_{i5}$(i=1, 2, 3, j=1, 2). Each of the finger mechanisms with one DOF is formed by a planar four bar mechanism which is composed of a frame, a claw w_i and a active rod L_i. The three finger mechanisms are distributed uniformly on m at point o_i in the same circumference and can coordinative operate for grabbing or excluding the object with complicated shape. Comparing with the existing 6-DOF PMs, the proposed 6-DOF PM with 3Q_i possesses following merits.

(1) Each of planar limbs Q_i only includes revolute joints R and prismatic joint P, therefore, it is simple in structure and is easy manufacturing.

(2) Since all R in each of 3Q_i are parallel mutually, each of r_{ij} is only subjected a linear force along its axis. Thus, the hydraulic translational actuator can be used for increasing a capability of large load bearing. In addition, a bending moment and a rotational torque between the piston rod and the cylinder can be avoided.

(3) The revolute joint R has a larger capability of load bearing than that of spherical joint S; R has higher precision than S under large cyclic loading because backlash of R can be eliminated more easily than that of S. The workspace can be increased due to R having larger rotation range than S before interference.

3 Kinetostatics of PM with 3 Planar Limbs

3.1 Displacement analysis

The derivation of displacement formulae of the proposed PM is a prerequisite for solving velocity, acceleration and statics of finger in (B).

The kinetostatics model of the 6-DOF PM with 3 planar limbs is shown in Fig. 2. Let X_o, Y_o, Z_o be the position components of m at o in (B). The coordinates of b_i of m in (m) and B_i of B in (B) are expressed as follows[24]:

$$\boldsymbol{B}_i=\frac{E}{2}\begin{pmatrix}\pm q\\-1\\0\end{pmatrix},\ \boldsymbol{B}_2=\begin{pmatrix}0\\E\\0\end{pmatrix},$$
$$^{m}\boldsymbol{b}_i=\frac{e}{2}\begin{pmatrix}\pm q\\-1\\0\end{pmatrix},\ ^{m}\boldsymbol{b}_2=\begin{pmatrix}0\\e\\0\end{pmatrix},\ \boldsymbol{o}=\begin{pmatrix}X_o\\Y_o\\Z_o\end{pmatrix},\qquad(1)$$

where E is the distance from B_i to O, e is the distance from b_i (i=1, 2, 3) to o. As i=1, q is $3^{1/2}$; as i=3, q is $-3^{1/2}$. This condition is also available for Eq. (4), Eq. (5) and Eq. (7) with q.

Let φ be one of (α, β, γ). Set $s_\varphi=\sin\varphi$, $c_\varphi=\cos\varphi$, $t_\varphi=\tan\varphi$. b_i and z of m in (B) are expressed as follows [24]:

$$\boldsymbol{b}_i=\boldsymbol{R}_m^B\,{}^m\boldsymbol{b}_i+\boldsymbol{o}\ \ (i=1,\,2,\,3),$$

$$\boldsymbol{R}_m^B=\begin{pmatrix} x_l & y_l & z_l\\ x_m & y_m & z_m\\ x_n & y_n & z_n\end{pmatrix}=\tag{2}$$

$$\begin{pmatrix} c_\alpha c_\beta c_\gamma-s_\alpha s_\gamma & -c_\alpha c_\beta s_\gamma-s_\alpha c_\gamma & c_\alpha s_\beta\\ s_\alpha c_\beta c_\gamma+c_\alpha s_\gamma & -s_\alpha c_\beta s_\gamma+c_\alpha c_\gamma & s_\alpha s_\beta\\ -s_\beta c_\gamma & s_\beta s_\gamma & c_\beta\end{pmatrix},$$

where $\boldsymbol{R}_m^B$ is a rotation matrix from (m) to (B) in order ZYZ; x_l, x_m, x_n y_l, y_m, y_n z_l, z_m, z_n are nine orientation parameters of m [24].

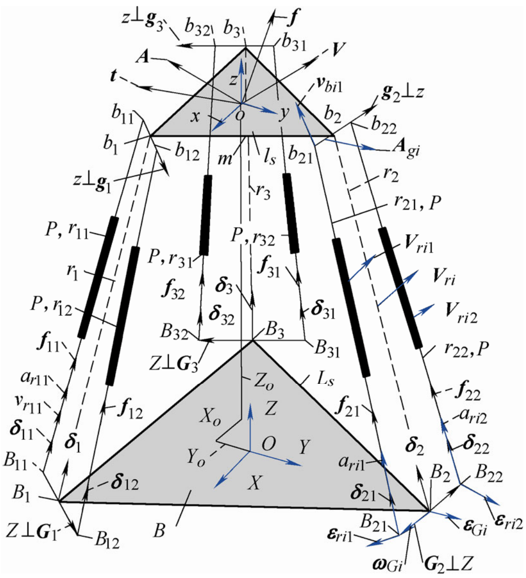


Fig. 2. Kinetostatics model 6-DOF PM with 3 planar limbs

b_i of m in (B) are expressed by a transformed matrix based on Eqs. (1), (2) as follows [24]:

$$\boldsymbol{b}_i=\frac{1}{2}\begin{pmatrix}\pm qex_l-ey_l+2X_o\\ \pm qex_m-ey_m+2Y_o\\ \pm qex_n-ey_n+2Z_o\end{pmatrix},\ \boldsymbol{b}_2=\begin{pmatrix} ey_l+X_o\\ ey_m+Y_o\\ ey_n+Z_o\end{pmatrix},\tag{3}$$
$$q=\sqrt{3},\ e=ql_s/3,\ E=qL_s/3.$$

Let $\boldsymbol{r}_i(i$=1, 2, 3) be the vector from B_i to b_i, $\boldsymbol{e}_i$ be the vector from o to b_i. They are derived as follows:

$$\boldsymbol{r}_i=\begin{pmatrix} r_{ix}\\ r_{iy}\\ r_{iz}\end{pmatrix}=\frac{1}{2}\begin{pmatrix}\pm q(ex_l-E)-ey_l+2X_o\\ \pm qex_m-ey_m+2Y_o+E\\ \pm qex_n-ey_n+2Z_o\end{pmatrix},\ \boldsymbol{e}_2=e\begin{pmatrix} y_l\\ y_m\\ y_n\end{pmatrix},$$
$$\boldsymbol{r}_2=\begin{pmatrix} r_{2x}\\ r_{2y}\\ r_{2z}\end{pmatrix}=\begin{pmatrix} ey_l+X_o\\ ey_m+Y_o-E\\ ey_n+Z_o\end{pmatrix},\ \boldsymbol{e}_i=\frac{e}{2}\begin{pmatrix}\pm qx_l-y_l\\ \pm qx_m-y_m\\ \pm qx_n-y_n\end{pmatrix}.\tag{4}$$

Let $\boldsymbol{G}_{0i}$ and $\boldsymbol{G}_i(i$=1, 2, 3) be the vector of G_i and its unit vector. Based on the geometric condition, it is known that both $\boldsymbol{G}_{0i}$ and $\boldsymbol{r}_i$ locate in the same plan P_i and there are $P_i\perp B$. Thus, $\boldsymbol{G}_{0i}$, $\boldsymbol{G}_i$ can be derived by Eq. (4) as follows:

$$\boldsymbol{G}_{01}=2\begin{pmatrix} r_{1x}\\ r_{1y}\\ 0\end{pmatrix},\ \boldsymbol{G}_{02}=\begin{pmatrix} r_{2x}\\ r_{2y}\\ 0\end{pmatrix},\ \boldsymbol{G}_{03}=2\begin{pmatrix} r_{3x}\\ r_{3y}\\ 0\end{pmatrix},\ \boldsymbol{G}_i=\frac{\boldsymbol{G}_{0i}}{|\boldsymbol{G}_{0i}|}.\tag{5}$$

Let $\boldsymbol{g}_{0i}$ and $\boldsymbol{g}_i$ be the vector and the unit vector of the upper beam g_i. Based on the geometric condition, it is known that both $\boldsymbol{g}_{0i}$ and $\boldsymbol{r}_i$ locate in the same plan P_i and there are $P_i\perp B$ and $\boldsymbol{g}_{0i}\perp\boldsymbol{z}$, ei. $\boldsymbol{g}_{0i}\cdot\boldsymbol{z}$=0. Thus, $\boldsymbol{g}_{0i}$ and $\boldsymbol{g}_i$ are derived from Eq. (3) as follows:

$$\boldsymbol{g}_{01}=\begin{pmatrix} 2r_{1x}\\ 2r_{1y}\\ g_{01z}\end{pmatrix},\ \boldsymbol{g}_{02}=\begin{pmatrix} 2r_{2x}\\ 2r_{2y}\\ g_{02z}\end{pmatrix},\ \boldsymbol{g}_{03}=\begin{pmatrix} 2r_{3x}\\ 2r_{3y}\\ g_{03z}\end{pmatrix},\ \boldsymbol{g}_i=\frac{\boldsymbol{g}_{0i}}{|\boldsymbol{g}_{0i}|},$$
$$g_{0iz}=\pm qex_n-ey_n+\frac{\pm qEz_l-2X_oz_l-2Y_oz_m-Ez_m}{z_n},$$
$$g_{02z}=ey_n-(X_oz_l+Y_oz_m-Ez_m)/z_n.\tag{6}$$

Let $B_{i1}B_i$=B_iB_{i2}=G, b_ib_{i1}=b_ib_{i2}=g, $\boldsymbol{r}_{ij}$ be the vector from B_{ij} to b_{ij}. $\boldsymbol{r}_{ij}$ are expressed as follows:

$$\begin{gathered}\boldsymbol{B}_{i1}\boldsymbol{B}_i=\boldsymbol{B}_i\boldsymbol{B}_{i2}=G\boldsymbol{G}_i,\\ \boldsymbol{g}_{i1}=\boldsymbol{b}_i\boldsymbol{b}_{i1}=-g\boldsymbol{g}_i,\ \boldsymbol{g}_{i2}=\boldsymbol{b}_i\boldsymbol{b}_{i2}=g\boldsymbol{g}_i,\\ \boldsymbol{r}_{i1}=\boldsymbol{B}_{i1}\boldsymbol{B}_i+\boldsymbol{B}_i\boldsymbol{b}_i+\boldsymbol{g}_{i1}=\boldsymbol{r}_i+G\boldsymbol{G}_i-g\boldsymbol{g}_i,\\ \boldsymbol{r}_{i2}=\boldsymbol{B}_i\boldsymbol{b}_i-\boldsymbol{B}_i\boldsymbol{B}_{i2}+\boldsymbol{g}_{i2}=\boldsymbol{r}_i-G\boldsymbol{G}_i+g\boldsymbol{g}_i.\end{gathered}\tag{7}$$

Let $\boldsymbol{\delta}_i$ be the unit vector of $\boldsymbol{r}_i$, let $\boldsymbol{\delta}_{ij}(i$=1, 2, 3; j=1, 2) be the unit vector of $\boldsymbol{r}_{ij}$. The formulae for solving r_i, r_{ij}, $\boldsymbol{r}_{ij}$, $\boldsymbol{\delta}_i$ and $\boldsymbol{\delta}_{ij}$ are derived from Eqs. (5)–(7) as follows:

$$\begin{gathered}\boldsymbol{r}_{ij}=\boldsymbol{r}_i+(-1)^j\left(g\frac{\boldsymbol{g}_{0i}}{|\boldsymbol{g}_{0i}|}-G\frac{\boldsymbol{G}_{0i}}{|\boldsymbol{G}_{0i}|}\right),\\ \boldsymbol{g}_{ij}=\boldsymbol{b}_i\boldsymbol{b}_{ij}=(-1)^jg\boldsymbol{g}_i,\ \boldsymbol{G}_{ij}=\boldsymbol{B}_i\boldsymbol{B}_{ij}=(-1)^jG\boldsymbol{G}_i,\\ \boldsymbol{\delta}_i=\frac{\boldsymbol{r}_i}{r_i},\ \boldsymbol{\delta}_{ij}=\frac{\boldsymbol{r}_{ij}}{r_{ij}},\ r_i^2=r_{ix}^2+r_{iy}^2+r_{iz}^2,\\ r_{ij}^2=r_{ijx}^2+r_{ijy}^2+r_{ijz}^2\ (i=1,\,2,\,3,\,j=1,\,2).\end{gathered}\tag{8}$$

3.2 Conceptions of kinematics

The derivation of velocity formulae of the proposed PM is a key issue to establish the statics model and acceleration model of the proposed PM and its fingers in {B}. Suppose there are a vector ζ and a skew-symmetric matrix $\hat{\zeta}$ or $s(\zeta)$. They must satisfy[1, 24]

$$\zeta\times=\hat{\zeta}=s(\zeta),\ \hat{\zeta}^{\mathrm{T}}=-\hat{\zeta},\ -\hat{\zeta}^2=\boldsymbol{E}-\zeta\zeta^{\mathrm{T}},\tag{9}$$

where ζ may be one of vectors ($\boldsymbol{e}_i$, $\boldsymbol{\delta}_i$, $\boldsymbol{e}_{ij}$, $\boldsymbol{\delta}_{ij}$, $\boldsymbol{R}_{i1}$, $\boldsymbol{R}_{i2}$, $\boldsymbol{R}_{i4}$, $\boldsymbol{R}_{i5}$, i=1, 2, 3, j=1, 2).

Let $\boldsymbol{V}$, $\boldsymbol{A}$, $\boldsymbol{v}$, $\boldsymbol{\omega}$, $\boldsymbol{a}$, $\boldsymbol{\varepsilon}$ be the general forward velocity, acceleration, translational velocity, angular velocity, translational acceleration, angular accelerations of m at o, respectively. They are expressed as follows:

$$\boldsymbol{V}=\begin{pmatrix}\boldsymbol{v}\\ \boldsymbol{\omega}\end{pmatrix},\ \boldsymbol{A}=\begin{pmatrix}\boldsymbol{a}\\ \boldsymbol{\varepsilon}\end{pmatrix}. \tag{10}$$

Let $\boldsymbol{v}_{bi}$ be a velocity of m at b_i, $\boldsymbol{v}_{bij}$ be a velocity vector of the upper beam g_i at b_{ij}, $\boldsymbol{\omega}_{gi}$ be the angular velocity of g_i; $\boldsymbol{\omega}_{ri}$ be the angular velocity of r_i, $\boldsymbol{\omega}_{rij}$ be the angular velocity of r_{ij}, ω_{gij} be the scalar angular velocity of g_i about r_{ij} at b_{ij}, v_{ri} be the scalar velocity along r_i, v_{rij} be the input scalar velocity along r_{ij}. Let ω_{i1} and $\boldsymbol{R}_{i1}$ be a scalar angular velocity of the lower beam G_i about B at B_i and its unit vector; ω_{i2} and $\boldsymbol{R}_{i2}$ be a scalar angular velocity of r_i about G_i at B_i and its unit vector; ω_{i3} and $\boldsymbol{R}_{i3}$ be the scalar angular velocity of g_i about r_i at b_i and its unit vector and there is $\boldsymbol{R}_{i3}\|\boldsymbol{R}_{i2}$. Let ω_{i4} and $\boldsymbol{R}_{i4}$ be the scalar angular velocity of vertical rod r_{vi} about m at b_i and its unit vector. Let ω_{i5} and $\boldsymbol{R}_{i5}$ be the scalar angular velocity of g_i about r_{vi} at b_i and its unit vector. They can be expressed as follows[1, 2]:

$$\begin{gathered}\boldsymbol{R}_{i2}=\boldsymbol{R}_{i3}=\boldsymbol{R}_{i1}\times\boldsymbol{\delta}_i/|\boldsymbol{R}_{i1}\times\boldsymbol{\delta}_i|,\ \boldsymbol{R}_{i5}=\boldsymbol{g}_{i1}/|\boldsymbol{g}_{i1}|,\\ \boldsymbol{R}_{i1}=\boldsymbol{Z},\ \boldsymbol{R}_{i4}=\boldsymbol{z},\ \boldsymbol{R}_{i6}=\boldsymbol{R}_{i4}\times\boldsymbol{R}_{i5},\boldsymbol{R}'_{i4}=\boldsymbol{\omega}\times\boldsymbol{R}_{i4},\\ \boldsymbol{R}'_{i5}=\boldsymbol{\omega}_{gi}\times\boldsymbol{R}_{i5},\ \boldsymbol{\omega}_{ri}=\omega_{i1}\boldsymbol{R}_{i1}+\omega_{i2}\boldsymbol{R}_{i2},\\ \boldsymbol{\omega}_{gi}=\boldsymbol{\omega}+\omega_{i4}\boldsymbol{R}_{i4}+\omega_{i5}\boldsymbol{R}_{i5}=\boldsymbol{\omega}_{rij}+\omega_{gij}\boldsymbol{R}_{i2},\\ \boldsymbol{v}_{bi}=v_{ri}\bullet\boldsymbol{\delta}_i+\boldsymbol{\omega}_{ri}\times\boldsymbol{r}_i,\ v_{ri}=\boldsymbol{v}_{bi}\bullet\boldsymbol{\delta}_i,\ \boldsymbol{r}_i=r_i\boldsymbol{\delta}_i,\\ \boldsymbol{v}_{bij}=\boldsymbol{v}_{rij}+\boldsymbol{\omega}_{rij}\times\boldsymbol{r}_{ij}+\omega_{i1}\boldsymbol{R}_{i1}\times\boldsymbol{G}_{ij},\\ v_{rij}=\boldsymbol{v}_{bij}\bullet\boldsymbol{\delta}_{ij},\ \boldsymbol{r}_{ij}=r_{ij}\boldsymbol{\delta}_{ij}.\end{gathered} \tag{11}$$

3.3 Angular velocity $\boldsymbol{\omega}_{ri}$ of r_i and velocity v_{ri} along r_i

From the line 4 formula of Eq. (11), it leads to

$$\begin{gathered}\omega_{i1}\boldsymbol{R}_{i1}\times\boldsymbol{r}_i+\omega_{i2}\boldsymbol{R}_{i2}\times\boldsymbol{r}_i=\boldsymbol{\omega}_{ri}\times\boldsymbol{r}_i=\boldsymbol{v}_{bi}-v_{ri}\bullet\boldsymbol{\delta}_i=\\ (\boldsymbol{\delta}_i\bullet\boldsymbol{\delta}_i)\boldsymbol{v}_{bi}-(\boldsymbol{v}_{bi}\bullet\boldsymbol{\delta}_i)\boldsymbol{\delta}_i=-\hat{\boldsymbol{\delta}}_i^2(\boldsymbol{v}-\hat{\boldsymbol{e}}_i\boldsymbol{\omega}).\end{gathered} \tag{12}$$

Dot multiply the both sides of Eq. (12) by $\boldsymbol{R}_{i2}$ and $\boldsymbol{R}_{i1}$, respectively, it leads to

$$\begin{gathered}\omega_{i1}(\boldsymbol{R}_{i1}\times\boldsymbol{r}_i)\bullet\boldsymbol{R}_{i2}=\boldsymbol{R}_{i2}^{\mathrm{T}}\hat{\boldsymbol{\delta}}_i^2(-\boldsymbol{v}+\hat{\boldsymbol{e}}_i\boldsymbol{\omega}),\\ \omega_{i2}(\boldsymbol{R}_{i2}\times\boldsymbol{r}_i)\bullet\boldsymbol{R}_{i1}=\boldsymbol{R}_{i1}^{\mathrm{T}}\hat{\boldsymbol{\delta}}_i^2(-\boldsymbol{v}+\hat{\boldsymbol{e}}_i\boldsymbol{\omega}).\end{gathered} \tag{13}$$

Let $D_1=\boldsymbol{r}_i\cdot(\boldsymbol{R}_{i1}\times\boldsymbol{R}_{i2})$, $\boldsymbol{D}_2=\boldsymbol{R}_{i1}\boldsymbol{R}_{i2}^{\mathrm{T}}-\boldsymbol{R}_{i2}\boldsymbol{R}_{i1}^{\mathrm{T}}$. From Eq. (13), it leads to

$$\begin{gathered}\omega_{i1}=\frac{-\boldsymbol{R}_{i2}^{\mathrm{T}}\hat{\boldsymbol{\delta}}_i^2(\boldsymbol{v}-\hat{\boldsymbol{e}}_i\boldsymbol{\omega})}{(\boldsymbol{R}_{i1}\times\boldsymbol{r}_i)\bullet\boldsymbol{R}_{i2}}=\boldsymbol{J}_{\omega i1}\boldsymbol{V},\\ \omega_{i2}=\frac{-\boldsymbol{R}_{i1}^{\mathrm{T}}\hat{\boldsymbol{\delta}}_i^2(\boldsymbol{v}-\hat{\boldsymbol{e}}_i\boldsymbol{\omega})}{D_1},\boldsymbol{J}_{\omega i1}=\frac{\boldsymbol{R}_{i2}^{\mathrm{T}}}{D_1}\begin{pmatrix}\hat{\boldsymbol{\delta}}_i^2 & -\hat{\boldsymbol{\delta}}_i^2\hat{\boldsymbol{e}}_i\end{pmatrix}.\end{gathered} \tag{14}$$

The angular velocity $\boldsymbol{\omega}_{ri}$ of r_i is derived from line 3 formula of Eq. (11) and Eq. (14) as follows:

$$\boldsymbol{\omega}_{ri}=\boldsymbol{J}_{\omega ri}\boldsymbol{V},\boldsymbol{J}_{\omega ri}=\frac{\boldsymbol{D}_2}{D_1}\begin{pmatrix}\hat{\boldsymbol{\delta}}_i^2 & -\hat{\boldsymbol{\delta}}_i^2\hat{\boldsymbol{e}}_i\end{pmatrix}. \tag{15}$$

The velocity v_{ri} along r_i is derived from line 5 formula of Eq. (11) as follows:

$$v_{ri}=\boldsymbol{J}_{vi}\boldsymbol{V},\boldsymbol{J}_{vi}=\begin{pmatrix}\boldsymbol{\delta}_i^{\mathrm{T}} & -(\hat{\boldsymbol{e}}_i\boldsymbol{\delta}_i)^{\mathrm{T}}\end{pmatrix}_{1\times 6}. \tag{16}$$

3.4 Angular velocity $\boldsymbol{\omega}_{bi}$ of g_i and $\boldsymbol{\omega}_{rij}$ of r_{ij}

Dot multiply the both side of Eq. (11) by $\boldsymbol{R}_{i6}$, ω_{i3} and $\boldsymbol{\omega}_{gi}$ are derived based on ($\boldsymbol{R}_{i6}\perp\boldsymbol{R}_{i4}$, $\boldsymbol{R}_{i6}\perp\boldsymbol{R}_{i5}$) and Eq. (15) as follows:

$$\begin{gathered}(\boldsymbol{\omega}+\omega_{i4}\boldsymbol{R}_{i4}+\omega_{i5}\boldsymbol{R}_{i5})\bullet\boldsymbol{R}_{i6}=(\boldsymbol{\omega}_{ri}+\omega_{i3}\boldsymbol{R}_{i3})\bullet\boldsymbol{R}_{i6},\\ \omega_{i3}=\frac{\boldsymbol{R}_{i6}^{\mathrm{T}}(\boldsymbol{\omega}-\boldsymbol{\omega}_{ri})}{\boldsymbol{R}_{i3}\bullet\boldsymbol{R}_{i6}},\ \boldsymbol{D}_3=\frac{\boldsymbol{R}_{i3}\boldsymbol{R}_{i6}^{\mathrm{T}}}{\boldsymbol{R}_{i3}\bullet\boldsymbol{R}_{i6}},\\ \boldsymbol{\omega}_{gi}=\boldsymbol{\omega}_{ri}+\omega_{i3}\boldsymbol{R}_{i3}=\boldsymbol{\omega}_{ri}+\boldsymbol{D}_3(\boldsymbol{\omega}-\boldsymbol{\omega}_{ri})=\\ (\boldsymbol{E}_{3\times3}-\boldsymbol{D}_3)\boldsymbol{J}_{\omega ri}\boldsymbol{V}+\begin{pmatrix}\boldsymbol{0}_{3\times3} & \boldsymbol{D}_3\end{pmatrix}\boldsymbol{V}=\boldsymbol{J}_{\omega gi}\boldsymbol{V},\\ \boldsymbol{J}_{\omega gi}=(\boldsymbol{E}_{3\times3}-\boldsymbol{D}_3)\boldsymbol{J}_{\omega ri}+\begin{pmatrix}\boldsymbol{0}_{3\times3} & \boldsymbol{D}_3\end{pmatrix}.\end{gathered} \tag{17}$$

Dot multiply the both sides of Eq. (11) by $\boldsymbol{\delta}_{ij}$ (i=1, 2, 3; j=1, 2), based on $\boldsymbol{\delta}_{ij}\perp\boldsymbol{R}_{i2}$, it leads to

$$\boldsymbol{\omega}_{gi}\bullet\boldsymbol{\delta}_{ij}=\boldsymbol{\omega}_{rij}\bullet\boldsymbol{\delta}_{ij}+\omega_{gij}\boldsymbol{R}_{i2}\bullet\boldsymbol{\delta}_{ij}\Rightarrow\ \boldsymbol{\omega}_{gi}\bullet\boldsymbol{\delta}_{ij}=\boldsymbol{\omega}_{rij}\bullet\boldsymbol{\delta}_{ij}. \tag{18}$$

Cross multiply the both sides of Eq. (11) by $\boldsymbol{\delta}_{ij}$, it leas to

$$\begin{gathered}\boldsymbol{\delta}_{ij}\times(\boldsymbol{v}+\boldsymbol{\omega}\times\boldsymbol{e}_i+\boldsymbol{\omega}_{gi}\times\boldsymbol{g}_{ij})=\\ r_{ij}\boldsymbol{\omega}_{rij}-r_{ij}\boldsymbol{\delta}_{ij}(\boldsymbol{\omega}_{rij}\bullet\boldsymbol{\delta}_{ij})+\boldsymbol{\delta}_{ij}\times(\omega_{i1}\boldsymbol{R}_{i1}\times\boldsymbol{G}_{ij}).\end{gathered} \tag{19}$$

Substitute Eq. (18) into Eq. (19), angular velocity $\boldsymbol{\omega}_{rij}$ of r_{ij} (i=1, 2, 3; j=1, 2) is derived as follows:

$$\begin{gathered}\boldsymbol{\omega}_{rij}=\boldsymbol{J}_{\omega ij}\boldsymbol{V},\\ \boldsymbol{J}_{\omega ij}=[\hat{\boldsymbol{\delta}}_{ij}\left(\boldsymbol{E}-\hat{\boldsymbol{e}}_i\right)+(r_{ij}\boldsymbol{\delta}_{ij}\boldsymbol{\delta}_{ij}^{\mathrm{T}}-\hat{\boldsymbol{\delta}}_{ij}\hat{\boldsymbol{g}}_{ij})\boldsymbol{J}_{\omega gi}-\\ \hat{\boldsymbol{\delta}}_{ij}\hat{\boldsymbol{R}}_{i1}\boldsymbol{G}_{ij}\boldsymbol{J}_{\omega i1}]/r_{ij}.\end{gathered} \tag{20}$$

3.5 General input velocity $\boldsymbol{V}_r$ and forward velocity $\boldsymbol{V}$

From line 7 formula of Eq. (11), Eq. (17) and Eq. (20), v_{rij} (i=1, 2, 3; j=1, 2) is derived as follows:

$$v_{rij}=\boldsymbol{J}_{ij}\boldsymbol{V},\boldsymbol{J}_{ij}=\begin{pmatrix}\boldsymbol{\delta}_{ij}^{\mathrm{T}} & (\hat{\boldsymbol{e}}_i\boldsymbol{\delta}_{ij})^{\mathrm{T}}\end{pmatrix}+(\hat{\boldsymbol{g}}_{ij}\boldsymbol{\delta}_{ij})^{\mathrm{T}}\boldsymbol{J}_{\omega gi}. \tag{21}$$

The general input velocity $\boldsymbol{V}_r$ and the general forward velocity $\boldsymbol{V}$ are derived based on the principle of the virtual work [2, 24] and Eqs. (17)–(21) as follows:

$$\begin{aligned}&\boldsymbol{V}_r=\boldsymbol{J}_{6\times6}\boldsymbol{V},\\&\boldsymbol{V}=\boldsymbol{J}^{-1}\boldsymbol{V}_r,\\&i=1,2,3,\\&j=1,2,\end{aligned}\quad \boldsymbol{V}_r=\begin{pmatrix}v_{r11}\\v_{r12}\\v_{r21}\\v_{r22}\\v_{r31}\\v_{r32}\end{pmatrix},\ \boldsymbol{J}=\begin{pmatrix}\boldsymbol{J}_{11}\\\boldsymbol{J}_{12}\\\boldsymbol{J}_{21}\\\boldsymbol{J}_{22}\\\boldsymbol{J}_{31}\\\boldsymbol{J}_{32}\end{pmatrix},$$
$$\boldsymbol{J}_{\omega ri}=\boldsymbol{D}_2\left(\hat{\boldsymbol{\delta}}_i^2\quad -\hat{\boldsymbol{\delta}}_i^2\hat{\boldsymbol{e}}_i\right)/D_1,$$
$$\boldsymbol{J}_{\omega gi}=(\boldsymbol{E}_{3\times3}-\boldsymbol{D}_3)\boldsymbol{J}_{\omega ri}+\left(\boldsymbol{0}_{3\times3}\quad \boldsymbol{D}_3\right), \tag{22}$$

where $\boldsymbol{J}$ is a 6×6 Jacobian matrix of PM with $3Q_i$.

3.6 Acceleration of PM

The establishment of acceleration model of the proposed PM is a prerequisite to establish dynamics model of the proposed PM and its finger in (*B*) and to evaluate their dynamic characteristics. Let a_{rij} be the input scalar acceleration along r_{ij}, $\boldsymbol{a}_{bi}$ be the translational vector acceleration of *m* at b_i, $\boldsymbol{\varepsilon}_{gi}$ be the angular vector acceleration of g_i, $\boldsymbol{\varepsilon}_{rij}$ be the angular vector acceleration of r_{ij}. A standard formula for solving the general input acceleration $\boldsymbol{A}_r$ is derived as follows[24]:

$$\boldsymbol{A}_r=\boldsymbol{J}_{6\times6}\boldsymbol{A}+\boldsymbol{V}^T\boldsymbol{H}\boldsymbol{V},$$
$$\boldsymbol{A}=\boldsymbol{J}^{-1}(\boldsymbol{A}_r-\boldsymbol{V}^T\boldsymbol{H}\boldsymbol{V}),$$
$$a_{rij}=\boldsymbol{J}_{ij}\boldsymbol{A}+\boldsymbol{V}^T\boldsymbol{h}_{ij}\boldsymbol{V},\ \boldsymbol{h}_{ij}=\sum_{u=1}^{3}{}^{u}(\boldsymbol{h}_{ij})_{6\times6},$$
$$\boldsymbol{A}_r=\begin{pmatrix}a_{r11}\\a_{r12}\\a_{r21}\\a_{r22}\\a_{r31}\\a_{r32}\end{pmatrix},\ \boldsymbol{J}=\begin{pmatrix}\boldsymbol{J}_{11}\\\boldsymbol{J}_{12}\\\boldsymbol{J}_{21}\\\boldsymbol{J}_{22}\\\boldsymbol{J}_{31}\\\boldsymbol{J}_{32}\end{pmatrix},\ \boldsymbol{H}=\begin{pmatrix}\boldsymbol{h}_{11}\\\boldsymbol{h}_{12}\\\boldsymbol{h}_{21}\\\boldsymbol{h}_{22}\\\boldsymbol{h}_{31}\\\boldsymbol{h}_{32}\end{pmatrix}, \tag{23a}$$

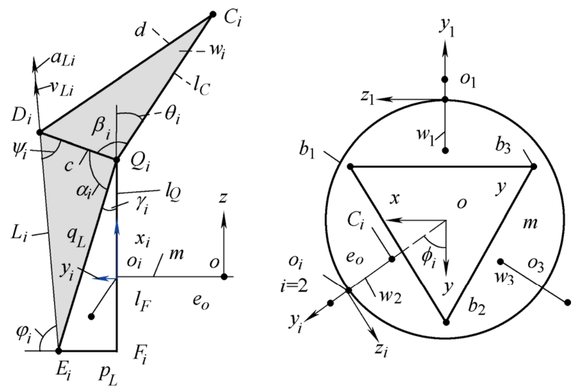
where $\boldsymbol{H}$ is a 6×6×6 Hessian matrix of 6-DOF PM with 3 Q_i, $\boldsymbol{h}_{ij}(i=1, 2, 3; j=1, 2)$ are the 6×6 sub-Hessian matrices of $\boldsymbol{H}$; ${}^{u}\boldsymbol{h}_{ij}(u=1, 2, 3)$ are the 6×6 sub-Hessian matrices of $\boldsymbol{h}_{ij}$. When given $\boldsymbol{V}_r$ and $\boldsymbol{A}_r$ of $r_{ij}(i=1, 2, 3; j=1, 2)$, $\boldsymbol{A}$ can be solved using Eq. (23a). ${}^{u}\boldsymbol{h}_{ij}$ are derived as follows:

$${}^{1}\boldsymbol{h}_{ij}=\begin{pmatrix}\boldsymbol{0}&\boldsymbol{0}_{3\times3}\\\boldsymbol{0}&\hat{\boldsymbol{e}}_i\hat{\boldsymbol{\delta}}_{ij}\end{pmatrix}-\begin{pmatrix}\boldsymbol{E}\\\hat{\boldsymbol{e}}_i\end{pmatrix}\hat{\boldsymbol{\delta}}_{ij}\boldsymbol{J}_{\omega ij}+\boldsymbol{J}_{\omega gi}^T\hat{\boldsymbol{g}}_{ij}(\hat{\boldsymbol{\delta}}_{ij}\boldsymbol{J}_{\omega gi}-\hat{\boldsymbol{\delta}}_{ij}\boldsymbol{J}_{\omega ij}),$$
$${}^{2}\boldsymbol{h}_{ij}=\left(\boldsymbol{0}_{3\times3}\quad \boldsymbol{d}_{ij}\right)-\boldsymbol{d}_{ij}\boldsymbol{J}_{\omega ri},$$
$${}^{3}\boldsymbol{h}_{ij}=\frac{\boldsymbol{J}_{\omega ri}^T}{D_1}\left\{\frac{\hat{\boldsymbol{\delta}}_i\hat{\boldsymbol{R}}_{i1}\hat{\boldsymbol{R}}_{i3}^2}{|\boldsymbol{R}_{i1}\times\boldsymbol{\delta}_i|}(\boldsymbol{c}_{ij}\boldsymbol{R}_{i1}\boldsymbol{E}-\boldsymbol{c}_{ij}^T\boldsymbol{R}_{i1}^T)\left(\hat{\boldsymbol{\delta}}_i^2\quad -\hat{\boldsymbol{\delta}}_i^2\hat{\boldsymbol{e}}_i\right)+\right.$$
$$r_i\hat{\boldsymbol{\delta}}_i\left(\frac{\hat{\boldsymbol{R}}_{i1}\hat{\boldsymbol{R}}_{i3}^2}{|\boldsymbol{R}_{i1}\times\boldsymbol{\delta}_i|}\hat{\boldsymbol{R}}_{i1}\boldsymbol{\delta}_i-\boldsymbol{R}_{i1}\times\boldsymbol{R}_{i2}\right)\boldsymbol{c}_{ij}\boldsymbol{J}_{\omega ri}-\boldsymbol{c}_{ij}^T\boldsymbol{\delta}_i^T(\boldsymbol{R}_{i1}\times\boldsymbol{R}_{i2})\boldsymbol{J}_{vi}+$$
$$\left.\hat{\boldsymbol{\delta}}_i(\boldsymbol{c}_{ij}\boldsymbol{D}_2\boldsymbol{\delta}_i+\boldsymbol{D}_2^T\boldsymbol{c}_{ij}^T\boldsymbol{\delta}_i^T)\left(\boldsymbol{E}_{3\times3}\quad -\hat{\boldsymbol{e}}_i\right)\right\}+$$
$$\frac{1}{D_1}\begin{pmatrix}\boldsymbol{0}_{3\times3}&\boldsymbol{0}_{3\times3}\\\boldsymbol{0}_{3\times3}&\hat{\boldsymbol{e}}_i s(\hat{\boldsymbol{\delta}}_i^2\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{c}_{ij}^{\mathrm{T}})\end{pmatrix},$$
$$\boldsymbol{d}_{ij}=\frac{1}{\boldsymbol{R}_{i3}\bullet\boldsymbol{R}_{i6}}\left\{\boldsymbol{J}_{\omega ri}^{\mathrm{T}}\frac{\hat{\boldsymbol{\delta}}_i\hat{\boldsymbol{R}}_{i1}\hat{\boldsymbol{R}}_{i3}^2}{|\boldsymbol{R}_{i1}\times\boldsymbol{\delta}_i|}\left[(\hat{\boldsymbol{g}}_{ij}\boldsymbol{\delta}_{ij})\boldsymbol{R}_{i6}^{\mathrm{T}}-\boldsymbol{R}_{i6}(\hat{\boldsymbol{g}}_{ij}\boldsymbol{\delta}_{ij})^{\mathrm{T}}\boldsymbol{D}_3\right]+\right.$$
$$\left.\left[\begin{pmatrix}\boldsymbol{0}\\\boldsymbol{E}\end{pmatrix}\hat{\boldsymbol{R}}_{i4}\hat{\boldsymbol{R}}_{i5}-\boldsymbol{J}_{\omega bi}^{\mathrm{T}}\hat{\boldsymbol{R}}_{i5}\hat{\boldsymbol{R}}_{i4}\right][(\hat{\boldsymbol{g}}_{ij}\boldsymbol{\delta}_{ij})^{\mathrm{T}}\boldsymbol{R}_{i3}-\boldsymbol{R}_{i3}(\hat{\boldsymbol{g}}_{ij}\boldsymbol{\delta}_{ij})^{\mathrm{T}}\boldsymbol{D}_3]\right\}. \tag{23b}$$

4 Kinematics of Finger Mechanism

4.1 Geometric relations and positions

A kinematics model of finger mechanism and their uniformly distribution on *m* are shown in Fig. 3. In (s_i), $x_i \| z$, $y_i | oo_i$ are satisfied. Let l_Q, l_F be the distances from o_i to Q_i, F_i, respectively. Let l_C, c, q_L be the distances from Q_i to C_i, D_i, E_i, respectively. Let θ_i be the angle between x_i and l_C. Let p_L be the distance from F_i to E_i.



(a) Mechanisms (b) Distribution

Fig. 3. Kinematics model of finger mechanisms and their distribution

Some key geometric relations are represented as

$$c_{\alpha i}=\frac{c^2+q_L{}^2-L_i^2}{2cq_L},\ c_{\beta i}=\frac{c^2+l_C{}^2-d^2}{2cl_C},$$
$$c_{\psi i}=\frac{c^2+L_i^2-q_L{}^2}{2cL_i},\ s_{\gamma i}=\frac{p_L}{q_L}, \tag{24}$$
$$\varphi_i=\alpha_i+\gamma_i+\psi_i-90°,\ \theta_i=\alpha_i+\beta_i+\gamma_i-180°.$$

The position vectors of $K_i(K=C, Q, D, E, F)$ in (s_i), (m) and (B) are derived as follows:

$${}^{m}\boldsymbol{K}_i={}^{m}_{si}\boldsymbol{R}\,{}^{si}\boldsymbol{K}_i+{}^{m}\boldsymbol{o}_i,\ \boldsymbol{K}_i={}^{B}_{m}\boldsymbol{R}\,{}^{m}\boldsymbol{K}_i+\boldsymbol{o},$$
$${}^{si}\boldsymbol{C}_i=\begin{pmatrix}l_Cc_{\theta i}+l_Q\\-l_Cs_{\theta i}\\0\end{pmatrix},\ {}^{si}\boldsymbol{Q}_i=\begin{pmatrix}l_Q\\0\\0\end{pmatrix},$$
$${}^{si}\boldsymbol{D}_i=\begin{pmatrix}L_is_{\varphi i}-l_F\\L_ic_{\varphi i}+p_L\\0\end{pmatrix},\ {}^{si}\boldsymbol{E}_i=\begin{pmatrix}-l_F\\p_L\\0\end{pmatrix},\ {}^{si}\boldsymbol{F}_i=\begin{pmatrix}-l_F\\0\\0\end{pmatrix},$$

$$
{}^{m}\boldsymbol{o}_1=\begin{pmatrix}0\\-e_o\\0\end{pmatrix},{}^{m}\boldsymbol{o}_2=\frac{q}{2}\begin{pmatrix}e_o\\e_o\\0\end{pmatrix},{}^{m}\boldsymbol{o}_3=\frac{q}{2}\begin{pmatrix}-e_o\\e_o\\0\end{pmatrix},
$$
$$
{}_{s1}^{m}\boldsymbol{R}=\begin{pmatrix}0&0&1\\0&-1&0\\1&0&0\end{pmatrix},\ {}_{si}^{m}\boldsymbol{R}=\frac{1}{2}\begin{pmatrix}0&\pm q&-1\\0&1&\pm q\\2&0&0\end{pmatrix},
\tag{25}
$$

where i=2, q is $3^{1/2}$; i=3, q is $-3^{1/2}$.

4.2 Velocity/acceleration of finger mechanism in (s_i)

Let ${}^{si}\boldsymbol{\delta}_{Li}$, ${}^{si}\boldsymbol{v}_{Li}$, ${}^{si}\boldsymbol{\omega}_{Li}$, ${}^{si}\boldsymbol{V}_{Li}$, ${}^{si}\boldsymbol{a}_{Li}$, ${}^{si}\boldsymbol{\varepsilon}_{Li}$, ${}^{si}\boldsymbol{A}_{Li}$ be the unit vector, the(translational, angular, general) velocity, and the (translational, angular, general) acceleration of L_i in (s_i). Let v_{Li}, ω_{Li}, a_{Li}, ε_{Li} be their scalar. Let ${}^{si}\boldsymbol{\omega}_{wi}$, ${}^{si}\boldsymbol{\varepsilon}_{wi}$, ω_{wi}, ε_{wi} be the angular velocity and acceleration of claw w_i in (s_i) and their scalar. They are derived as follows:

$$
\begin{gathered}
\omega_{Li}=\frac{v_{Li}}{L_i t_{\psi i}},\ \dot{\psi}_i=-\frac{L_i^{\,2}+q_L^{\,2}-c^2}{2cL_i^2\sqrt{1-q_L^2}}v_{Li},\\
\omega_{wi}=\frac{v_{Li}}{s_{\psi i}c},\ \varepsilon_{wi}=\frac{a_{Li}-\omega_{wi}c_{\psi i}\dot{\psi}_i c}{s_{\psi i}c},\\
\varepsilon_{Li}=\frac{a_{Li}-\omega_{Li}(v_{Li}t_{\psi i}+L_i\sec^2\psi_i\dot{\psi}_i)}{L_i t_{\psi i}},\\
{}^{si}\boldsymbol{\delta}_{Li}=\begin{pmatrix}s_{\varphi i}\\c_{\varphi i}\\0\end{pmatrix},\begin{matrix}{}^{si}\boldsymbol{v}_{Li}=v_{Li}\,{}^{si}\boldsymbol{\delta}_{Li},\\{}^{si}\boldsymbol{a}_{Li}=a_{Li}\,{}^{si}\boldsymbol{\delta}_{Li},\end{matrix}\ {}^{si}\boldsymbol{\omega}_{Li}=\begin{pmatrix}0\\0\\\omega_{Li}\end{pmatrix},\\
{}^{si}\boldsymbol{\varepsilon}_{Li}=\begin{pmatrix}0\\0\\\varepsilon_{Li}\end{pmatrix},\ {}^{si}\boldsymbol{\omega}_{wi}=\begin{pmatrix}0\\0\\\omega_{wi}\end{pmatrix},\ {}^{si}\boldsymbol{\varepsilon}_{wi}=\begin{pmatrix}0\\0\\\varepsilon_{wi}\end{pmatrix},\\
{}^{si}\boldsymbol{V}_{Li}={}^{si}\boldsymbol{J}_{Li}v_{Li},\ {}^{si}\boldsymbol{A}_{Li}=\begin{pmatrix}{}^{si}\boldsymbol{a}_{Li}\\{}^{si}\boldsymbol{\varepsilon}_{Li}\end{pmatrix},\\
{}^{si}\boldsymbol{J}_{Li}=\begin{pmatrix}s_{\varphi i}&c_{\varphi i}&0&0&0&1/L_i t_{\psi i}\end{pmatrix}^{\mathrm{T}}.
\end{gathered}
\tag{26}
$$

The velocity/acceleration of the claw at C_i are derived as

$$
\begin{aligned}
&{}^{si}\boldsymbol{v}_{Ci}={}^{si}\boldsymbol{\omega}_{wi}\times{}^{si}\boldsymbol{l}_{Ci},\ {}^{si}\boldsymbol{V}_{Ci}=\begin{pmatrix}{}^{si}\boldsymbol{v}_{Ci}\\{}^{si}\boldsymbol{\omega}_{Ci}\end{pmatrix}={}^{si}\boldsymbol{J}_{Ci}v_{Li},\\
&{}^{si}\boldsymbol{\omega}_{Ci}={}^{si}\boldsymbol{\omega}_{wi},\\
&{}^{si}\boldsymbol{J}_{Ci}=\begin{pmatrix}\boldsymbol{0}_{3\times3}&-{}^{si}\hat{\boldsymbol{l}}_{Ci}\\\boldsymbol{0}_{3\times3}&\boldsymbol{E}\end{pmatrix}\begin{pmatrix}0&0&0&0&0&\dfrac{1}{s_{\psi i}c}\end{pmatrix}^{\mathrm{T}},\\
&{}^{si}\boldsymbol{a}_{Ci}={}^{si}\boldsymbol{\varepsilon}_{wi}\times{}^{si}\boldsymbol{l}_{Ci}+{}^{si}\boldsymbol{\omega}_{wi}\times{}^{si}\boldsymbol{\omega}_{wi}\times{}^{si}\boldsymbol{l}_{Ci},\ {}^{si}\boldsymbol{\varepsilon}_{Ci}={}^{si}\boldsymbol{\varepsilon}_{wi}.
\end{aligned}
\tag{27}
$$

4.3 Kinematics of finger mechanism in {m} and {B}

Let $\boldsymbol{C}_i$, $\boldsymbol{v}_{Ci}$, $\boldsymbol{\omega}_{Ci}$, $\boldsymbol{V}_{Ci}$, $\boldsymbol{a}_{Ci}$, $\boldsymbol{\varepsilon}_{Ci}$, $\boldsymbol{A}_{Ci}$(i=1, 2, 3) be the position, the (translational, angular, general) velocities, and the (translational, angular, general) accelerations of fingertip C_i. They in (m), (B) are derived as follows:

$$
\begin{gathered}
{}^{m}\boldsymbol{C}_i={}_{si}^{m}\boldsymbol{R}\,{}^{si}\boldsymbol{C}_i+{}^{m}\boldsymbol{o}_i,\ {}^{m}\boldsymbol{v}_{Ci}={}_{si}^{m}\boldsymbol{R}\,{}^{si}\boldsymbol{v}_{Ci},\\
{}^{m}\boldsymbol{\omega}_{Ci}={}_{si}^{m}\boldsymbol{R}\,{}^{si}\boldsymbol{\omega}_{Ci},\ {}^{m}\boldsymbol{V}_{Ci}={}^{m}\boldsymbol{J}_{Ci}v_{Li},\\
{}^{m}\boldsymbol{a}_{Ci}={}_{si}^{m}\boldsymbol{R}\,{}^{si}\boldsymbol{a}_{Ci},\ {}^{m}\boldsymbol{\varepsilon}_{Ci}={}_{si}^{m}\boldsymbol{R}\,{}^{si}\boldsymbol{\varepsilon}_{Ci},\\
{}^{m}\boldsymbol{A}_{Ci}={}_{si}^{m}\boldsymbol{k}\,{}^{si}\boldsymbol{A}_{Ci},\ {}^{m}\boldsymbol{J}_{Ci}={}_{si}^{m}\boldsymbol{k}\,{}^{si}\boldsymbol{J}_{Ci},\\
\boldsymbol{e}_{Ci}={}_{m}^{B}\boldsymbol{R}\,{}^{m}\boldsymbol{C}_i,\ {}_{si}^{B}\boldsymbol{k}={}_{m}^{B}\boldsymbol{k}\,{}_{si}^{m}\boldsymbol{k},\\
{}_{m}^{B}\boldsymbol{k}=\begin{pmatrix}{}_{m}^{B}\boldsymbol{R}&\boldsymbol{0}_{3\times3}\\\boldsymbol{0}_{3\times3}&{}_{m}^{B}\boldsymbol{R}\end{pmatrix},\ {}_{si}^{m}\boldsymbol{k}=\begin{pmatrix}{}_{si}^{m}\boldsymbol{R}&\boldsymbol{0}_{3\times3}\\\boldsymbol{0}_{3\times3}&{}_{si}^{m}\boldsymbol{R}\end{pmatrix},\\
\boldsymbol{v}_{Ci}=\boldsymbol{v}+\boldsymbol{\omega}\times{}_{m}^{B}\boldsymbol{R}\,{}^{m}\boldsymbol{C}_i+{}_{m}^{B}\boldsymbol{R}\,{}^{m}\boldsymbol{v}_{Ci},\\
\boldsymbol{\omega}_{Ci}=\boldsymbol{\omega}+{}_{m}^{B}\boldsymbol{R}\,{}^{m}\boldsymbol{\omega}_{Ci},(i=1,2,3),\\
\boldsymbol{V}_{Ci}=\boldsymbol{J}_{Ci\_V_r}\boldsymbol{V}_r+\boldsymbol{J}_{Ci\_v_{Li}}v_{Li},\\
\boldsymbol{J}_{Ci\_V_r}=\begin{pmatrix}\boldsymbol{E}&-\hat{\boldsymbol{e}}_{Ci}\\\boldsymbol{0}_{3\times3}&\boldsymbol{E}\end{pmatrix}\boldsymbol{J}^{-1},\ \boldsymbol{J}_{Ci\_v_{Li}}={}_{si}^{B}\boldsymbol{k}\,{}^{si}\boldsymbol{J}_{Ci},\\
\boldsymbol{a}_{Ci}=\boldsymbol{a}+\boldsymbol{\varepsilon}\times{}_{m}^{B}\boldsymbol{R}\,{}^{m}\boldsymbol{C}_i+\boldsymbol{\omega}\times\boldsymbol{\omega}\times{}_{m}^{B}\boldsymbol{R}\,{}^{m}\boldsymbol{C}_i+\\
2\boldsymbol{\omega}\times{}_{m}^{B}\boldsymbol{R}\,{}^{m}\boldsymbol{v}_{Ci}+{}_{m}^{B}\boldsymbol{R}\,{}^{m}\boldsymbol{a}_{Ci},\\
\boldsymbol{\varepsilon}_{Ci}=\boldsymbol{\varepsilon}+\boldsymbol{\omega}\times{}_{m}^{B}\boldsymbol{R}\,{}^{m}\boldsymbol{\omega}_{Ci}+{}_{m}^{B}\boldsymbol{R}\,{}^{m}\boldsymbol{\varepsilon}_{Ci},\\
\boldsymbol{A}_{Ci}=\begin{pmatrix}\boldsymbol{E}&-\hat{\boldsymbol{e}}_{Ci}\\\boldsymbol{0}_{3\times3}&\boldsymbol{E}\end{pmatrix}\boldsymbol{A}+{}_{si}^{B}\boldsymbol{k}\,{}^{si}\boldsymbol{A}_{Ci}+\\
\begin{pmatrix}2\hat{\boldsymbol{\omega}}\,{}_{m}^{B}\boldsymbol{R}&\boldsymbol{0}_{3\times3}\\\boldsymbol{0}_{3\times3}&\hat{\boldsymbol{\omega}}\,{}_{m}^{B}\boldsymbol{R}\end{pmatrix}{}^{m}\boldsymbol{V}_{Ci}+\begin{pmatrix}\hat{\boldsymbol{\omega}}^2\boldsymbol{e}_{Ci}\\\boldsymbol{0}_{3\times1}\end{pmatrix}.
\end{gathered}
$$

5 Statics of 6-DOF PM with 3Q_i and 3 Fingers

Let $\boldsymbol{F}_a$ be the general active wrench applied to the 6-DOF PM with 3 planar limbs, F_{Li} be the active force of L_i in finger mechanism, $\boldsymbol{F}_{Ci}$ be workload wrench applied on C_i. Let $\boldsymbol{F}_C$ and $\boldsymbol{V}_n$ be the general workload wrench and general input velocity of 6-DOF PM with 3 planar limbs and 3 finger mechanisms. They are represented as follows:

$$
\boldsymbol{V}_n=\begin{pmatrix}\boldsymbol{V}_r\\v_{L1}\\v_{L2}\\v_{L3}\end{pmatrix},\boldsymbol{V}_r=\begin{pmatrix}v_{r11}\\v_{r12}\\v_{r21}\\v_{r22}\\v_{r31}\\v_{r32}\end{pmatrix},\ \begin{matrix}\boldsymbol{V}_L=\begin{pmatrix}v_{L1}\\v_{L2}\\v_{L3}\end{pmatrix},\\\boldsymbol{V}_C=\begin{pmatrix}\boldsymbol{V}_{C1}\\\boldsymbol{V}_{C2}\\\boldsymbol{V}_{C3}\end{pmatrix},\end{matrix}\ \boldsymbol{V}_{Ci}=\begin{pmatrix}v_{Cix}\\v_{Ciy}\\v_{Ciz}\\\omega_{Cix}\\\omega_{Ciy}\\\omega_{Ciz}\end{pmatrix},
\tag{28a}
$$

$$
\boldsymbol{F}_a=\begin{pmatrix}F_{r11}\\F_{r12}\\F_{r21}\\F_{r22}\\F_{r31}\\F_{r32}\end{pmatrix},\ \begin{matrix}\boldsymbol{F}_L=\begin{pmatrix}F_{L1}\\F_{L2}\\F_{L3}\end{pmatrix},\ \boldsymbol{F}_C=\begin{pmatrix}\boldsymbol{F}_{C1}\\\boldsymbol{F}_{C2}\\\boldsymbol{F}_{C3}\end{pmatrix},\\\boldsymbol{F}_{Ci}=\begin{pmatrix}\boldsymbol{f}_{ci}\\\boldsymbol{t}_{ci}\end{pmatrix}.\end{matrix}
\tag{28b}
$$

When neglected mass and inertia moment of moving links, the statics equation is derived based on the principle of virtual power and Eqs. (25), (32) as follows:

$$
\begin{gathered}
\boldsymbol{F}_a^{\mathrm{T}}\boldsymbol{V}_r+\boldsymbol{F}_L^{\mathrm{T}}\boldsymbol{V}_L+\boldsymbol{F}_C^{\mathrm{T}}\boldsymbol{V}_C=0,\\
\begin{pmatrix}(\boldsymbol{F}_a^{\mathrm{T}})_{1\times6}&F_{L1}&F_{L2}&F_{L3}\end{pmatrix}\boldsymbol{V}_n=-\begin{pmatrix}\boldsymbol{F}_{C1}^{\mathrm{T}}&\boldsymbol{F}_{C2}^{\mathrm{T}}&\boldsymbol{F}_{C3}^{\mathrm{T}}\end{pmatrix}\boldsymbol{J}_C\boldsymbol{V}_n,
\end{gathered}
$$

$$\boldsymbol{J}_C = \begin{pmatrix} \boldsymbol{J}_{C1\_V_r} & \boldsymbol{J}_{C1\_v_{L1}} & & \\ \boldsymbol{J}_{C2\_V_r} & & \boldsymbol{J}_{C2\_v_{L2}} & \\ \boldsymbol{J}_{C3\_V_r} & & & \boldsymbol{J}_{C3\_v_{L3}} \end{pmatrix}. \tag{29}$$

The formulae for solving $\boldsymbol{F}_a$ and $F_{Li}(i=1, 2, 3)$ are derived as below:

$$\boldsymbol{F}_a = -\sum_{i=1}^{3} \boldsymbol{J}^{\mathrm{T}}_{Ci\_V_r} \boldsymbol{F}_{Ci},\ F_{Li} = \boldsymbol{J}^{\mathrm{T}}_{Ci\_v_{L1}} \boldsymbol{F}_{Ci}. \tag{30}$$

6 Solved Examples of 6-DOF PM with 3 Planar Limbs and 3 Fingers

The pose parameters X_o, Y_o, Z_o, α, β, γ are given in Fig. 1. The geometric parameters of finger, the input velocity of PM, the workloads exerted on fingers are given in Table 1. A program is compiled in Matlab based on the derived relative analytic formulae in sections 3–5 and given parameters.

Table 1. Given parameters of finger mechanism, input velocity of PM and workloads applied on fingertips

Parameter	Value
l_s/mm	120
L_s/mm	240
l_C/mm	121.4
c/mm	42.83
$2g_1$, $2G_1$/mm	50, 80
l_Q/mm	71.5
l_F/mm	49.5
p_L/mm	34.4
e_o/mm	78.6
d/mm	128.28
$\boldsymbol{f}_{Ci}$/ N	$[0\ 0\ -1000]^{\mathrm{T}}$
$\boldsymbol{t}_{Ci}$/(N·m)	$[0\ 0\ 10]^{\mathrm{T}}$
v_{rij}	$14.4t$, $15t$, $5.2t$, $5.6t$, $7t$, $6t$
ϕ_1, ϕ_2, ϕ_3/(°)	180, 60, 60
v_{L1}, v_{L2}, v_{L3}/(mm • s^{-1})	2, 2, 1

The input displacement, velocity, acceleration of PM and finger mechanisms are given, see Fig. 4(a) and Table 1. The output displacement, velocity, acceleration of 6-DOF PM with 3 planar limbs are solved, see Fig. 4(b)–Fig. 4(d). The output displacements, velocity, acceleration of fingertip C_i are solved, see Fig. 5(a)–Fig. 5(d). The static active forces $F_{rij}(i=1, 2, 3; j=1, 2)$ of 6-DOF PM are solved, see Fig. 5(e). The static active forces $F_{Li}(i=1, 2, 3)$ of finger mechanisms are solved, see Fig. 5(f).

The characteristics of the proposed manipulator are analyzed based on solved results as follows.

(1) When $r_{ij}(i=1, 2, 3; j=1, 2)$ are varied within 280–550 mm(see Fig. 4a), the displacement components of m are varied within –100→220, –80→–270, 240→290 mm for X_o, Y_o, Z_o, respectively, see Fig. 4(b). The orientation components of m are varied within 40→170°, 18→5°, –50→–140° for α, β, γ, respectively, see Fig. 4b. It implies that the proposed manipulator has a quite large position and orientation workspace.

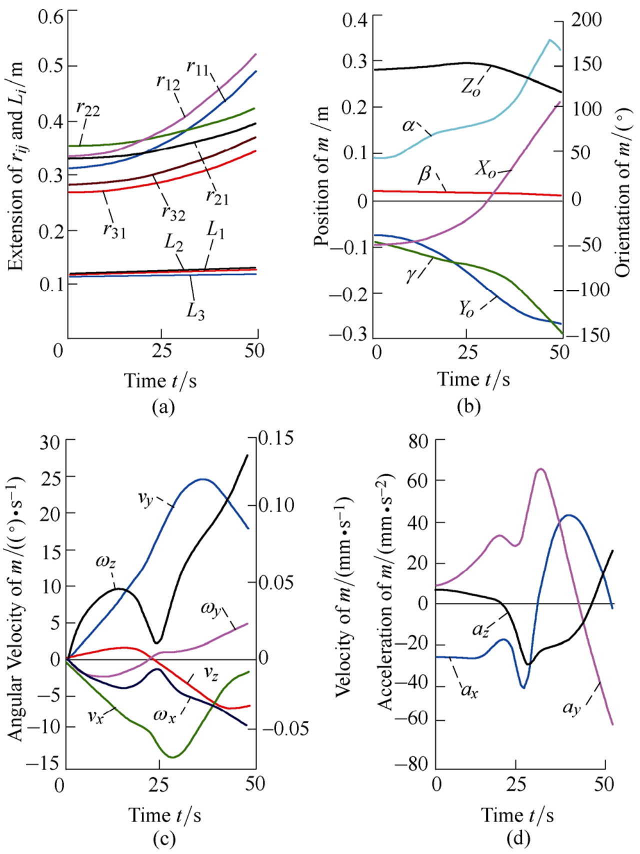


Fig. 4 Analytic kinematics solutions of PM with 3 planar limbs

(2) The displacement components of fingertip are much larger than that of m, see Fig. 5(a). It implies that the workspaces of fingers are much larger than that of m. The angular velocity components of fingertip are similar to that of m, see Fig. 5(d). It implies that the position workspaces of fingers are much larger than that of m, the orientation workspaces of fingers are similar to that of m.

(3) When the center o of m is close to Z, the static active forces of PM are lowered. When o is far from Z, the static active forces of PM are increased largely, see Fig. 5(e).

(4) When the displacement, velocity, and acceleration of active legs r_{ij} and L_i are varied smoothly, the displacement, translational velocity and acceleration of m and finger tips are varied smoothly in a large range; the orientations, angular velocity and acceleration of m are varied smoothly in a large range. The active forces of r_{ij} and L_i are varied smoothly in a large range. It implies that the proposed manipulator has good characteristics of kinematics and statics.

The analytic solutions are verified by the simulation solutions in advanced CAD software. The maximum absolute errors between analytic and simulation kinematics solutions of fingertips are shown in Table 2. The maximum relative errors between analytic and simulation active forces solutions are shown in Table 3. It is proved by the error solutions that the derived analytic formulae are correct.

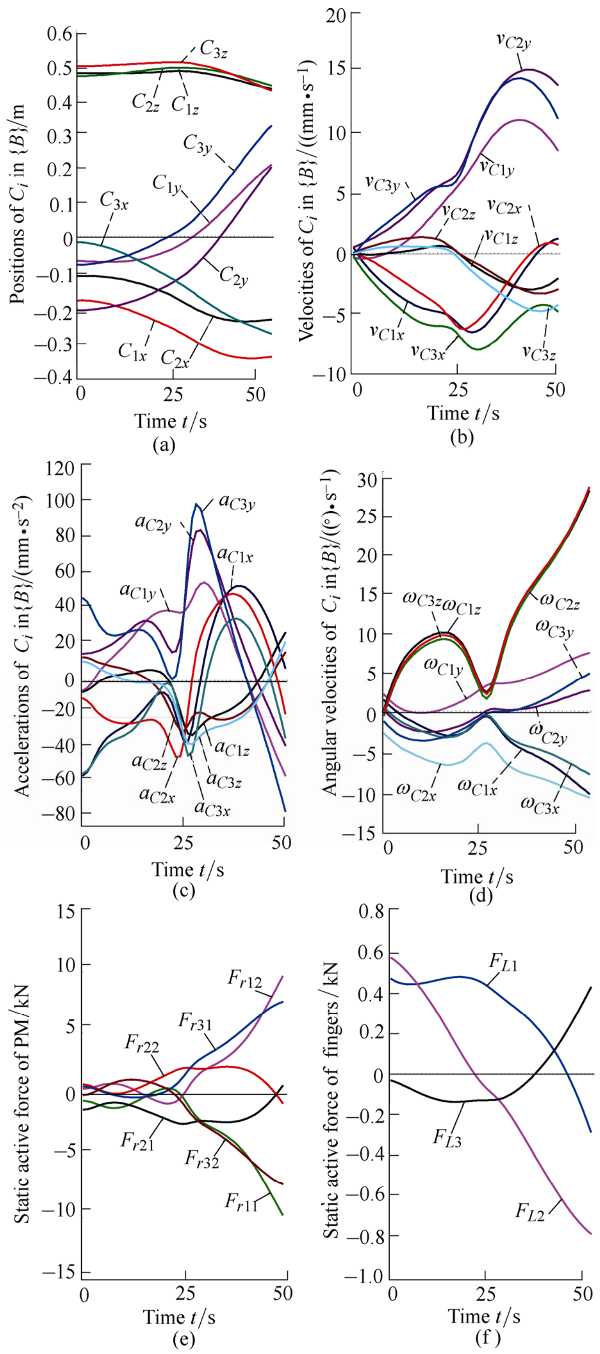


Fig. 5. Analytic solutions of kinematics and statics of 6-DOF PM with 3 finger mechanisms

Table 2. Maximum absolute errors between analytic and simulation kinematics solutions of fingertips

Absolute errors	i	1	2	3
Position/μm	ΔC_{ix}	0.401 1	0.000 6	0.063 2
	ΔC_{iy}	0.121 2	0.000 2	0.391 2
	ΔC_{iz}	0.133 4	0.000 1	0.161 3
Translational velocity/(μm • s^{-1})	Δv_{Cix}	4.30	0.81	8.01
	Δv_{Ciy}	1.65	2.32	1.04
	Δv_{Ciz}	2.25	0.72	0.28
Angular velocity/ 10^{-3}((°) • s^{-1})	$\Delta\omega_{Cix}$	1.72	2.40	0.03
	$\Delta\omega_{Ciy}$	2.61	1.96	0.15
	$\Delta\omega_{Ciz}$	2.22	0.83	0.11
Translational acceleration/ (μm • s^{-2})	Δa_{Cix}	3.81	0.51	6.19
	Δa_{Ciy}	1.39	1.38	3.18
	Δa_{Ciz}	0.55	1.17	1.88
Angular acceleration/ 10^{-4}((°) • s^{-2})	$\Delta\varepsilon_{Cix}$	1.52	1.12	0.61
	$\Delta\varepsilon_{Ciy}$	0.16	1.19	0.24
	$\Delta\varepsilon_{Ciz}$	0.85	0.89	0.14

Table 3. Maximum relative errors between analytic and simulation active forces solutions

Relative error of active forces of PM/%					
ΔF_{r11}	ΔF_{r12}	ΔF_{r21}	ΔF_{r22}	ΔF_{r31}	ΔF_{r32}
0.000 95	0.001 50	0.001 300	0.001 60	0.000 54	0.001 60

Relative error of active forces of fingers/%		
ΔF_{L1}	ΔF_{L2}	ΔF_{L3}
0.091	0.018	0.047

7 Conclusions

(1) A novel 6-DOF parallel manipulator(PM) with 3 planar limbs and equipped with 3 one-DOF fingers is proposed and its structure characteristics and merits are analyzed. The formulae for solving its kinetostatics are derived.

(2) When given the input displacement, velocity, acceleration of the proposed PM and 3 fingers, its output displacement, velocity, acceleration can be solved by using derived formulae. When given the workload applied on the fingertips, the coordinated static active forces applied on proposed PM can be solved using derived formulae. The analytic solutions of coordinated kinematics and statics for the proposed parallel manipulator are verified by its simulation solutions.

(3) The proposed PM has higher rigidity, more DOFs, and more room for arranging multi-finger mechanisms without interference among active legs and finger mechanisms. Each of active legs is only the subjected to a linear force along active leg, the active leg and has a large capability of load bearing.

(4) The proposed PM equipped with three finger mechanisms has potential applications for of forging operator, manufacturing and fixture of parallel machine tool, assembly cells, CT-guided surgery, health recover and training of human neck or waist, and micro–nano operation of bio-medicine, and rescue missions, industry pipe inspection. Theoretical formulae and results provide foundation for its structure optimization, control, manufacturing and applications.

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