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Constraint Force Analysis of Metamorphic Joints Based on the Augmented Assur Groups

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Abstract: In order to obtain a simple way for the force analysis of metamorphic mechanisms, the systematic method to unify the force analysis approach of metamorphic mechanisms as that of conventional planar mechanisms is proposed. A force analysis method of metamorphic mechanisms is developed by transforming the augmented Assur groups into Assur groups, so that the force analysis problem of metamorphic mechanisms is converted into the force analysis problems of conventional planar mechanisms. The constraint force change rules and values of metamorphic joints are obtained by the proposed method, and the constraint force analysis equations of revolute metamorphic joints in augmented Assur group RRRR and prismatic metamorphic joints in augmented Assur group RRPR are deduced. The constraint force analysis is illustrated by the constrained spring force design of paper folding metamorphic mechanism, and its metamorphic working process is controlled by the spring force and geometric constraints of metamorphic joints. The results of spring force show that developped design method and approach are feasible and practical. By transforming augmented Assur groups into Assur groups, a new method for the constraint force analysis of metamorphic joints is proposed firstly to provide the basis for dynamic analysis of metamorphic mechanism.

Keywords: metamorphic joints, constraint forces, augmented Assur groups, metamorphic mechanism

1 Introduction

The metamorphic mechanisms were proposed by DAI, et al[1], and were studied by the scholars all over the world. YAN, et al[2], presented topological representation of variable joints. The structural synthesis method of metamorphic mechanisms was developed by DAI, et al[3] using matrix operation of the configuration transformations. LIU, et al[4], proposed a new metamorphic way by utilizing the coupled links, the linkage relationship, and the characteristics of their kinematic pairs based on the research results on metamorphic ways of metamorphic mechanisms. DING, et al[5], proposed a new design concept of metamorphic mechanisms according to the research results on topological and metamorphic characteristics of metamorphic mechanisms with the identical links, symmetrical constructions, and assembly. WANG, et al[6], introduced a theoretical foundation and structural synthesis method of metamorphic mechanisms. LI, et al[7], developed the Assur group combinatory theory ,applied this concept to metamorphic mechanisms, and proposed the concept of augmented Assur group(AAG). The structural combinatorial theory based on the Assur group connects Assur group with the driving links, and/or other Assur group and the frame, which makes the synthesis process intuitive, convenient and practical. LI, et al[8], proposed an idea of metamorphic cyclogram and equivalent resistance gradient model of metamorphic mechanisms to investigate the relationship between the kinematic status, resistant forces and constraint forms of metamorphic joints, and to find a new design approach for practical constrained metamorphic mechanisms.

The general constrained metamorphic processes are performed by converting the multi-DOF source metamorphic mechanism into a single-DOF mechanism by constrains of metamorphic joints to form one of the working stages and to keep the working configuration. Then, release the former constrained metamorphic joint free and constrain other metamorphic joint to change the working configuration and form another working stage. The movement of constrained metamorphic joints of metamorphic mechanism is restricted and controlled by kinematic geometry arrangements, geometric and/or force constraints, designated profiles of links and/or joints in the metamorphic working configurations. It is the basic parameter in the design work that determining the constraint force change rules and values to satisfy the working configurations and sequences. The constraint

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forces of metamorphic joints are key parameters for the expected metamorphic processes.

The structural combinatorial theory based on the Assur group is a kind of structural design and force analysis approach of planar mechanisms. A classical approach for structural synthesis can be traced to ASSUR[9] and the concept of Assur group(AG) which made contributions in the field of structural synthesis and kinematic and dynamic analysis of planar mechanisms. MANOLESCU, VERHO, MRUTHYUNJAYS and SOHN, et al[10–13], studied the structural analysis of kinematic chains and mechanisms applying the concept of group, binary chains and dual graphs respectively. GALLETTI[14] presented a modular approach to planar linkage kinematic analysis, LI and HONG[15] proposed a method for computer identifying and modeling of Assur group of planar linkages. CHUENCHOM and KOTA[16] used adjustable dyads. CHU and CAO[17–19] used an improved graph to study type synthesis of 1-DOF 6-bar and 8-bar linkages, including the use of component group or module of the Assur group. Structural of planar mechanisms with dyads was designed by VEEGER[20]. TANG and SUN[21] introduced a method of computer-aided combination of Assur groups, and SHAI[22] proposed topological synthesis of planar mechanisms through Assur graphs.

The structure theory and analysis method based on AG (or dyad) is one of the mostly used ways of composing and analyzing of planar kinematic chains, and it can be easy computerized.

However, there is no proposed symmetric expression approach which can analysis the constraint forces of metamorphic joints with the way of conventional planar mechanisms up to now. The AAG will be degenerated into equivalent AG in the constrained metamorphic presses. On the other hand, the kinematic and dynamic analysis of metamorphic working configurations can be studied based on degenerated AG to provide a convenient and efficient way for the kinematic and dynamic analysis of metamorphic mechanism.

A systematic way for the constraint forces analysis of metamorphic joints was developed based on the AAG in this paper, and a unified approach and efficient way for the kinematic and dynamic analysis of metamorphic mechanism were provided as that of conventional planar mechanism.

2 Analysis Approach of Metamorphic Mechanisms Based on the AAG

2.1 Assur group

According to the structure theory of mechanisms based on AG, any planar mechanism is formed by adding an element of AG to driver link, the frame and/or the former AG . Class II Assur groups are mostly used in the planar mechanism design. Class II AG is formed by two binary links with three joints of revolute(R) type and prismatic(P) type. An AG of three revolute R-joints is called as RRR-element in the group. The five structural forms of class II AG are shown in Fig. 1.

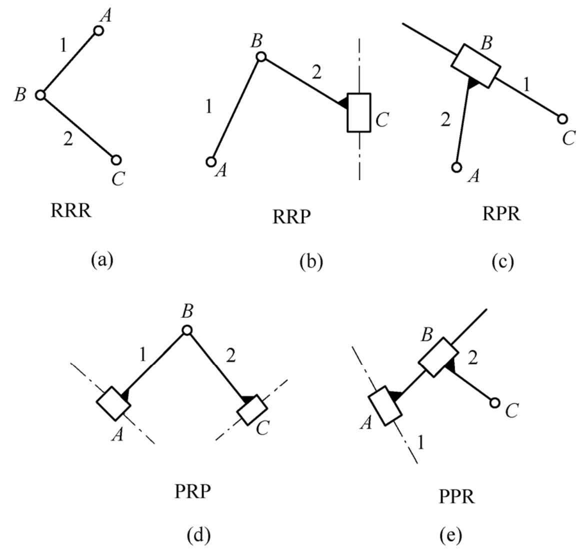

Fig. 1. Five structural forms of the class II Assur group

2.2 Class II augmented Assur group

If an additional binary link and a R/P joint are inserted into the class II AG, the mobility of the augmented group becomes one instead of the zero of class II AG which is called augmented Assur group of Class II. The nine structural forms of class II AAG are shown as in Fig. 2.

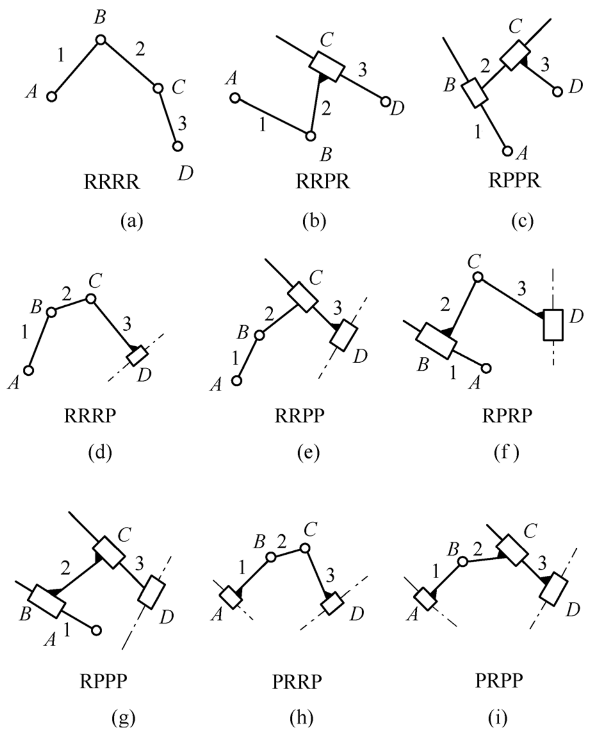

Fig. 2. Nine structural forms of class II augmented Assur group

One of the joints of AAG should be constrained during the corresponding metamorphic working stages of constraint metamorphic processes, and then the constrained AAG will be degenerated into AG[7].

2.3 Working processes of metamorphic mechanism

There is only one metamorphic joint keeps moving (working) at a time in corresponding working stage while the other(s) keeps static in the metamorphic process. The interchanges from moving to static of metamorphic joints are controlled by the constraints or constrained forces by the designed structure of metamorphic joints. Fig. 3 is a 2-DOF constrained metamorphic mechanism, which consists of driving link and an augmented Assur group (RRRP). To constrain the metamorphic joint C of augmented Assur group, the two links are annexed into a united one by the spring force. The augmented Assur group RRRP is degenerated into RRP group to form one of the working configurations as shown in Fig 4(a). When the mechanism of Fig 4(a) moves to the geometrical constraint point I, the metamorphic joint E is constrained and the constraint of joint C is released in the same time, thus the augmented Assur group RRRP is degenerated into RRR to form another working configuration as shown in Fig 4(b). Therefore, the kinematics and dynamics of multi-DOF constrained metamorphic mechanism can be analyzed by the Assur groups respectively in traditional ways.

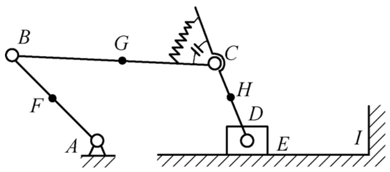

Fig. 3. 2-DOF constrained metamorphic mechanism

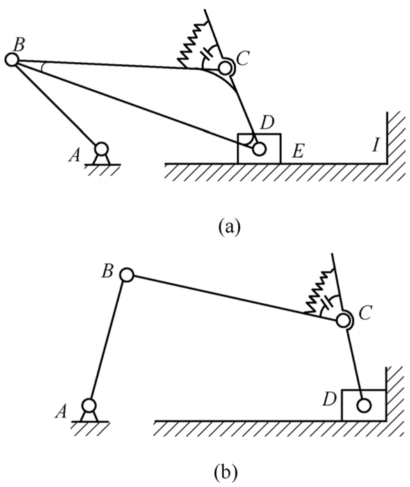

Fig. 4. Working-stage of the constrained metamorphic mechanism

3 Constraint Force Analysis of Metamorphic Joints

3.1 Constraint force analysis of revolute metamorphic joints of RRRR group

3.1.1 Force analysis of RRR Assur group

Force analysis sketch map of RRR group is shown in Fig. 5, points 5 and 6 are the centers of gravity of links (1) and (2) respectively. Reference coordinate frame, inertial forces and couples are also known in Fig. 5.

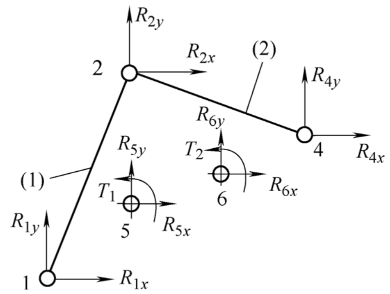

Fig. 5. Force analysis of the RRR group

The reaction forces of the R-joints are deduced by analysis of the group as follows:

$$\begin{aligned}
&R_{4x} = (-Ap_{42x} + BP_{41x})/C,\\
&R_{4y} = (-AP_{42y} + BP_{41y})/C,\\
&A = -(T_{51} + T_{61} + T_1 + T_2),\\
&B = -(T_{62} + T_2),\\
&C = P_{42x}R_{41y} - P_{42y}P_{41x},\\
&R_{2x} = -(R_{4x} + F_{6x}),\\
&R_{2y} = -(R_{4y} + F_{6y}),\\
&R_{1x} = -(R_{2x} + F_{5x}),\\
&R_{1y} = -(R_{2y} + F_{5y}).
\end{aligned} \tag{1}$$

Where R_{ix} and R_{iy} denote the reaction forces of joint i in x and y coordinates respectively. F_{ix} and F_{iy} denote the forces acted in point i in x and y coordinates respectively. The T_{ij} denotes the moment of point j to point i, and T_i denotes the moment acted in point i. P_{ijx} and P_{ijy} denote the distance between point i and point j in x and y coordinates respectively.

3.1.2 Force analysis of augmented Assur group RRRR

In the metamorphic process, the equivalent Assur groups from RRRR are shown in Fig. 6.

A typical structural form and force diagrams of the revolute metamorphic joint of RRRR are shown as in Fig. 7.

In Fig. 7, the positive direction of force is shown as in the positive direction of horizontal and vertical axes, and counterclockwise is positive for torque. Distance d between points 2 and 4 can be calculated by the geometric relationship

$$d = \sqrt{(P_{4x} - P_{2x})^2 + (P_{4y} - P_{2y})^2}, \tag{2}$$

and β denotes the angle between l_1 and l_2,

$$\cos\beta = \frac{l_1^2 + l_2^2 - d^2}{2l_1 l_2} = \frac{d_1^2 + d_2^2 - d_{mk}^2}{2d_1 d_2}. \tag{3}$$

d_{mk} can be calculated according to Eq. (3):

$$d_{mk}=\sqrt{d_1^2+d_2^2-\frac{d_1d_2(l_1^2+l_2^2-d^2)}{l_1l_2}}, \tag{4}$$

and θ denotes the angle between a straight line represented by points m and k and l_2,

$$\cos\theta=\frac{d_{mk}^2+d_1^2-d_2^2}{2d_{mk}l_1}, \tag{5}$$

$$\theta=\arccos\left(\frac{2d_1^2-\dfrac{d_1d_2(l_1^2+l_2^2-d^2)}{l_1l_2}}{2\sqrt{d_1^2+d_2^2-\dfrac{d_1d_2(l_1^2+l_2^2-d^2)}{l_1l_2}}\,d_1}\right). \tag{6}$$

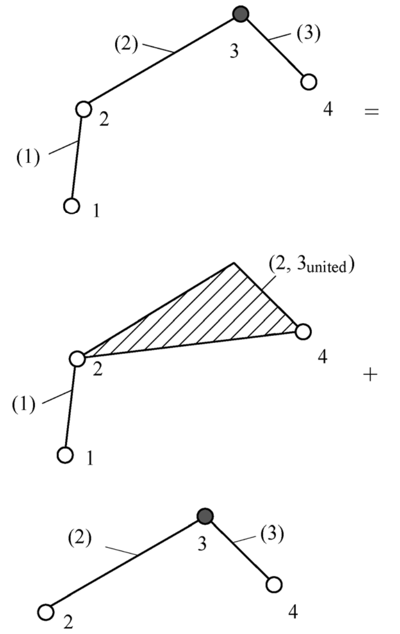

Fig. 6. Rigidization process of RRRR group

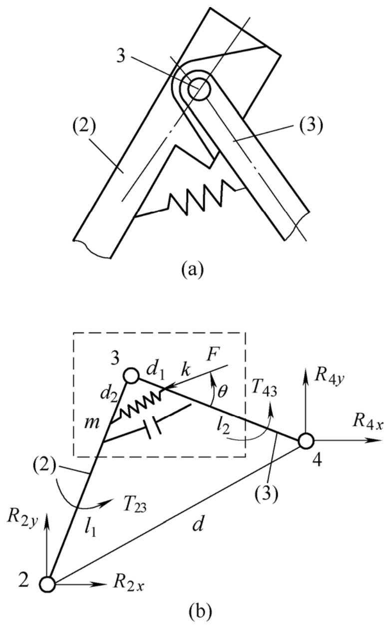

Fig. 7. Structure and force diagrams of the revolute metamorphic joint

The spring is fixed on point k of link (3) and point m of link (2), d_1 denotes the distance from point k to point 3, and d_2 denotes the distance from point m to point 3. θ denotes the angle between the direction of spring force F and link (3), T_{i3} denotes torque of the force on point i (i=2, 4) to point 3.

$$\begin{cases}T_{23}=P_{23}R_{2y}-P_{23y}R_{2x},\\ T_{43}=P_{43x}R_{4y}-P_{43y}R_{4x},\end{cases} \tag{7}$$

$$\Delta T=T_{23}-T_{43}. \tag{8}$$

When $\Delta T>0$, link (2) rotates counterclockwise relative to link (3) theoretically. Actually, links (2) and (3) maintain a constant angle due to the geometrical constraints in Fig. 7(a), and then the spring force F is equal to the initial value. When $\Delta T<0$, link (2) rotates clockwise relative to link (3) theoretically due to the release of the geometrical constraints.

$$F=\frac{-\Delta T}{d\sin\theta}. \tag{9}$$

The spring force F is

$$F=\frac{\left(P_{23x}R_{2y}-P_{23y}R_{2x}\right)-\left(P_{43x}R_{4y}-P_{43y}R_{4x}\right)}{d\sin\theta}. \tag{10}$$

3.2 Constraint force analysis of prismatic metamorphic joints of RRPR group

The equivalent Assur group from RRPR are shown in Fig. 8.

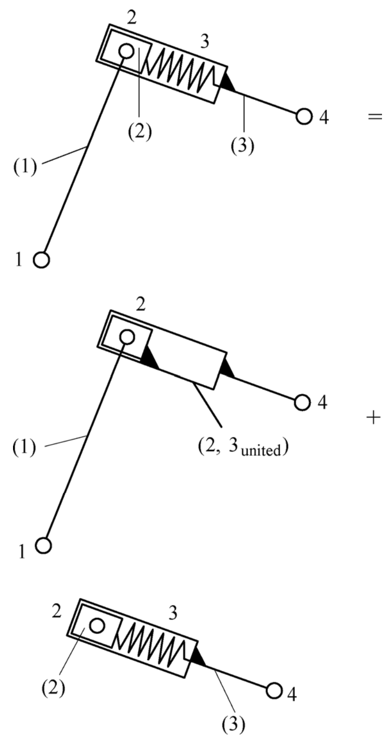

Fig. 8. Degenerated process of RRPR group

According to static analysis of dynamic force of RRR group, reaction forces (R_{2x}, R_{2y}, R_{4x}, R_{4y}) of revolute joints 2 and 4 are deduced shown as in Eq. (1). The structure and force diagrams of the prismatic metamorphic joint are shown as in Fig. 9.

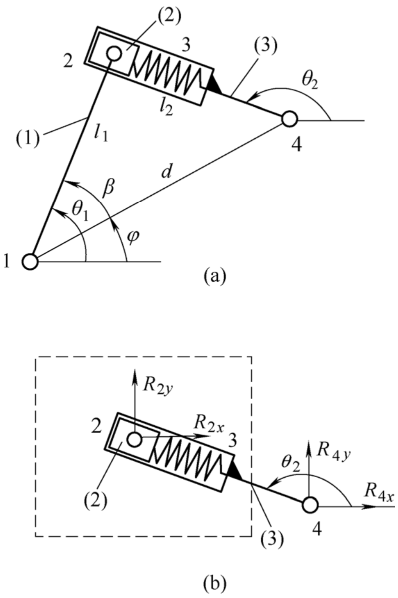

Fig. 9. Structure and force diagrams of the prismatic metamorphic joint

l_1 and l_2 denote the lengths of links (1) and (3) respectively, θ_1 and θ_2 are the orientation angle of links (1) and (3) respectively. The distance d between points 1 and 4 can be expressed in Eq. (11) by geometrical relationship:

$$d=\sqrt{\left(P_{4x}-P_{1x}\right)^2+\left(P_{4y}-P_{1y}\right)^2}, \tag{11}$$

$$\beta=\arccos\left(\frac{l_1^1+d^2-l_2^2}{2dl_1}\right), \tag{12}$$

$$\varphi=\arctan\left(\frac{P_{4y}-P_{1y}}{P_{4x}-P_{1x}}\right). \tag{13}$$

Where β denotes the angle between a straight line represented by points 1 and 4 and l_1, and φ denotes the angle between the line and horizontal axis.

$$\theta_1=\beta+\varphi. \tag{14}$$

The coordinate values of point 2 are

$$\begin{cases}P_{2x}=P_{1x}+l_1\cos\theta_1,\\ P_{2y}=P_{1y}+l_1\sin\theta_1.\end{cases} \tag{15}$$

The orientation angle θ_2 of link (2) is

$$\theta_2=\arctan\left(\frac{P_{2y}-P_{4y}}{P_{2x}-P_{4x}}\right). \tag{16}$$

When link 2 moves relative to link 3, the spring force is

$$\Delta F=\left(R_{4x}-R_{2x}\right)\cos\theta_2+\left(R_{4y}-R_{2y}\right)\sin\theta_2. \tag{17}$$

When $\Delta F>0$, links (2) and (3) have a relatively squeezing trend, so the spring force $F=\Delta F$; when $\Delta F<0$, links (2) and (3) have a motion trend of relative separation. Because of the geometrical constraints, links (1) and (2) keep relatively static, so the spring does not work at this time, i.e. the spring force $F=0$.

4 Resistance Parameters Design of Metamorphic Joints

4.1 Equivalent resistance gradient model

There is only one metamorphic joint keeps moving (working) at a time in corresponding working stage while the other(s) keeps static in the metamorphic process. The interchanges from moving to static of metamorphic joints are controlled by the constraints or constrained forces by the designed structure of metamorphic joints. A new idea of equivalent resistance coefficient is proposed to describe the working statuses and constraint resistance characteristics of the joints, also to compare the constrained forces between revolute and prismatic metamorphic joints. It is defined as: the ratio of the force and/or torque in moving direction provided by the constraint of the metamorphic joint to the force and/or torque in the moving direction acted on the metamorphic joint in the working process.

$$f_{\mathrm{e}}\left(\theta_i\right)=\frac{\boldsymbol{F}_{\mathrm{c}}\left(\theta_i\right)}{\boldsymbol{F}\left(\theta_i\right)}=\frac{\boldsymbol{T}_{\mathrm{c}}\left(\theta_i\right)}{\boldsymbol{T}\left(\theta_i\right)}, \qquad i=1,2,\cdots,m, \tag{18}$$

where $f_{\mathrm{e}}(\theta_i)$—Equivalent resistance coefficient of metamorphic joint,

θ_i—Displacement of driver in corresponding working stages,

m—Number of the working stages,

$\boldsymbol{F}_{\mathrm{c}}(\theta_i)$,$\boldsymbol{T}_{\mathrm{c}}(\theta_i)$—Resistant force and resistant torque in moving direction provided by the constraint of metamorphic joint,

$\boldsymbol{F}(\theta_i)$, $\boldsymbol{T}(\theta_i)$—Actual force and torque acted on the metamorphic joint in the moving direction during the working process.

The moving sequences of metamorphic joints should be proportional to the equivalent resistant forces according to the law of minimum resistance of kinematics. In the corresponding working stages, the equivalent resistance coefficient of moving metamorphic joint is smaller than that of static metamorphic joint, i.e. the equivalent resistance gradient of the metamorphic joints in the working stages of constrained metamorphic process should be

$$f_{\mathrm{em}}\left(\theta_i\right)\leqslant 1 \quad \text{and} \quad f_{\mathrm{es}}\left(\theta_i\right)\geqslant 1. \tag{19}$$

The equivalent resistance gradient sketch of 2DOF metamorphic mechanism can be formed as shown in Fig. 10 according to Eq. (19).

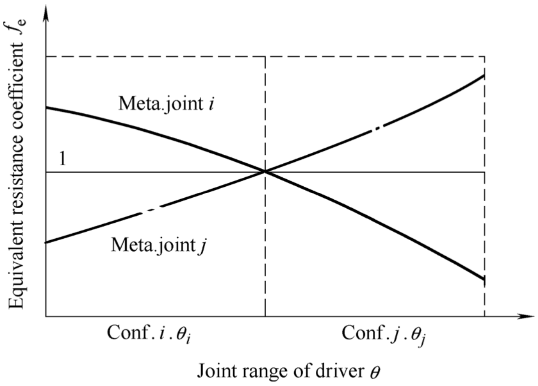

Fig. 10. Equivalent resistance gradient sketch of 2DOF metamorphic mechanism

4.2 Constrained metamorphic mechanisms for the paper folding process

Folding process diagram of a cardboard is shown in Fig. 11.

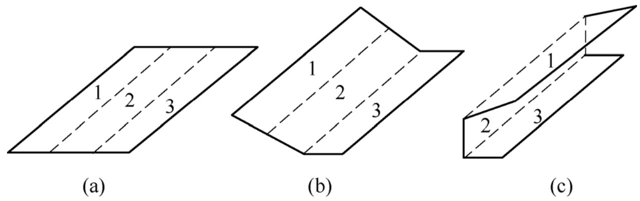

Fig. 11. Process of the folding paper

One of the structural diagram of 2DOF source constrained metamorphic mechanism, which can achieve the paper folding process, is shown as in Fig. 3. Joints *A*, *B* and *D* are general revolute joints. Joint *C* is revolute metamorphic joint with geometric constraint controlled by spring force and joint *E* is prismatic metamorphic joint with geometric constraint. After the spring force of metamorphic joint *C* constrains two links(*BC*, *CE*) to become a united one, the 5-bar metamorphic mechanism becomes a crank-slider mechanism which forms one working configuration. The slider keeps static when it moves to the geometric constraint I as shown in Fig. 3. Under the driving torque, constrained metamorphic joint *C* generates relative moving, the source metamorphic mechanism becomes into crank-rocker mechanism, which forms the other working configuration. Metamorphic working process is controlled by the spring force and geometric constraints of metamorphic joints *C* and *E*.

Configuration transforming of the paper folding metamorphic mechanism is shown in Fig. 12. In working configuration I, Joint *C* is revolute metamorphic joint with geometric constraint controlled by spring force, and the spring is under press when the mechanism starts moving. Metamorphic joint *C* constrains two links(*BC*, *CE*) to become a united one, which degenerates the 2-DOF source metamorphic mechanism into a single-DOF crank-slider mechanism with determined movement. During working configuration II, the slider will be fixed as a part of the frame when it moves to the geometric constraint I. Under the driving torque, links *BC* and *CD* generates relative moving, i.e. constrained metamorphic joint *C* changes into general revolute joint. The source metamorphic mechanism becomes into crank-rocker mechanism by constraining the relative moving of prismatic joint *E*. Horizontal and vertical folding of paper is achieved by the metamorphic process of metamorphic mechanism which includes configuration transforming and configuration keeping.

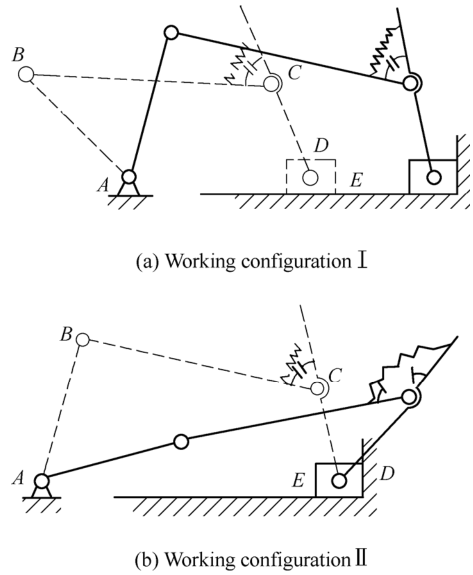

Fig. 12. Working-stage of the constrained metamorphic mechanism

4.3 Constraint force change of metamorphic joints

The moving sequences of metamorphic joints should be proportional to the equivalent resistant forces according to the law of minimum resistance of kinematics. In the working stages of constrained metamorphic process, the constraint force change of the metamorphic joints should satisfy the equivalent resistance gradient as shown in Fig. 10 according to Ref. [8]. The equivalent resistance gradient of the mechanism in Fig. 3 should satisfy the rules as shown in Fig. 13 according to Eqs. (18), (19) and Fig. 10.

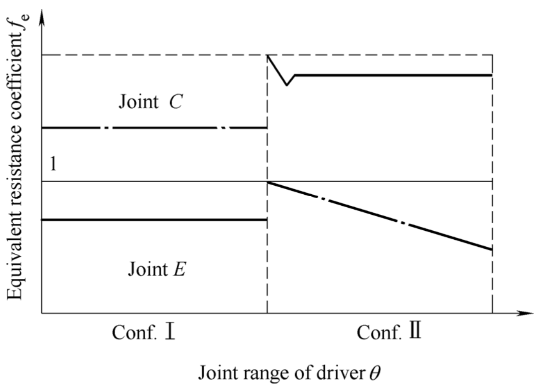

Fig. 13. Equivalent resistance gradient of the metamorphic joints

4.4 Spring force analysis of metamorphic joints

Kinematic parameters of the source metamorphic mechanism(Fig. 3) are shown in Table 1. Points *F, G, H* and *D* are the centers of gravity of links *AB, BC, CD* and slider respectively. *JAB, JBC* and *JCD* denote moments of inertia to the mass centers of links *AB, BC* and *CD* respectively. MinAE is the initial distance between points *A* and *E*, MaxAE is the distance between points *A* and *E* when the slider is constrained by geometric constraints as shown in Fig. 12(b). During the horizontal movement process of slider, a space fixed force acted on it towards horizontally leftward is represented as *F*, and *F*=5(N). The angular velocity and acceleration of driving link are denoted as ω_1 and ε_1 respectively, $\omega_1=180((°)/s)$ and $\varepsilon_1=0$.

Table 1. Kinematic and structural parameters of the source metamorphic mechanism

Parameter	Value	Parameter	Value
Length L_{AB}/mm	70	Length L_{AF}/mm	35
Length L_{BC}/mm	120	Length L_{BG}/mm	60
Length L_{CD}/mm	70	Length MinAE/mm	93
Length L_{CH}/mm	35	Length MaxAE/mm	151
Mass M_{AB}/g	1000	Mass slider/g	1000
Mass M_{CD}/g	1000	Mass M_{BC}/	1500
Mom.of ine. J_{BC}/(kg • m^2)	0.7	Mom.of ine. J_{AB}/(kg • m^2)	0.3
Mom.of ine. J_{CD}/(kg • m^2)	0.5		

For working configuration I, augmented Assur group RRRP degenerates into RRP group under the spring force and geometric constraint of metamorphic joint *C*. After overcoming the resistance of spring force, metamorphic joint *C* begins to move in working configuration II, the annexed links(*BC* and *CD*) converts into RRR group as shown in Fig. 14.

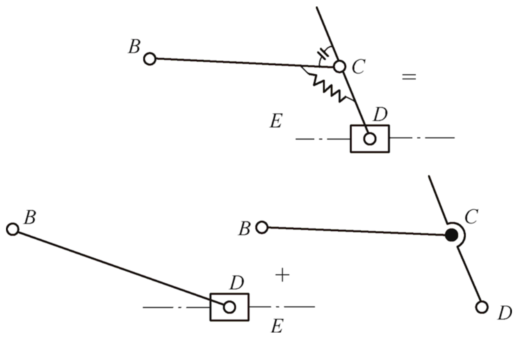

Fig. 14. Degenerating process of RRRP group

The kinematic analysis and static analysis for dynamics of degenerated Assur groups RRP are calculated by the basic groups analysis method. The displacement, velocity and acceleration curves of slider are shown in Fig. 15 during the working configuration I. *s*[4][1], *vp*[4][1] and *ap*[4][1] present displacement, velocity and acceleration curves of slider respectively.

The reaction forces of revolute joints *B* and *D* are shown as in Fig. 16(a) and Fig. 16(b) respectively. In Fig. 16, *fr*[*i*][1] and *fr*[*i*][2] (*i*=2, 4) denote reaction force in horizontal and vertical direction respectively.

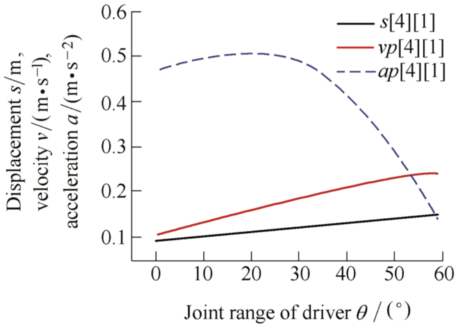

Fig. 15. Displacement, velocity and acceleration of the slider

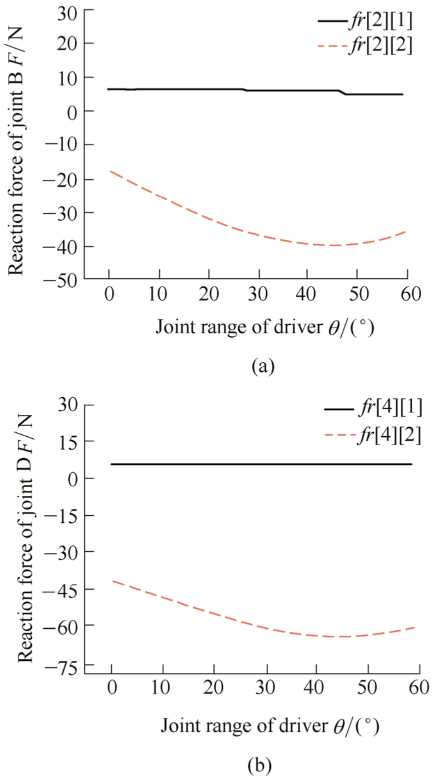

Fig. 16. Reaction force of revolute joints *B* and *D*

According to the Eqs. (2)–(8), the torque acted on metamorphic joint *C* is obtained shown as in Fig. 17. In which, T_1 denotes the torque of force acted on joint *B* to point *C* during the process of the horizontal folding paper of the mechanism, T_2 denotes the torque of force acted on joint *D* to point C, and $T_3=T_1-T_2$. When $T_3>0$, it indicates *BC* and *CD* squeeze each other.

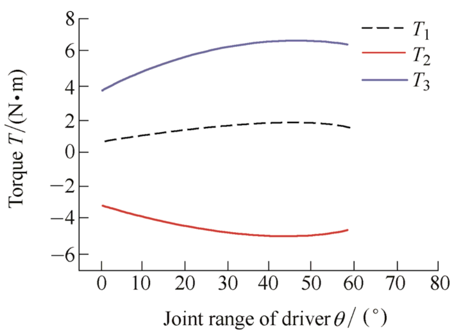

Fig. 17. Torque of revolute joint *C*

Based on the force analysis result, when the angular displacement of driving link is 46°, the torque value of T_3 reaches its maximum value T_{max}=6.641N/m. According to the maximum torque, spring force needed is designed to ensure the metamorphic joint *C* static for configuration keeping stage. In accordance with Eq. (9), the spring force is calculated as shown in Fig. 18. Corresponding to the maximum torque at 46° which is the angular displacement of driving link, the maximal spring force F_{max}=442.499 N is calculated theoretically. For practical application, bigger spring force value such as F_{max}=500 N is obtained to ensure safety according to Eq. (19). In Fig. 17 and Fig. 18, the driving joint angle starts from zero, which means the driving link *AB* starts turning form the horizontal position.

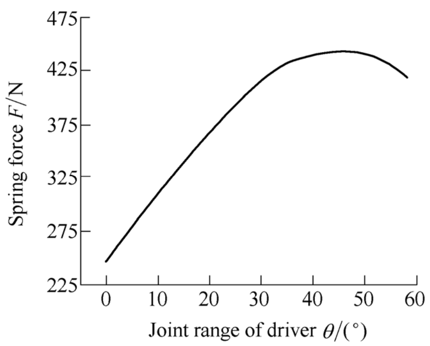

Fig. 18. Spring force diagram

Fig. 19 shows the relationship between the force *f* in moving direction provided by the constraint of metamorphic joint *E* and the force *fr*[4][1] in moving direction acted on joint *E* in the working configuration I. During the configuration keeping stage of the process of horizontal folding paper, metamorphic joint *E* keeps moving due to $f < fr[4][1]$.

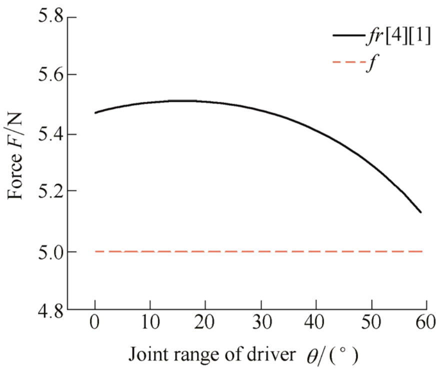

Fig. 19. Force characteristic of the slider

According to the resistance characteristic of metamorphic joints *C* and *E*, the equivalent resistance gradient sketch in working configuration I can be obtained. During working configuration II, geometric constraint of metamorphic joint *E* provides infinite constraint force, so whose equivalent resistance coefficient is calculated according to Eq. (18); resistant force provided by the constraint of metamorphic joint *C* is the spring force, which is determined on the stiffness and the deformation of the spring in the practical designing, and the equivalent resistance gradient sketch of metamorphic joint *C* and *E* in working configuration II can be obtained finally. Fig. 20 shows the resistance characteristic curve of metamorphic joint *C* and *E* in working configuration I and II. The metamorphic mechanism with the structural forms of constraint joints in Fig. 3 can achieve the practical metamorphic process according to Eq. (19).

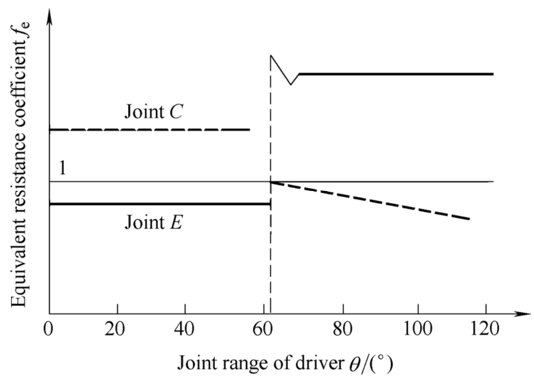

Fig. 20. Resistance characteristic curve of metamorphic joints

5 Conclusions

(1) A unified approach for the constraint force analysis of metamorphic joints of metamorphic mechanisms is developed by transforming the augmented Assur groups into Assur groups.

(2) The constraint force analyzing equations of augmented Assur group RRRR and RRPR are deduced from the Assur group formulations used in the conventional mechanisms to provide a convenient and efficient way for the kinematic and dynamic analysis of metamorphic mechanism.

(3) The constraint force analysis method is approved in the design of the paper folding metamorphic mechanisms.

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