

Large Deformation Analysis and Synthesis of Elastic Closed-loop Mechanism Made of a Certain Spring Wire Described by Free Curves

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Abstract: Recently novel mechanisms with compact size and without many mechanical elements such as bearing are strongly required for medical devices such as surgical operation devices. This paper describes analysis and synthesis of elastic link mechanisms of a single spring beam which can be manufactured by NC coiling machines. These mechanisms are expected as disposable micro forceps. Smooth Curvature Model(SCM) with 3rd order Legendre polynomial curvature functions is applied to calculate large deformation of a curved cantilever beam by taking account of the balance between external and internal elastic forces and moments. SCM is then extended to analyze large deformation of a closed-loop curved elastic beam which is composed of multiple free curved beams. A closed-loop elastic link is divided into two free curved cantilever beams each of which is assumed as serially connected free curved cantilever beams described with SCM. The sets of coefficients of Legendre polynomials of SCM in all free curved cantilever beams are determined by taking account of the force and moment balance at connecting point where external input force is applied. The sets of coefficients of Legendre polynomials of a nonleaded closed-loop elastic link are optimized to design a link mechanism which can generate specified output motion due to input force applied at the assumed dividing point. For example, two planar micro grippers with a single pulling input force are analyzed and designed. The elastic deformation analyzed with proposed method agrees very well with that calculated with FEM. The designed micro gripper can generate the desired pinching motion. The proposed method can contribute to design compact and simple elastic mechanisms without high calculation costs.

Keywords: elastic link mechanism, closed-loop, spring beam, large deformation analysis, smooth curvature model, force balance equation, synthesis

1 Introduction

In general, mechanisms in industrial assembling robots require high precision and high rigidity. However, there exist some mechanisms which require flexibility, compactness, lubrication free or low cost especially for medical devices, food machines and so on.

In order to realize such properties, some compliant mechanisms which utilize elastic deformation of links have been developed. Almost of them are based on topological optimization. For examples, FRECKER, et al^[1-2], and KOTA, et al^[3-4], assumed a mechanism as a truss structure and optimized arrangement of truss beam to be removed. TAI, et al^[5], optimized meshed structures with genetic algorithm. XU, et al^[6], also optimized arrangement of beams in skeleton structures. In these optimizations, FEM analysis is used to calculate elastic deformation of structure, therefore the calculation cost is high. And topological design essentially requires complex processing such as electrical discharge machining or chemical etching.

JANG, et al^[7], assumed a planar parallel mechanism composed of combinations of a rigid link and flexure hinges at both ends of the rigid link as standardizes elements and optimized properties of hinges. YI, et al^[8], also designed micro planar parallel manipulator with flexure hinges. However they took only deformations of hinges into account. WU, et al^[9-10], calculated stiffness and natural frequency of planar parallel manipulator. They also dealt with deformation of joints.

On the other hand, some researchers have dealt with large bending deformations of elastic links in mechanisms. BELENDEZ, et al^[11], analyzed the large deformation of cantilever beam based on exact differential equation of bending and experimentally validated the deformation. YU, et al^[12], and SU, et al^[13], used pseud-rigid-body model which approximated elastic link as rigid link and flexure hinges. MACCHELLI, et al, tried to derive general procedure to analyze deformation of mechanism with rigid and elastic links^[14]. NALLATHAMABI, et al^[15], analyzed large bending deformation of circular curved beams. However these analyses required complicated calculations.

One of authors has also proposed novel elastic link mechanism made of a certain spring wire^[16]. The elastic

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link mechanism shown in Fig. 1 is composed of straight beam part and torsional coil spring part and is manufactured from a certain spring wire with a continuous bending processing by an NC coiling machine as shown in Fig. 2. Therefore the elastic link mechanism can be mass-produced at low cost and is then disposable.

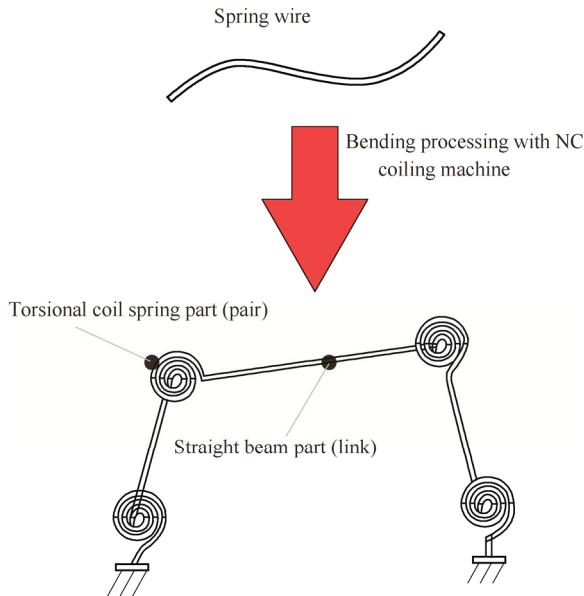


Fig. 1. Closed-loop elastic link mechanism made of a certain spring wire

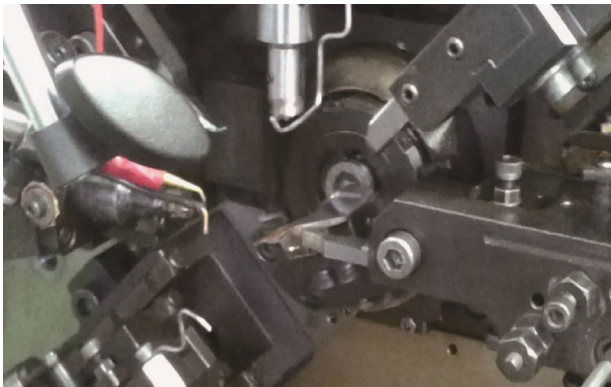


Fig. 2. NC coiling machine

The torsional coil spring part played a role of revolute pair and could be analyzed as circular arc beam. The straight beam part was assumed as Euler-Bernoulli beam and its large bending deformation was calculated with an iterative method. Resultantly the motion of the mechanism then could be analyzed. The synthesis method of the mechanism was also proposed and a planar micro gripper shown in Fig. 3 was then synthesized^[16].

However, the synthesized mechanism had a problem that it was easily deformed out of ideal plane due to thickness of torsional coil spring parts as seen in Fig. 3. And straight beam parts were not effective for large motion of the mechanism.

Thus in order to overcome the problem, we extend the elastic link mechanism made of a certain spring wire so as to compose of multiple free curved beams without torsional

coil springs. For this purpose, the Smooth Curvature Model^[17-18] is applied and extended so as to represent large deformation of free curved beams. The synthesis method with optimization is also proposed and examined.

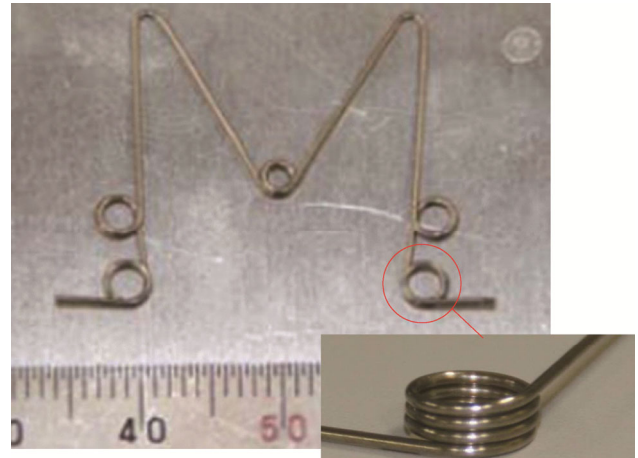


Fig. 3. Planar micro gripper with torsional coil springs and straight beams

2 Analysis of Closed-loop Elastic Link Mechanism of Free Curved Beams

2.1 Smooth curvature model

Let assume an elastic link mechanism as a closed-loop that is composed of multiple free curved cantilever beams serially connected. The Smooth Curvature Model(SCM) formulated by ODHNER, et al^[17-18] was adopted to analyze a large deformation of a free curved cantilever beam as shown in Fig. 4.

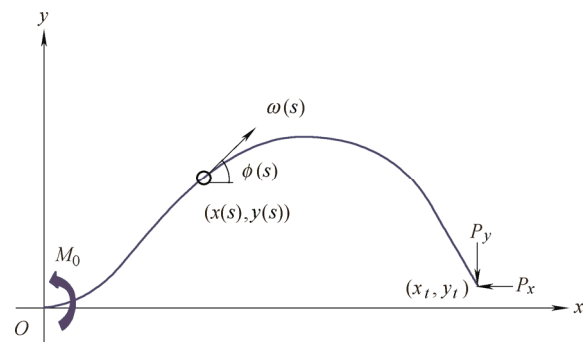


Fig. 4. Free curved cantilever beam

The curvature, ω , of the free curved cantilever beam with a length L can be represented as function of longitudinal position, s , as

$$\omega(s) = \omega^*(s) + \frac{M_0 + P_y x(s) - P_x y(s)}{EI}, \quad (1)$$

where ω^* is the curvature function of initial shape of the beam, M_0 is reaction moment at fixed position, $\mathbf{P}_E = (P_x \ P_y)^T$ is external force applied at the tip of beam, E is young's modulus, and I is moment of inertia of area. $\phi(s)$ is a

bending angle at point, $(x(s), y(s))$, on the beam. The strain energy, U , of the beam can be calculated as

$$U = \int_0^L \frac{M(s)^2}{2EI} ds = \int_0^L \frac{EI}{2} [\omega(s) - \omega^*(s)]^2 ds. \quad (2)$$

The curvature is then approximated with Legendre polynomials as

$$\omega(s) = \frac{\alpha_1}{L} + \frac{\alpha_2}{L} \left(\frac{2s}{L} - 1 \right) + \frac{\alpha_3}{L} \left(\frac{6s^2}{L^2} - 6 \frac{2s}{L} + 1 \right) + \dots \quad (3)$$

where $\alpha = (\alpha_1 \ \alpha_2 \ \alpha_3 \ \dots)^T$ is coefficient for Legendre polynomials. The curvature of initial shape of the beam is also represented with the coefficient for Legendre polynomials, $\alpha^* = (\alpha_1^* \ \alpha_2^* \ \alpha_3^* \ \dots)^T$ as

$$\omega^*(s) = \frac{\alpha_1^*}{L} + \frac{\alpha_2^*}{L} \left(\frac{2s}{L} - 1 \right) + \frac{\alpha_3^*}{L} \left(\frac{6s^2}{L^2} - 6 \frac{2s}{L} + 1 \right) + \dots \quad (4)$$

By substituting Eqs.(3) and (4) with terms by 3rd order into Eq.(2), the strain energy, U , can be easily calculated as^[5]

$$U = \frac{EI}{2L} \left[(\alpha_1 - \alpha_1^*)^2 + \frac{(\alpha_2 - \alpha_2^*)^2}{3} + \frac{(\alpha_3 - \alpha_3^*)^2}{5} \right]. \quad (5)$$

Since the increment of strain energy is equal to the increment of work given by external force, the following equation holds:

$$\nabla_{\alpha} U - \nabla_{\alpha} (\phi_t M) - \nabla_{\alpha} (x_t P_y) - \nabla_{\alpha} (y_t P_x) = 0, \quad (6)$$

where ∇_{α} denotes partial differentiation with coefficient for Legendre polynomials, α , and the bending angle and translational displacement at a point on the curved beam can be calculated with the following equations:

$$\phi(s) = \int \omega(s) ds, \quad (7)$$

$$x(s) = \int \cos \phi(s) ds, \quad (8)$$

$$y(s) = \int \sin \phi(s) ds. \quad (9)$$

Then Eq. (6) becomes a system of three nonlinear equations with respect to $\alpha_1, \alpha_2, \alpha_3$. By numerically solving Eq. (6) with Newton-Raphson method^[19], coefficients, $\alpha_1, \alpha_2, \alpha_3$ can be easily determined.

2.2 Multiple smooth curvature model

Because the Smooth Curvature Model can be applied only to a certain free curved cantilever beam, the model should be extended so as to calculate large deformation of closed-loop elastic link mechanism.

Fig. 5 shows a closed-loop elastic link mechanism which is assumed to be composed of n free curved beams where an external force, $P_E = (P_E \cos \theta_P \ P_E \sin \theta_P)^T$ acts at beam m . In order to apply the prementioned smooth curvature model, the closed-loop beam is divided to two curved cantilever beams at force acting point, (x_p, y_p) , as shown in Fig. 6. The smooth curvature model is applied to each beam, then the curvature of each beam is represented with three coefficients of after deforming. The strain energy of all beams is calculated with the coefficients, a system of nonlinear equations can then be derived. Also force, (P_x, P_y) , and moment, M , acting at (x_p, y_p) are assumed as new variables to balance force and moment of the closed-loop beam.

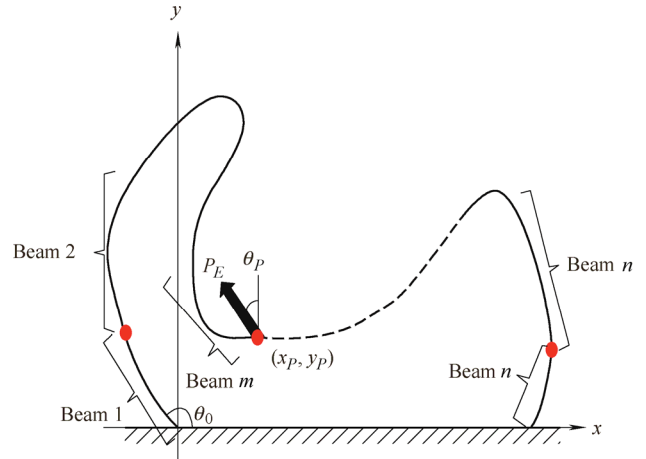


Fig. 5. Closed-loop elastic link mechanism composed of multiple free curved beams

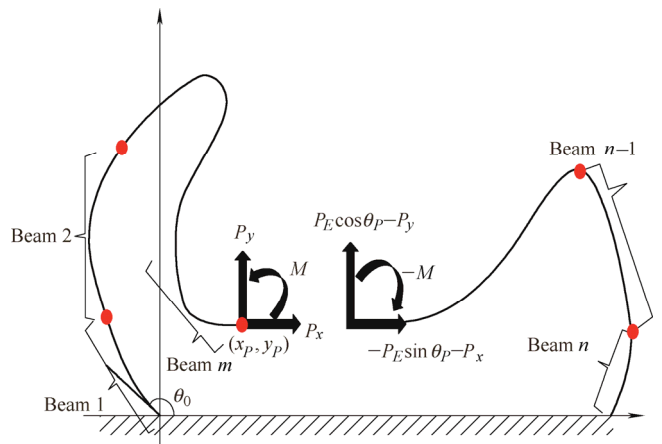


Fig. 6. Force and moment balance of divided free curved cantilever beams

From the balance of increment of strain energy and work by external force, a system of nonlinear equations for left side free curved cantilever beam composed of m free curved beams can be derived as follows:

$$\nabla_{\alpha} U_i - \nabla_{\alpha} (\phi_p M) - \nabla_{\alpha} (x_p P_y) - \nabla_{\alpha} (y_p P_x) = 0, \quad (i=1, \dots, m). \quad (10)$$

Another system of nonlinear equations for right side free curved cantilever beam composed of $n-m$ free curved beams can also be derived as

$$\nabla_{\alpha} U_i + \nabla_{\alpha} (\phi_p M) + \nabla_{\alpha} (x_p P_y) + \nabla_{\alpha} (y_p P_x) = 0, \quad (i=m+1, \dots, n). \quad (11)$$

Eqs. (10) and (11) denote $3n$ equations. Then boundary conditions on position, (x_p, y_p) , and posture θ_p of two beams at the dividing point are added. Resultantly, one can obtain a system of $3m+3$ nonlinear equations with $3m+3$ variables of α, P_x, P_y, M . By solving this system of equations with Newton-Raphson method^[19], one can determine the deformed shape of the closed-loop elastic link mechanism. We call this calculation models as the Multiple Smooth Curvature Model.

2.3 Example of analysis

For an example, a large deformation analysis of a planar micro gripper shown in Fig. 7 was carried out with the proposed method. The micro gripper is a closed-loop spring wire with symmetrical shape and is assumed to be composed of 4 curved cantilever beams. The specifications of free curved cantilever beams of left side of the micro gripper are listed in Table 1. The coefficients of Legendre polynomials for the initial shape without load are listed in Table 2.

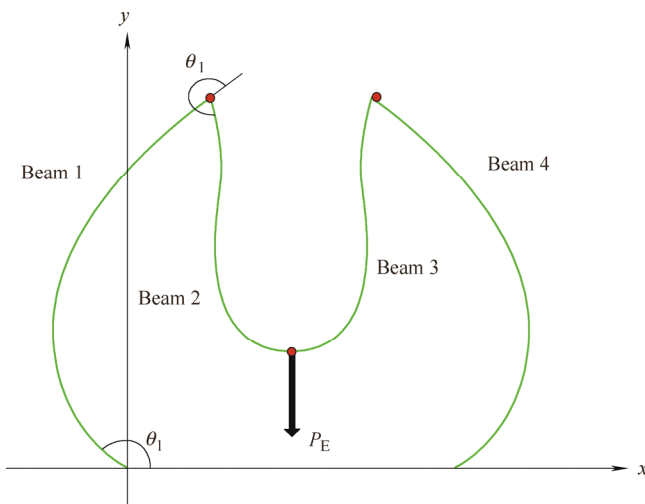


Fig. 7. A planar micro gripper to be analyzed

Table 1. Specifications of a planar micro gripper

L_1/mm	L_2/mm	θ_1/mm	θ_2/rad	d/mm	E/GPa	P/N
19.4	12.4	2.62	4.49	0.40	190	2.0

Table 2. Coefficients of Legendre polynomial in initial shape

	α_1^*	α_2^*	α_3^*
Beam 1	-1.96	1.61	0.00
Beam 2	1.13	4.08	0.00

Fig. 8 shows the result of large deformation analysis. Green line denotes the initial shape. Red line and blue plots denote the deformed shapes calculated with the proposed multiple smooth curvature model and finite element model, respectively. The result calculated with the proposed method agrees quite well with that calculated with FEM. It is thus confirmed that the proposed method is effective and useful.

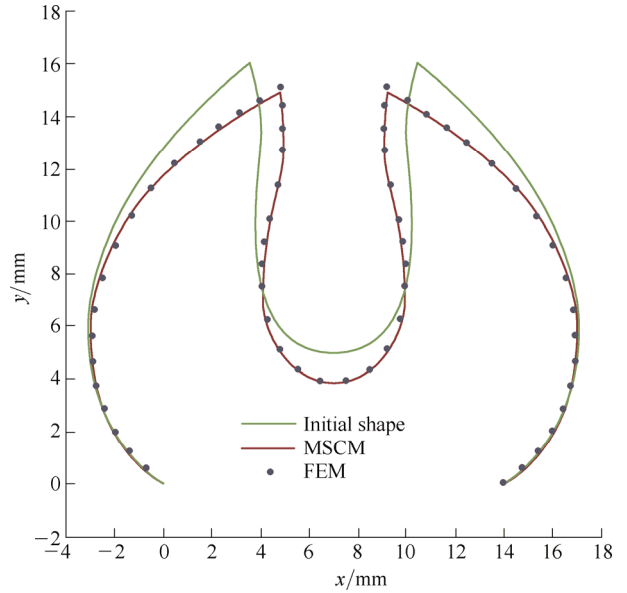


Fig. 8. Example of the analyzed deformation

2.4 Experiment

Fig. 9 is a photograph of a prototype of the planar micro gripper analyzed in section 2.3. The free curved beams in the gripper are approximated as circular beams and then are processed with an NC coiling machine.

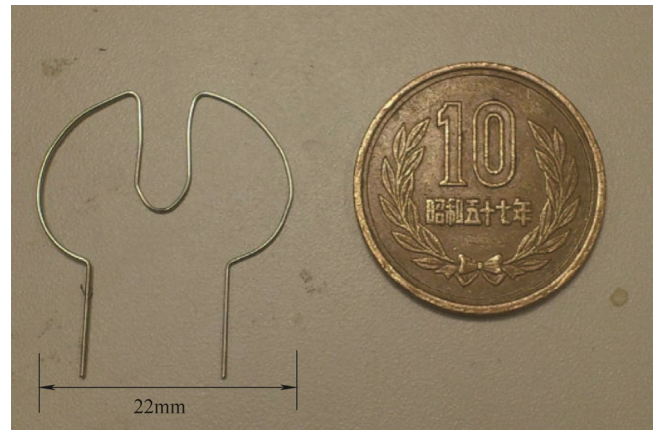
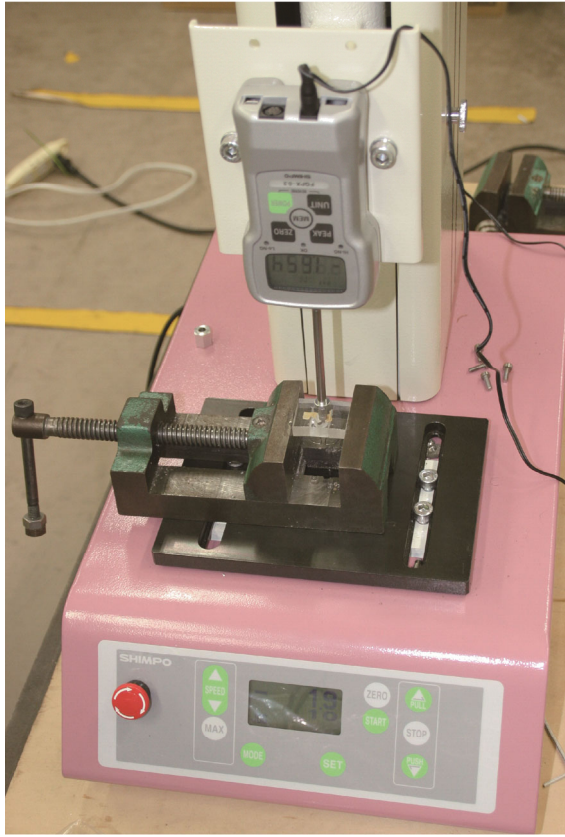


Fig. 9. Prototype of micro gripper

Straight parts at the both ends of the prototype are inserted in slits of an acrylic block as seen in Fig. 10(b). Then the block with the prototype is fixed with a vise as seen in Fig. 10(a). The relation between then input force, P_E , and the elastic displacement, Δy_{in} , at input point was measured with a digital force gauge with a hook and a precision linear positioner to give pulling force to the prototype as shown in Fig. 10(a).



(a) Force gauge and loading apparatus



(b) Elastic link mechanism fixed with an acrylic block

Fig. 10. Experimental setup

Fig. 11 shows the experimental result with the calculated one. Blue and red plots denote the measured results in loading and unloading conditions, respectively. A green line denotes the result calculated with the proposed multiple smooth curvature model.

The measured elastic displacement is larger than the calculated displacement. The theoretical displacement calculated with finite element model agreed quite well with that calculated with the proposed multiple smooth curvature model.

Therefore, this large error may be due to the difference between the theoretical model and the actual prototype. Although the both ends of the theoretical model, namely ends of circular beams, are directly fixed with a frame, the both ends of the prototype are connected with straight parts and then the straight parts are fixed with a frame. Therefore the bending stiffness at both ends of the prototype

decreases and the elastic displacement of the prototype becomes large.

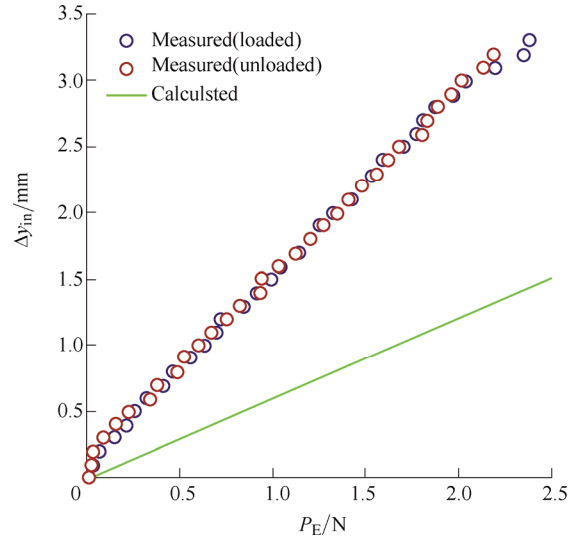


Fig. 11. Measured and calculated elastic displacement of a prototype

3 Synthesis of Closed-loop Elastic Link Mechanism

3.1 Synthesis of curved cantilever beam

Based on the proposed large deformation analysis, we tried to synthesize a closed-loop elastic link mechanism which can generate the desired output displacement. At first, a free curved cantilever beam was synthesized with optimization.

A straight cantilever beam is assumed as an original shape. Coefficients of Legendre polynomials and length of the beam, $\xi = (\alpha_1 \ \alpha_2 \ \alpha_3 \ L)^T$, are set as design variables. Of course, the design variables of the straight cantilever beam as original shape can be represented as $\xi = (0 \ 0 \ 0 \ L)^T$. The design variables are optimized so that the cantilever beam can generate the desired tip elastic displacement, Δy_{des} , in transverse direction due to applied force, P , in transverse direction. Position, (x_{tip0}, y_{tip0}) , and posture angle, ϕ_{tip0} , at the tip when force is not applied are also specified.

Therefore an objective function is set as

$$\Phi(\xi) = w_1 [x_{tip0} - x_{tip}(\xi)]^2 + w_2 [y_{tip0} - y_{tip}(\xi)]^2 + w_3 [\phi_{tip0} - \phi_{tip}(\xi)]^2 + w_4 [\Delta y_{des} - \Delta y(\xi)]^2, \quad (12)$$

where $x_{tip}(\xi), y_{tip}(\xi), \phi_{tip}(\xi), \Delta y(\xi)$, and $w_1 - w_4$ denote tip position, tip posture angle, the output displacement in transverse direction and weights, respectively, and they can be calculated with the smooth curvature model. The gradient method^[20] with Jacobian matrix obtained with difference approximation was used for optimization.

Fig. 12 shows one example of the calculated deformation of the obtained curved cantilever beam which is optimized to generate 0.15 m of output deformation for 1.0 N of applied force. As seen in the figure, the designed curved cantilever can generate the desired output displacement while tip position and posture angle keep zeros when load is not applied.

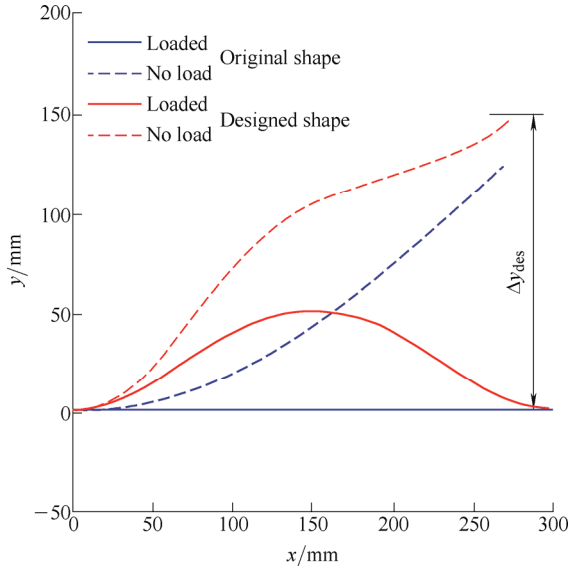


Fig. 12. Deformation of designed cantilever beam

3.2 Synthesis of closed-loop link mechanism of multiple curved beams

Next, closed-loop elastic link mechanism was synthesized as same as the synthesis of free curve cantilever beam mentioned in last section. For an example, we dealt with a planar micro gripper composed of 8 free curved beams. Fig. 13 shows the original shape of the mechanism to be optimized. In the figure, beams 1, 3, 6 and 8 are straight beams and beams 2, 4, 5 and 7 are circular arc beams. The gripper has symmetrical structure and the input force, P_E , is applied in $-y$ direction.

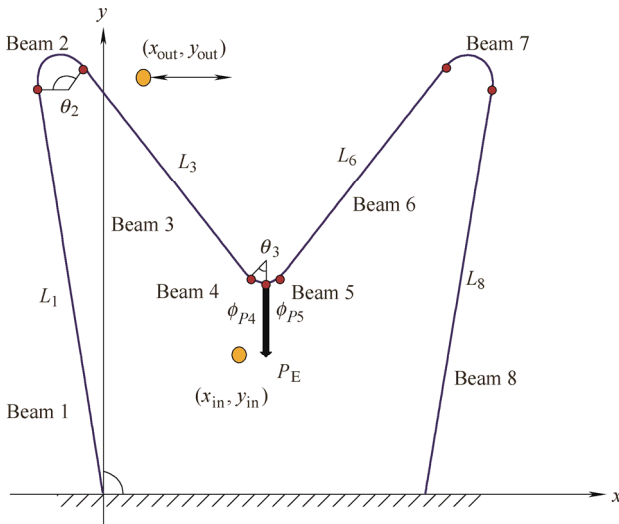


Fig. 13. Initial shape of planar micro gripper to be optimized

By assuming a tip of gripper as output point, the desired displacement of the output point is then specified. Beams 1, 3, 6 and 8 should be changed from straight beams to free curved beams while beams 2, 4, 5 and 7 keep their shape. Lengths L_1, L_3, L_6 and L_8 , and the coefficients, $\alpha_1^*, \alpha_3^*, \alpha_6^*$ and α_8^* for Legendre polynomial of the free curved beams are set as design variables. The square error between desired output displacement and actual output displacement which can be calculated with the proposed multiple smooth curvature model is set as an objective function. Gradient method with equality constraints that input point keeps its position at center of the mechanism and that the bending angle at the input point keeps zero was applied to optimize the design variables.

Resultantly, the objective function to be minimized is as follows:

$$\begin{aligned} \Phi(\xi) = & w_1 [x_{in0} - x_{in}(\xi)]^2 + w_2 [y_{in0} - y_{in}(\xi)]^2 + \\ & w_3 [x_{out0} - x_{out}(\xi)]^2 + w_4 [y_{out0} - y_{out}(\xi)]^2 + \\ & w_5 [\phi_{P4}(\xi) + \phi_{P5}(\xi) - \pi]^2 + w_6 [\Delta y_{des} - \Delta y(\xi)]^2, \end{aligned} \quad (13)$$

where ϕ_{P4} and ϕ_{P5} are tip angle of free curved beams 1–4 and 8–5, respectively.

Fig. 14 shows the deformation of the synthesized micro gripper. It was confirmed that the synthesized micro gripper could generate the desired output displacement that the tip of the gripper moves along a circular arc to the center of the gripper so as to pinch an object.

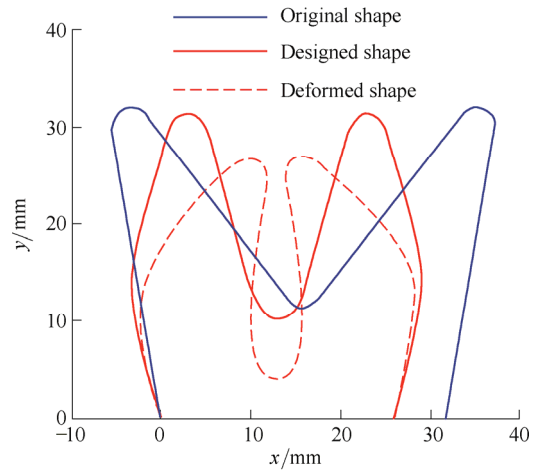


Fig. 14. Result of synthesis

4 Conclusions

(1) The large deformation of a free curved cantilever beam can be calculated with Smooth Curvature Model in which curvature is approximately represented as function of Legendre polynomials with respect to longitudinal length of the beam.

(2) An elastic link mechanism can be assumed as a closed-loop beam composed of multiple free curved

cantilever beams with a constant cross-section. Multiple Smooth Curvature Model is proposed and analyzed by solving a system of nonlinear equations with respect to coefficients of Legendre polynomials and reaction force and moment acting at connecting point.

(3) A micro gripper with a prismatic input is analyzed with the proposed method. The analyzed result agreed very well with that obtained with FEM analysis with iteration while experimental result measured with a prototype had large error due to modeling error at connecting point with large variation of curvature.

(4) The synthesis method to obtain initial shape of closed-loop elastic link mechanisms composed of multiple free curved beams by using optimization under the constraint of shape was proposed. A planar micro compliant gripper was then synthesized and confirmed that it could generate desired output motion with FEM large deformation analysis.

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