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# Novel Evaluation Method of Vehicle Suspension Performance Based on Concept of Wheel Turn Center

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**Abstract:** The current research of suspension performance evaluation is mixed in the evaluation of vehicle handling and ride comfort. However, it is lack of a direct and independent evaluation method for suspension performance. In this paper, a novel wheel turn center method is proposed to evaluate the suspension performance. This method is based on the concept and application of wheel turn center (WTC) and sprung mass turn center (SPTC). The vehicle body and each wheel are regarded to be independent rigid bodies and have their own turn centers which reflect respective steering motions and responses. Since the suspension is the link between vehicle body and wheels, the consistence between the sprung mass turn center and the wheel turn center reflects the effect and performance of the suspension system. Firstly, the concept and appropriate calculation method of WTC and SPTC are developed. Then the degree of inconsistence between WTC and SPTC and the time that they achieve consistence, when the vehicle experiences from transient steering to steady steering state, are proposed to evaluate suspension performance. The suspension evaluation tests are conducted under different vehicle velocities and lateral accelerations by using CarSim software. The simulation results show that the inconsistence of steering motion between vehicle body and wheels are mainly at high speeds and low lateral accelerations. Finally, based on the proposed evaluation indexes, the influences of different suspension characteristic parameters on suspension performance and their matches to improve steering coordination are discussed. The proposed wheel turn center method provides a guidance and potential application for suspension evaluation and optimization.

**Keywords:** suspension, evaluation, wheel turn center, sprung mass turn center

## 1 Introduction

In the vehicle pre-development stage, the evaluation and optimization of suspension system is a crucial part[1]. Since the suspension system is closely related to vehicle handling and ride comfort, the design of suspension is always a compromise and comprehensive event[2–3]. It must take into account of many factors for different functions, such as excellent vehicle handling or good ride comfort.

Many researchers have made great effort in the suspension design and optimization. SCHULLER, et al[4], optimized the key suspension parameters including spring stiffness, the damping of shock absorber, etc, by genetic algorithm under standard steering conditions. The evaluation criterions they used were handling indexes. HWANG, et al[5–6], conducted a design sensitivity analysis and optimization for suspension systems. A comprehensive weighted index which considered camber, toe, etc, was proposed to optimize suspension parameters. HABIBI, et al[7], optimized the suspension parameters by the target of suspension roll steer characteristics. To improve vehicle handling, many studies have been made on the optimization of suspension based on the target kinematic and compliance(K&C) characteristics. JIN[8] studied the ideal K&C characteristics based on the evaluation indexes of handing and ride comfort, then the suspension hard point and bushing stiffness were optimized. PRANAVA, et al[9], studied a twist beam suspension design method for the optimization of vehicle handling and rollover behavior. YONG-SUB, et al[10], developed a suspension design process by which K&C target curves can be achieved systemically and automatically. AHMADIAN, et al[11], presented the test results for evaluating kinematics characteristics of heavy truck suspensions, which provided a basis for the evaluation of suspension performance. However, the compliance characteristics were not concerned. Recently, with the development of semi-active and active suspension, WANG, et al[13–14], BOURMISTROVA, et al[15], and RAJAGOPAL, et al[16], studied the design and optimization method of them to improve ride comfort. Such researches mainly focused on the optimization of the controller parameters to minimize the objective function.

Since vehicle handling and ride comfort are affected by suspension system greatly, the evaluation of suspension

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performance is mixed in the evaluation of them. However, the direct and independent evaluation method for suspension performance hasn't been matured and consummated.

It is well-known that wheels are not fixed in the vehicle body and the suspension system is the link and coordination component between the vehicle body and wheels. Due to the effect of elastic components of the suspension system, the wheel and vehicle body which it links are different rigid bodies and they have their respective turn center. In this paper, the concept of wheel turn center (WTC) and sprung mass turn center (SPTC) are proposed and the indexes based on them are developed to directly evaluate and analyze suspension performance from the prospective of steering motion coordination, based on the consistence variation regularity between SPTC and WTC. The performance of suspension is reflected and evaluated by the degree of inconsistence between SPTC and WTC and the time that they achieve consistence when the steering wheel angle is unchanged.

## 2 Concept of Wheel Turn Center

### 2.1 Classic vehicle turn center

In the steering system design and handling analysis of the vehicle, the classic vehicle turn centers include the Ackermann turn center and vehicle turn center of the bicycle model[17], as shown in Fig. 1.

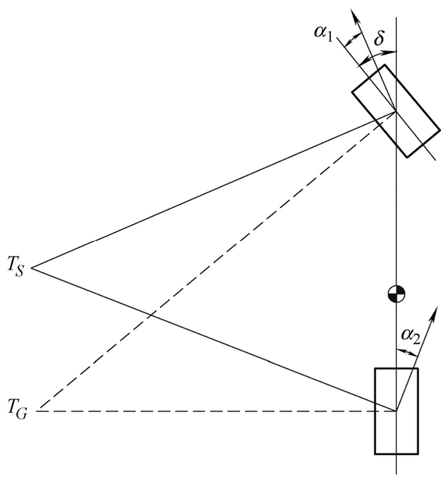


Fig. 1. Classic vehicle turn center

The Ackermann turn center ($T_G$) is shown in Fig. 2[18]. It is the intersection of the perpendiculars of all wheel rotational planes, which is on the projection of the rear axle centerline. Theoretically, the vehicle turns around the Ackermann turn center to reduce tire wear and improve steering motion coordination. However, it is the ideal vehicle turn center at low speeds since tire slip angles are ignored in the Ackermann principle.

Since tires are not rigid, the wheel rotational plane and the actual tire motion direction are not consistent. With the increase of vehicle speeds and lateral accelerations, tire slip angles occur. In the bicycle model, the vehicle turn center ($T_S$) is determined by the intersection of perpendiculars of wheel slip angles of the front and rear axle, as shown in Fig. 1. Therefore, when tires produce slip angles, $T_S$ and $T_G$ are not consistent.

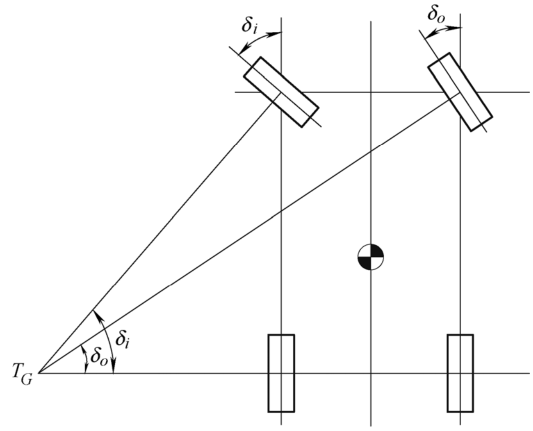


Fig. 2. Ackermann turn center

Since the bicycle model ignores the influences of the suspension system and steering system, in other words, the vehicle is assumed as a rigid body, it has limitations in evaluating the suspension performance.

### 2.2 Concept of instantaneous wheel turn center

Since there are many flexible components that cannot be ignored between the vehicle body and wheels, wheels and vehicle body are not an identical rigid body, and the wheel motions are influenced by suspension effects. Therefore, vehicle body and every wheel should be treated to be independent rigid bodies, and the vehicle body and each wheel have their own instantaneous turn center accordingly.

The concept of instantaneous WTC is proposed to comprehensively describe the independent motion of each wheel during steering. WTC is an extension of the classic vehicle turn centers introduced and shown in Fig. 1. The concept of WTC is based on the theory that the motions of wheels during steering are the spatial motion of a rigid body. When the instantaneous center of each wheel is projected in the ground *XY* plane, it is the defined instantaneous wheel turn center.

The wheel turn center can reflect the actual steering motion center of each wheel in real time. It has taken into consideration of the flexible components' influences on the wheel motion during steering, such as bushings, stiffness of the suspension guiding mechanism, et al. In the wheel turn center theory, the sprung mass is also treated to be an independent rigid body and has its own turn center. The consistence of SPTC and WTC reflects the effect and performance of the suspension system.

The comparisons of Ackermann turn center, vehicle turn center of the bicycle model and wheel turn center are shown in Fig. 3 where $C_{11}, C_{12}, C_{21}, C_{22}$ are each wheel turn center, respectively. Ackermann turn center is the center of steering mechanism kinematics. The turn center of the bicycle model considers tire side elasticity, but it has limitations in evaluating the suspension performance. WTC and SPTC are a conceptual extension of the traditional vehicle turn center. The steering motion of them can be described by their respective turn center, and the suspension performance can be analyzed and evaluated by their consistence characteristics.

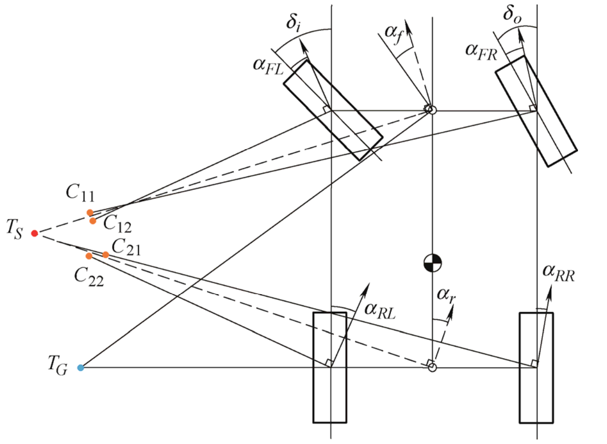


Fig. 3. Ackermann turn center, vehicle turn center of the bicycle model and wheel turn center

## 3 Calculation Methodology of Wheel Turn Center

It is well-known that for the planar motion of a rigid body, any points of a rigid body are rotated around a point whose velocity is zero, which is called the instantaneous center of velocities ($P$) [19], as shown in Fig. 4. Any points of the rigid body can be considered to rotate around $P$. In other words, the instantaneous center of velocities is the center of planar motion of the rigid body. Generally, the vehicle turns around the instantaneous center of velocities during steering[20].

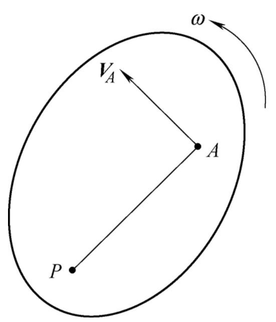


Fig. 4. Instantaneous center of velocities

A simple calculation method for WTC is the rigid body planar kinematics. In Fig. 4, $\omega$ is the angular speed of the planar motion of the rigid body; $V_A$ denotes a selected point on the rigid body. The instantaneous position of $P$ relative to $A$ can be easily determined by Eq. (1):

$$AP=\frac{\boldsymbol{V}_A}{\omega}, \tag{1}$$

If wheel steering motions are simplified as the planar motion of a rigid body, the instantaneous wheel turn center can be easily obtained by Eq. (1), as shown in Fig. 5. The plane $V$ is the reference planar motion of the wheel and passes through the wheel center ($C$).

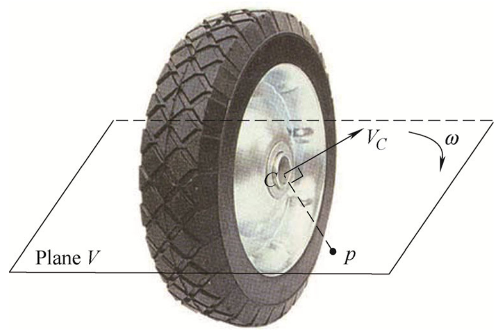


Fig. 5. Wheel turn center by planar motion kinematics

However, a wheel has roll and pitch motions which cannot be ignored. Using the rigid body spatial kinematics to calculate the wheel turn center is more accurate than planar kinematics. According to the classical mechanics, the instantaneous center of the spatial rotation of a rigid body is an instantaneous axis[21–22]. It always parallels to the spatial angular speed, so its position and direction are influenced by rotation motions of the rigid body. Here the wheel center ($C$) is chosen as the reference point to determine the position of instantaneous center, which can be calculated by rigid body spatial kinematics, as shown in Fig. 6. The instantaneous axis is a straight line on the rigid body and the velocities of its arbitrary points are equal[21]. The spatial velocity of any points on the rigid body can be easily calculated once the position of instantaneous axis is determined[22], so the instantaneous axis is the center of spatial motions of the rigid body. In the following, how to determine WTC based on rigid body spatial kinematics is introduced.

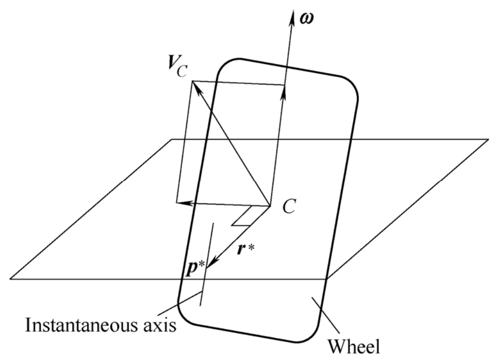


Fig. 6. Instantaneous axis

The spatial velocity of an arbitrary point of a rigid body can be expressed as below:

$$\boldsymbol{V}=\boldsymbol{V}_C+\boldsymbol{\omega}\times\boldsymbol{\rho}, \tag{2}$$

where $\boldsymbol{\omega}$ is the spatial angular speed of the rigid body; $\boldsymbol{V}_C$ is the spatial velocity of a selected reference point on the rigid body. In this paper, the rim center ($C$) or the sprung mass center are selected and they are respective reference points. $\boldsymbol{\rho}$ is the position vector from $C$ to any points of the rigid body.

Since the velocity of the instantaneous axis is parallel to $\boldsymbol{\omega}$, if existing a point $\boldsymbol{p}^*$ whose velocity $\boldsymbol{V}^*$ is parallel to $\boldsymbol{\omega}$, Eq. (3) can be obtained:

$$\boldsymbol{\omega}\times\boldsymbol{V}^* = \boldsymbol{0}. \tag{3}$$

Thus $\boldsymbol{r}^*$ is the position vector of the instantaneous axis, $\boldsymbol{\rho} = \boldsymbol{r}^*$.

Eq. (3) can be easily rewritten as Eq. (4) by Eq. (2):

$$\boldsymbol{\omega}\times\boldsymbol{V}^* = \boldsymbol{\omega}\times\boldsymbol{V}_C + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}^*) = \boldsymbol{0}. \tag{4}$$

Obviously, $\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}^*) = w\bullet\boldsymbol{r}^*\omega - \boldsymbol{\omega}^2\boldsymbol{r}^*$. Eq. (4) can be rewritten into Eq. (5) accordingly:

$$\boldsymbol{\omega}\times\boldsymbol{V}_C + \boldsymbol{\omega}\bullet\boldsymbol{r}^*\boldsymbol{\omega} - \boldsymbol{\omega}^2\boldsymbol{r}^* = \boldsymbol{0}. \tag{5}$$

Since it is needed to determine the instantaneous axis position corresponding to reference point $C$, $\boldsymbol{r}^*$ is in the plane which passes through $C$ and is perpendicular to $\boldsymbol{\omega}$ ($\boldsymbol{\omega}\bullet\boldsymbol{r} = \boldsymbol{0}$). Finally, $\boldsymbol{r}^*$ can be obtained by Eq. (6). The position point $\boldsymbol{p}^*$ is the determined instantaneous wheel turn center.

$$\boldsymbol{r}^* = \frac{\boldsymbol{\omega}\times\boldsymbol{V}_C}{\boldsymbol{\omega}^2}. \tag{6}$$

In order to analyze the distribution of SPTC and WTC, specifying a reference plane is needed, and all wheel turn centers are expressed in the reference plane. The reference plane must be fixed in the vehicle and cannot be influenced by vehicle roll and pitch motion. The $XY$ plane of intermediate axis system ($IAS$) defined in SAE J670 is chosen as the reference plane for WTC and SPTC. It is a right-handed orthogonal axis system whose $X$ and $Y$ axes are parallel to the ground plane and its coordinate origin coincides with the origin of vehicle coordinate system. Fig. 7 shows the position of the wheel turn center presented in the reference plane. $\boldsymbol{p}^*$ is the position point of the instantaneous axis corresponding to the reference point. $\boldsymbol{r}_{OP}{}^r$ and $\boldsymbol{r}_{OP}{}^e$ represent the position vector of the instantaneous axis expressed in the intermediate axis system and the earth-fixed coordinate system, respectively. $p$ is the final coordinate of the wheel turn center in the reference plane ($XY$ plane of $IAS$), which can be obtained by the projection of $\boldsymbol{r}_{OP}{}^r$ in the reference plane. Therefore, once $\boldsymbol{r}_{OP}{}^r$ is obtained, the coordinate of the wheel turn center can be determined.

The calculation of $\boldsymbol{r}_{OP}{}^r$ is introduced as follows. If all motion vectors of the wheel are expressed in the earth-fixed coordinate system, $\boldsymbol{r}_{OP}{}^r$ can be determined by coordinate transformation between the earth-fixed coordinate system and the intermediate axis system on the basis of the calculated $\boldsymbol{r}_{OP}{}^e$.

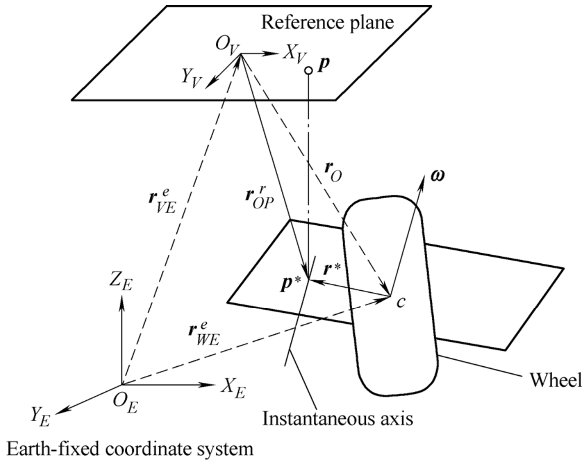


Fig. 7. Wheel turn center in the reference plane

The relationship between $\boldsymbol{r}_{OP}{}^r$ and $\boldsymbol{r}_{OP}{}^e$ can be obtained by Eq. (7):

$$\boldsymbol{r}_{OP}{}^r = \boldsymbol{A}^{re}\boldsymbol{r}_{OP}{}^e, \tag{7}$$

where $\boldsymbol{A}^{re}$ is the direction cosine matrix, $\boldsymbol{A}^{re} = (\boldsymbol{A}^{er})^{-1}$. $\boldsymbol{A}^{er}$ is the direction cosine matrix in which $IAS$ is relative to the earth-fixed coordinate system, as shown in Eq. (8) where $\psi, \varphi, \theta$ represent the Euler angle (roll, pitch and yaw angle of the vehicle), respectively. $s\psi$ means $\sin\psi$ and $c\psi$ means $\cos\psi$, respectively.

$$A^{er} = \begin{bmatrix} c\psi c\theta & -s\psi c\varphi + c\psi s\theta s\varphi & s\psi s\varphi + c\psi s\theta c\varphi \\ s\psi c\theta & c\psi c\varphi + s\psi s\theta s\varphi & -c\psi s\varphi + s\psi s\theta c\varphi \\ -s\theta & c\theta s\varphi & c\psi c\varphi \end{bmatrix}. \tag{8}$$

And $\boldsymbol{r}_{OP}{}^e$ in Eq. (7) can be calculated by Eq. (9):

$$\boldsymbol{r}_{OP}{}^e = \boldsymbol{r}_O{}^e - \boldsymbol{r}^*, \tag{9}$$

where $\boldsymbol{r}^*$ is the position vector of the instantaneous axis relative to $C$ in the earth-fixed coordinate system, which is calculated by Eq. (6); $\boldsymbol{r}_O{}^e$ is the relative position vector between the wheel reference point and the vehicle reference point in the earth-fixed coordinate system, which can be calculated by Eq. (10):

$$\boldsymbol{r}_O{}^e = \boldsymbol{r}_{VE}{}^e - \boldsymbol{r}_{OE}{}^e, \tag{10}$$

where $\boldsymbol{r}_{VE}{}^e$ is the position vector of vehicle reference point in the earth-fixed coordinate system; $\boldsymbol{r}_{WE}{}^e$ is the position vector of the wheel reference point in the earth-fixed coordinate system. Both of them can be obtained from the vehicle model directly.

Finally, based on Eqs. (6)–(10), $\boldsymbol{r}_{OP}{}^{r}$ can be calculated by Eq. (7), thus the position and coordinate of wheel turn center can be obtained. The sprung mass turn center can also be calculated by this method above accordingly, and only the reference point for calculation is replaced by the sprung mass center.

## 4 Method and Indexes of Suspension Performance Evaluation

It can be seen from the previous analysis that each wheel and vehicle body turn around their respective turn center during steering. If WTC and SPTC are quite far away from each other during steering, their steering motion and trajectory are quite different. In other words, the steering motion of the vehicle is not coordinated, which will have a negative influence on the whole vehicle handling and cause abnormal wear of suspension components. Since the suspension system is the link between the wheels and the vehicle body, the suspension characteristics influence the wheel motions and tire forces, thereby affecting the whole vehicle steering motion response. Therefore, the suspension design plays a crucial role in improving steering coordination performance.

Since the effect and quality of suspension are reflected by the response of vehicle and wheel motions, this paper proposes the suspension evaluation indexes and methodology based on the consistence of WTC and SPTC from the perspective of steering coordination. SPTC represents the steering performance response of high energy vehicle body platform. Accordingly, WTC represents the steering performance response of chassis platform, as shown in Fig. 8. The consistence reflects the function and effect of the suspension system.

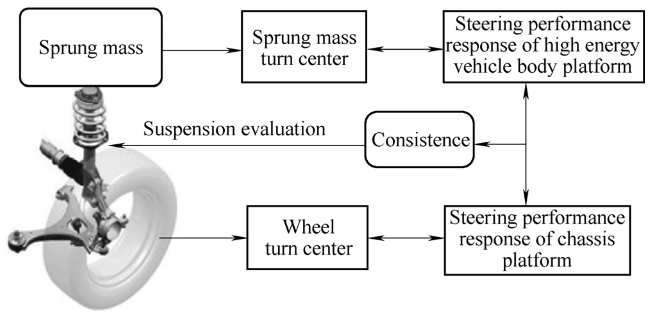


Fig. 8. Suspension evaluation basis

During transient steering, due to the relative motions between the vehicle body and wheels and influences of suspension elastic components, the vehicle body and each wheel are distinct rigid bodies and have their respective turn center accordingly. The WTC and SPTC are inconsistent. When the vehicle is in a steady steering state, the position of vehicle body and wheels is relatively fixed. The whole vehicle can acceptably be regarded as a rigid body, therefore, the turn centers of vehicle body and wheels are consistent in steady states.

The simulation studies have validated the analysis above. Based on the *E* class sedan in CarSim, a step steer test is conducted. The steering wheel angle is shown in Fig. 9 and the vehicle speed is 90 km/h. When the vehicle is in steady state, the speed and the angular speed of each wheel are unchanged. Therefore, the turn centers of each wheel are consistent. It will take some time for the vehicle to achieve steady state. The time experienced is transient steering process and the WTC, SPTC are disperse. Fig. 10 shows the distributions of sprung mass turn center and each wheel turn center in the reference plane. L1, L2, R1, R2, CM denote the left front wheel, left rear wheel, right front wheel, right rear wheel, sprung mass, respectively. It can be seen that the inconsistence between wheel turn centers and sprung mass turn center is large during transient steering. When the vehicle is in the steady steering state, all wheel turn centers and sprung mass turn center achieve consistence. In other words, from the transient steering to steady steering, each wheel turn center and sprung mass turn center will experience the process from inconsistence during transient steering to consistence when the vehicle achieves steady steering state, as shown in Fig.10. From this figure, it can also be seen that there exists a time interval when all turn centers are converged and consistent. The time interval reflects the speed that steering motion of vehicle body and wheels required for achieving consistence.

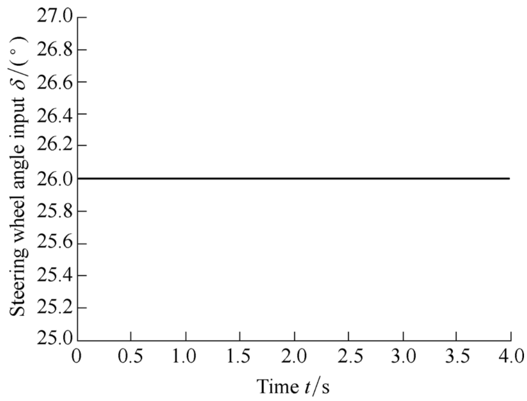


Fig. 9. Steering angle input

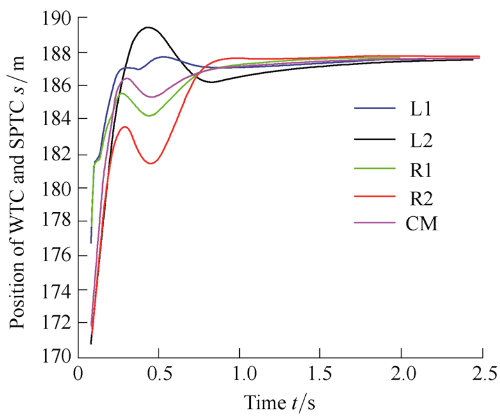


Fig. 10. The calculated SPTC and each WTC in *Y* direction

It can be concluded from above that the suspension coordination ability for the steering motion of wheels and sprung mass reflects in two aspects. One is the degree of inconsistence between SPTC and WTC during transient steering. The other is the time that SPTC and WTC achieve consistence when the steering wheel angle is unchanged. The corresponding evaluation indexes will be introduced as below, respectively.

From the previous analysis, it can be seen that SPTC and WTC are not consistent during transient steering due to the effect of suspension system. In this paper, the distance between WTC and SPTC is proposed to depict their inconsistence quantitatively. If there is a vehicle with $N$ wheels, the WTC of each wheel is $\{(x_1, y_2), (x_2, y_2), \cdots, (x_N, y_N)\}$ and the sprung mass turn center is $(x_{sp}, y_{sp})$, the relative distance between each WTC and SPTC can be calculated by Eq. (11):

$$R_j = \sqrt{(x_i - x_{sp})^2 + (y_i - y_{sp})^2}. \tag{11}$$

Obviously, a smaller $R_j$ during steering reflects better consistence between SPTC and WTC. In order to analyze the overall suspension performance including all suspension systems of the vehicle, a comprehensive index, which can reflect the main performance of all suspension systems of each wheel, is needed. Thus, the index $R_{\max}$ is proposed, which is the maximum value of $R_j$ of each wheel, as shown in Eq. (12):

$$R_{\max} = \max\{R_1, R_2, \cdots, R_N\}, \tag{12}$$

where $R_{\max}$ reflects the maximum inconsistence level of WTC and SPTC, in other words, the worst performance of each suspension during transient steering. If $R_{\max}$ is small, the consistence of WTC and SPTC is good, which reflects that the overall suspension performance of the vehicle is satisfactory.

When the steering wheel angle is unchanged and the vehicle achieves a steady state, the positions of the vehicle body and wheels are relatively fixed and SPTC and WTC are consistent. Therefore, each WTC and SPTC achieve consistence and $R_{\max}$ is miniscule (near zero). The time $\Delta T$ when $R_{\max}$ is steadily in a small range is another important index to evaluate the coordination time of the suspension system to adjust the steering motion of the vehicle body and wheels to achieve consistence:

$$\Delta T = T_s - T_i, \tag{13}$$

where $T_i$ represents the initial time of the suspension evaluation (the time when the steering wheel angle is unchanged); $T_s$ denotes the time when $R_{\max}$ is steadily in a small interval ($R_{\max} \in [0, \xi]$) which is defined as the consistence interval ($[0, \xi]$).

The evaluation method diagram is shown in Fig. 11.

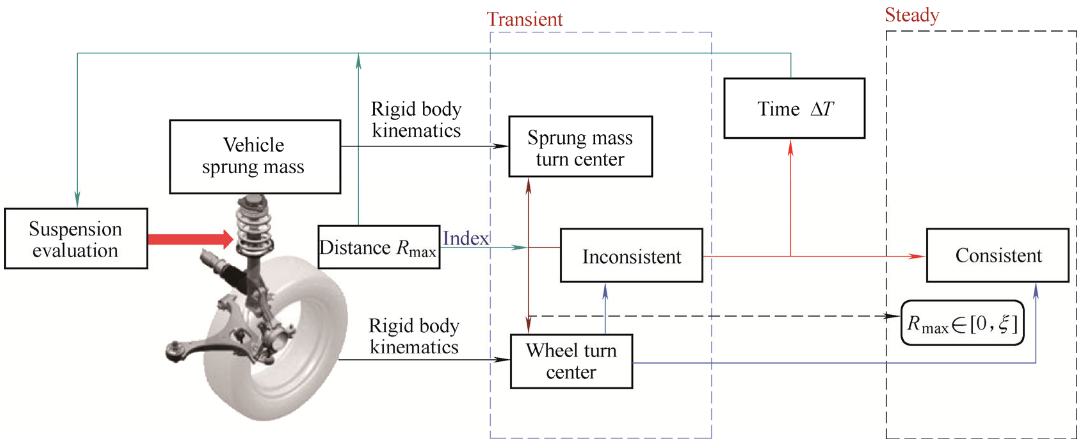


Fig. 11. Diagram of suspension evaluation method

In summary, $R_{\max}$ is the suspension evaluation index in transient steering, which depicts the maximum inconsistence between WTC and SPTC. Under the same vehicle condition, the smaller $R_{\max}$ reflects the better steering coordination between the vehicle body and each wheel. In addition, when the vehicle is in the steady steering state, $\Delta T$ reflects the speed that SPTC and WTC achieve consistence. A smaller $\Delta T$ reflects the faster speed to achieve consistence of steering motion between vehicle body and wheels. Both of them are the indexes to evaluate and optimize suspension systems from the perspective of steering coordination. The following section will show that proper matching of suspension parameters can reduce $R_{\max}$ and $\Delta T$ to improve suspension and steering performance.

## 5 Simulation and Analysis

### 5.1 A suspension evaluation example

A simulation test of suspension evaluation example is conducted to evaluate and analyze the suspension performance. CarSim is a commercial software with high precision and is widely used in vehicle virtual simulations. In this paper, the vehicle model for simulation is provided by CarSim. The *A* class car in CarSim is selected as the experimental vehicle. Fig. 12 shows the steering wheel angle input under a certain step steer test in which the vehicle speed is 120 km/h. When the vehicle achieves the steady state, the lateral acceleration is 0.1$g$. Fig. 13 shows the distance of SPTC and WTC ( $R_j$ ) of each wheel during steering. Since the lateral coordinate of WTC is large at the beginning of steering, the data are presented from 0.2 s. LF, RF, LR, RF, CM denote the left front wheel, right front wheel, left rear wheel, right rear wheel, sprung mass, respectively. Fig. 14 shows the time history of maximum value of them ($R_{max}$) and the time of it required for achieving consistence ($\Delta T$) which reflects the overall consistence of SPTC and WTC and coordination time of the suspension system to adjust the steering motion of vehicle body and wheels to achieve consistence, respectively.

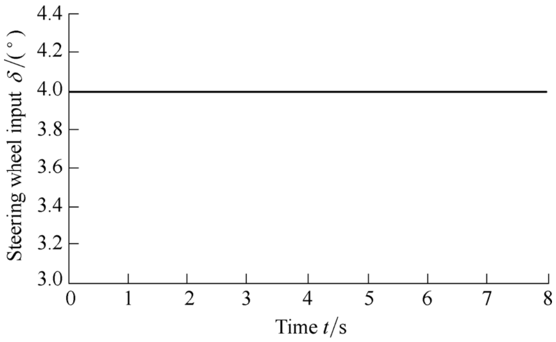


Fig. 12. Steering wheel angle input

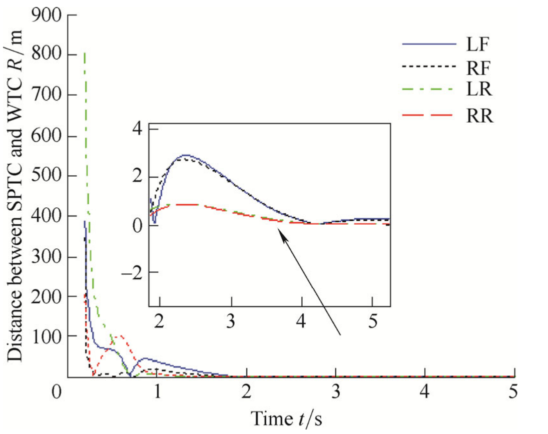


Fig. 13. Distance between SPTC and WTC of each wheel

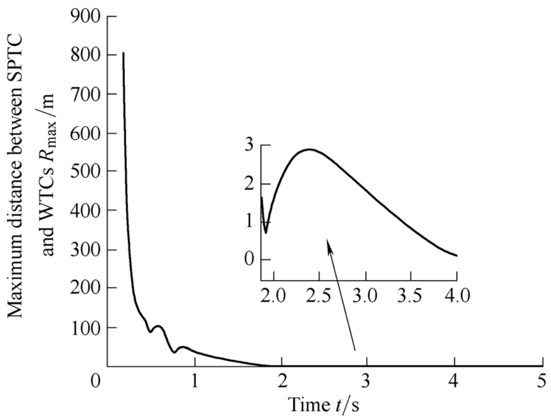


Fig. 14. Evaluation indexes $R_{max}$ and $\Delta T$

## 5.2 Results of $R_{max}$ and $\Delta T$ under different vehicle steering conditions

Under different vehicle steering conditions, the suspension system will demonstrate different performance and characteristics. In order to investigate the variation regularity of the evaluation indexes under different vehicle steering conditions, the step steer simulation tests are conducted under different vehicle velocities and lateral accelerations, and the simulation results of $R_{max}$ and $\Delta T$ are shown in Figs. 15–16. It can be seen that with the increase of vehicle velocity and decrease of lateral acceleration, both $R_{max}$ and $\Delta T$ increase greatly, which indicate that the consistence of SPTC and WTC becomes worse and needs more coordination time for the suspension system to adjust the steering motion between the vehicle body and wheels to achieve consistence. Table 1 shows $\Delta T$ under different vehicle velocities and lateral accelerations. The impact of suspension characteristics can be analyzed under different vehicle conditions according to the evaluation indexes.

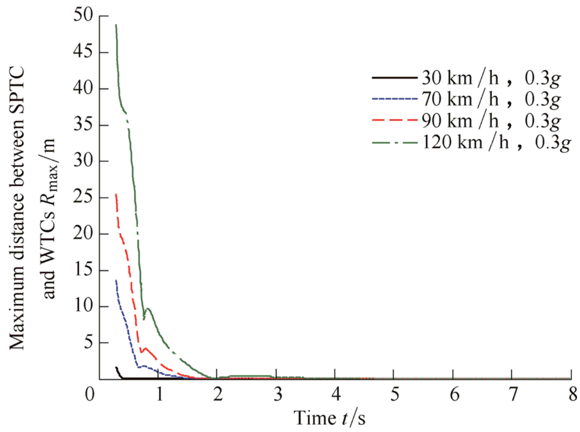


Fig. 15. $R_{max}$ under different vehicle velocities

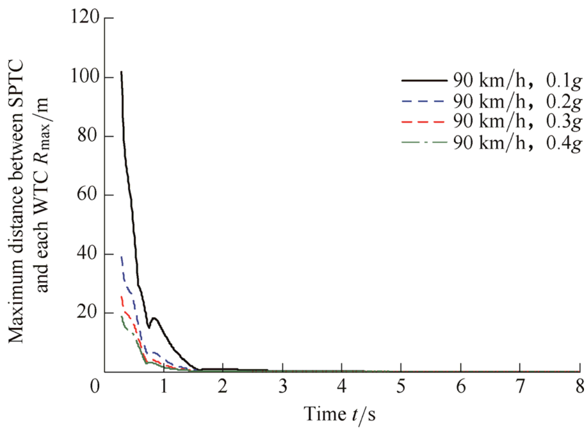


Fig. 16. $R_{max}$ under different vehicle lateral accelerations

**Table 1. Calculation results of the $\Delta T$ of A class car**

| Velocity $v$/(km • h$^{-1}$) | Acceleration $a$ | | | |
|---|---|---|---|---|
| | 0.1$g$ | 0.2$g$ | 0.3$g$ | 0.4$g$ |
| 30 | 1.675 | 1.345 | 0.419 | 0.306 |
| 70 | 2.903 | 1.567 | 1.517 | 1.478 |
| 90 | 3.541 | 2.761 | 2.279 | 1.635 |
| 120 | 4.089 | 3.876 | 3.689 | 3.495 |

It can be concluded from above that the inconsistence of steering motion between the body and wheels are mainly at high speeds and low lateral accelerations. No matter if it is a sports car or a comfortable car, under different vehicle velocities and lateral accelerations, $R_{max}$ and $\Delta T$ must be in an acceptable range, otherwise the vehicle response and steering coordination performance cannot be guaranteed. In the following, under certain steering conditions, the influence of suspension parameters on $R_{max}$ and $\Delta T$ and a proper matching of different parameters to improve suspension and steering coordination will be analyzed.

### 5.3 Proper matching of suspension parameters to improve the performance of suspension and steering coordination

This section analyzes the influence of suspension characteristic parameters and their matches on suspension performance reflected by $R_{max}$ and $\Delta T$ in a certain steering condition, such as spring stiffness, damping of shock absorber, suspension K&C characteristics. The vehicle speed is 90 km/h and the final steady lateral acceleration of the vehicle is 0.3$g$.

The spring and shock absorber are major components for adjusting the uncoordinated motion between vehicle body and wheels. By changing the value of spring stiffness of the front axle, it can be seen that $R_{max}$ is decreased and $\Delta T$ is shortened with the increase of spring stiffness, as shown in Fig. 17. The consistence of WTC and SPTC becomes better and they achieve consistence quickly. A more rigid spring adjusts the motion between body and wheels more effectively. Accordingly, with the increase of the value of shock absorber damping of the front axle, $R_{max}$ is decreased in the beginning but $\Delta T$ increases greatly, as shown in Fig. 18. Therefore, it is needed a trade-off between them, considering the variation of $R_{max}$ and $\Delta T$ comprehensively.

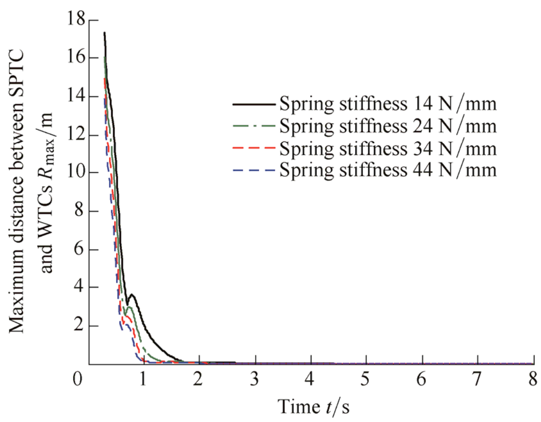


Fig. 17. Influence of different spring stiffness on $R_{max}$ and $\Delta T$

It can be drawn a conclusion from the analysis above that the matches of spring and damper parameters have a significant influence on $R_{max}$ and $\Delta T$. Appropriately increasing the stiffness of spring can decrease $R_{max}$ and shorten $\Delta T$, which is very beneficial for the steering coordination of the vehicle body and wheels. It also needs to point out that the stiffness of spring cannot be too large, otherwise it worsens vehicle ride comfort. The damping of shock absorber also has a great influence on $R_{max}$ and $\Delta T$. It is needed to design and match a compromise value of damping in order to balance a smaller $R_{max}$ and a shorter $\Delta T$.

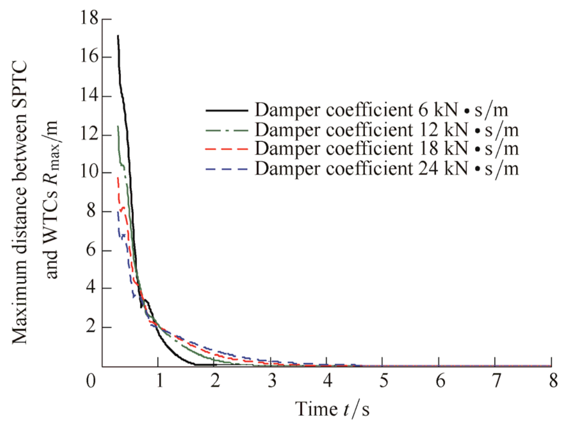


Fig. 18. Influence of different damps on $R_{max}$ and $\Delta T$

Since suspension guiding systems transfer the forces from wheels to the body, the matches of their characteristics of each wheel influence the steering coordination performance greatly, which reflects by suspension K&C characteristics. The influences of different suspension K&C characteristic parameters on $R_{max}$ and $\Delta T$ are shown in Figs. 19–24.

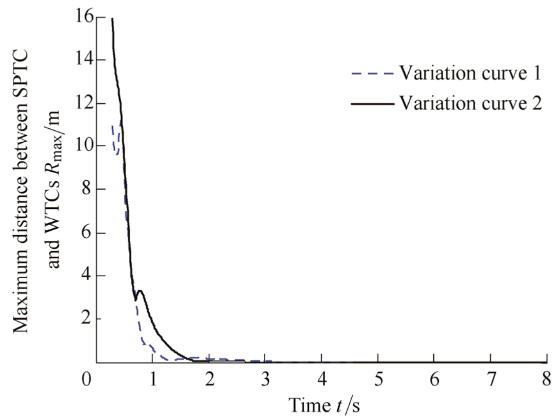


Fig. 19. Influence of different toe angle curves due to jounce

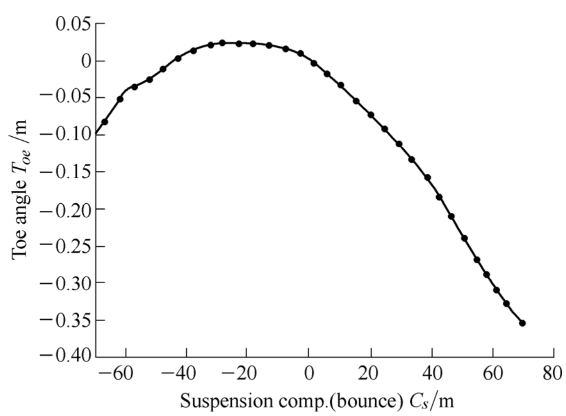


Fig. 20. Variation curve 1 of toe angle due to jounce

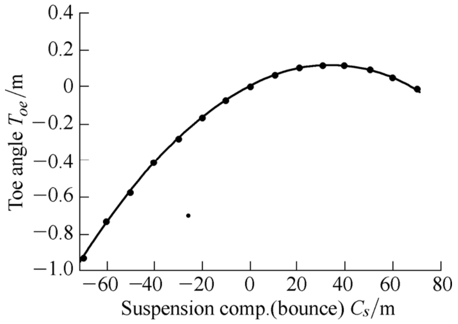


Fig. 21. Variation curve 2 of toe angle due to jounce

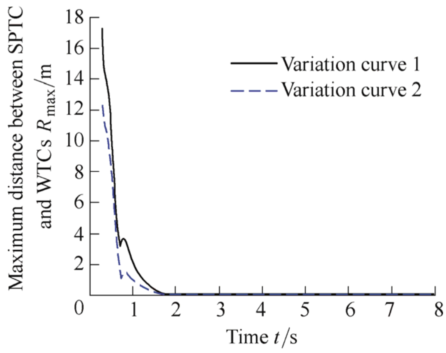


Fig. 22. Influence of different dive movement curves due to jounce

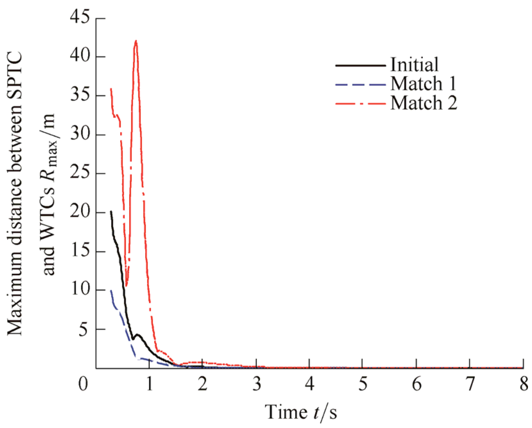


Fig. 23. Influences of different additional wheel angles caused by lateral force

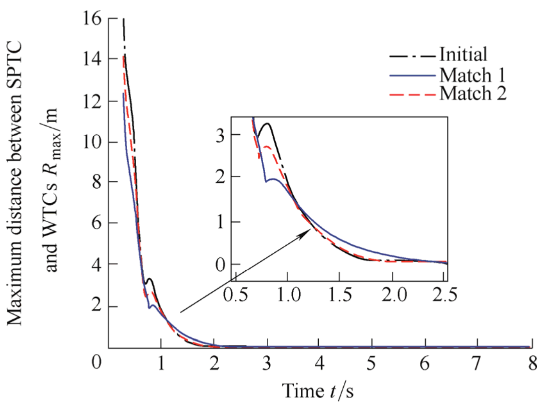


Fig. 24. Influence of different additional wheel angles caused by aligning moment

It can be seen from Fig. 19 that different toe angle variation curves due to jounce have a significant influence on $\Delta T$, and a proper curve can greatly reduce $R_{max}$ and shorten $\Delta T$. Fig. 19 and Fig. 20 show the default toe angle variation curve of the left front wheel of *A* class in CarSim (variation curve 1) and a modified curve (variation curve 2), respectively. Other wheels are similar which are not presented due to the space of the paper. Fig. 22 also shows the comparison of $R_{max}$ between the initial curve (variation curve 1) and the modified curve (variation curve 2) of dive movements due to jounce. According to $R_{max}$ and $\Delta T$, suspension and steering coordination performance can be improved by regulating and choosing an optimal variation curve.

Fig. 23 shows the time history of $R_{max}$ under different compliance coefficient matches (steer vs $F_y$) of front and rear axles shown in Table 2. It reflects the influences of different additional wheel angles caused by lateral force ($F_y$) on $R_{max}$ and $\Delta T$. It can be seen from Fig. 23 that match 2 is not recommended since $R_{max}$ and $\Delta T$ become larger. Match 1 reduces $R_{max}$ greatly as compared to the initial setting parameters and match 2. A meaningful conclusion can be drawn from above that appropriately reducing the compliance coefficient (steer vs $F_y$) of the front wheels and increasing that of the rear wheels can greatly improve steering coordination between the vehicle body and wheels.

**Table 2. Different matches of compliance coefficient (steer vs $F_y$) of front and rear axle**

| Match | Front axle | Rear axle |
|---|---|---|
| Initial | $-0.45\times10^{-3}$ | $-0.17\times10^{-4}$ |
| Match 1 | $-0.45\times10^{-4}$ | $-0.17\times10^{-3}$ |
| Match 2 | $-0.45\times10^{-2}$ | $-0.17\times10^{-5}$ |

Fig. 24 shows the time history of $R_{max}$ under different compliance coefficient (steer vs $M_z$) matches of front and rear axles shown in Table 3. It reflects the influence of different additional wheel angles caused by aligning moment ($M_z$) on $R_{max}$ and $\Delta T$. It can be seen that match 1 reduces $R_{max}$ but increases $\Delta T$. As compared to match 1, match 2 doesn't increase $\Delta T$ although the degree of reducing $R_{max}$ of it is smaller. Therefore, match 2 is a better combination and choice than match 1. Appropriately reducing compliance coefficient (steer vs $M_z$) of front wheels and increasing that of the rear wheels can improve steering coordination between the vehicle body and wheels.

**Table 3. Different matches of compliance coefficient (steer vs $M_z$) of front and rear axle**

| Match | Front axle | Rear axle |
|---|---|---|
| Initial | $0.13\times10^{-1}$ | $0.63\times10^{-2}$ |
| Match 1 | $0.13\times10^{-2}$ | $0.33\times10^{-1}$ |
| Match 2 | $0.45\times10^{-2}$ | $0.13\times10^{-1}$ |

It can be concluded from the analysis above that an optimal compromise design can be obtained based on the comprehensive optimal K&C characteristic curve and suspension stiffness and damping matches according to the indexes $R_{max}$ and $\Delta T$ from the perspective of steering coordination between vehicle body and wheels.

## 6 Conclusions

This paper proposes the concept and the calculation method of wheel turn center and develops the evaluation and analysis method for suspension performance. The following conclusions can be drawn from the study above.

(1) The consistence between SPTC and WTC is proposed to analyze the influence of suspension parameters including spring stiffness, damping of shock absorber, K&C characteristics, etc, on the vehicle steering coordination, providing an index and tool for suspension design and optimization.

(2) In the pre-development stage of vehicle, based on high-precision vehicle dynamics model, the suspension characteristic parameters can be designed and optimized to improve the steering coordination between vehicle body and wheels. Combined with other requirements of suspension design and vehicle performance, an optimal compromise design of suspension system can be obtained based on the comprehensive optimal K&C characteristic curves and matches of suspension stiffness and damping according to the indexes proposed in this paper.

(3) The inconsistence of steering motion between the vehicle body and wheels are mainly at high speeds and low lateral accelerations. No matter if it is a sports car or a comfortable car, the proposed indexes $R_{\max}$ and $\Delta T$ under different vehicle velocities and lateral accelerations must be within an acceptable range, otherwise the vehicle response and steering coordination performance cannot be guaranteed.

(4) According to the wheel turn center method, the further optimization and matching methodology for suspension system by using the vehicle model based on vehicle structure, such as hard point, bushing stiffness, etc, will be the emphasis of our future work.

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