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Synthesis of Branched Chains with Actuation Redundancy for Eliminating Interior Singularities of 3T1R Parallel Mechanisms

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Abstract: Although it is common to eliminate the singularity of parallel mechanism by adding the branched chain with actuation redundancy, there is no theory and method for the configuration synthesis of the branched chain with actuation redundancy in parallel mechanism. Branched chains with actuation redundancy are synthesized for eliminating interior singularity of 3-translational and 1-rotational(3T1R) parallel mechanisms. Guided by the discriminance method of hybrid screw group according to Grassmann line geometry, all the possibilities are listed for the occurrence of interior singularities in 3T1R parallel mechanism. Based on the linear relevance of screw system and the principles of eliminating parallel mechanism singularity with actuation redundancy, different types of branched chains with actuation redundancy are synthesized systematically to indicate the layout and the number of the branched chainsinterior with actuation redundancy. A general method is proposed for the configuration synthesis of the branched chains with actuation redundancy of the redundant parallel mechanism, and it builds a solid foundation for the subsequent performance optimization of the redundant actuation parallel mechanism.

Keywords: parallel mechanism, interior singularity, hybrid screw group, branched chain with actuation redundancy

1 Introduction

Parallel mechanisms have three kinds of singular state, platform singularity, actuation singularity[1], and branch singularity, and the first two belong to interior singularity of parallel mechanism[2]. Because of the existence of singular configuration, especially the influence of the interior singularity, the stiffness, bearing capacity and precision of the parallel mechanism will be greatly affected when it is in a singular position or near the singularity. Redundant actuation is a simple and effective way to eliminate the singularity of the parallel mechanism[3–8].

Scholars at home and abroad have carried out many researches on the theory and application of redundant actuation parallel mechanism, and made fruitful research achievements. According to the structure, the current redundant actuation parallel mechanism can be divided into three categories[9]. The first is to add an additional actuation on the non-active joint of the existing parallel mechanism. The second is to add one or more branched chains[10–13], and then have actuation on the joint of the added branches, and the third is a combination of the first two. Although the first type is relatively easy to construct, it is not convenient to install actuators. Moreover, there will be a larger inertia in the process of movement, which will affect the dynamic performance of the mechanism. The second type is adopted more commonly since it is convenient to install actuators, make the inverse kinematics easier, and partly solve the forward kinematics of the mechanism. The third type is rarely employed because of its complex structures. KIM[14] added actuation in the four rods of the parallel mechanism with six degree of freedom to avoid the singularity. The influence of redundant actuation on the performance improvement in parallel mechanism was analyzed by SHIN H, et al[15–16]. Static and stiffness of the redundant spherical parallel mechanism of 4-RRS were analyzed by QU et al[17]. CHOUDHURY, et al[18], added exercise branched chains to reduce the singular position of the workspace area. CHANG, et al[19], increased the freedom degree of movement and rotation in the operation platform of the 3-DOF parallel mechanism to form a 5-DOF hybrid machine tool. LIU, et al[20], discussed comprehensive performance of planar 2-DOF redundant parallel robot by adding a redundant branched chain to overcome the singular configuration and increase the cutter attitude angle range. WU, et al[21–22], studied a planar 3-DOF redundant parallel mechanism and applied to 4-DOF hybrid machine tool. NIU, et al[23], constructed a new type of redundant actuation parallel mechanism and analysed the dynamic

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modeling. KONG, et al[24], added a redundant branched chain in the 4-DOF parallel machine tool and concluded that the redundancy scheme is helpful to decrease agency clearance than the non-redundancy scheme. Self-calibration of kinematic parameters of a redundant branched parallel machine was stipulated with redundant information by LIU, et al[25]. YU, et al[26–27], researched the singularity of a class of parallel mechanism, and proposed a comprehensive method of flexible parallel mechanism. As we can see, most scholars only analyze a specific configuration of the redundant actuation parallel mechanism, they seldom study the configuration design of redundant actuation parallel mechanism, especially the system about redundant actuation synthesis and design aspects of the branched chains. In the construction of redundant actuation parallel mechanism to eliminate the interior singularity, there is a large randomness in the process. It will increase the difficulty of control after random adding redundant actuation, especially the parallel mechanism not suitable for adding redundant actuation. Therefore, it is the key to the research of redundant actuation parallel mechanism on how to add branched chains with redundant actuation to build reasonable redundant actuation parallel mechanism. However, there are no mature or perfect theories and methods for the synthesis of the branched chains with actuation redundancy of redundant actuation parallel mechanism.

To solve the above problems, for redundant actuation branched structure synthesis of redundant parallel mechanism, a general method is proposed to eliminate the singularity of the parallel mechanism without changing the degree of freedom of the mechanism. Using this method, the redundant branched chains are synthesized systematically to eliminate the interior singularities of 3T1R fully-symmetrical parallel mechanisms including: the structure of redundant actuation branched chains, the installed position and the number of redundant actuation chains. Finally, a 4-UPU parallel mechanism is taken as an example to illustrate the correctness of the proposed synthesis method. Thus, it lays the foundation for the subsequent performance optimization of the redundant parallel mechanism.

2 Analysis of Interior Singularity of 3T1R Parallel Mechanisms

2.1 Platform singularity of 3T1R parallel mechanisms

The constraint screws of the 3T1R PMs are two couples:

$$\$_i^r = (\boldsymbol{s}_i;\boldsymbol{s}_i^0) = (0,0,0;\,p_i,q_i,r_i)\ (i=1,2),$$

where $\$_i$ does not have practical meaning and $\$_i^0$ represents the direction of the axis of the couple. In Plücker coordinates, the matrix is composed of these two constraint couples is expressed as

$$\boldsymbol{S} = \begin{pmatrix} \$_1^r \\ \$_2^r \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & p_1 & q_1 & r_1 \\ 0 & 0 & 0 & p_2 & q_2 & r_2 \end{pmatrix}.$$

When the two constraint couples are linear dependent, $rank(\boldsymbol{S}) < 2$, the mechanism is at a platform singularity configuration.

The two constraint couples are linear dependent when $\$_2^r = a\$_1^r$ (a is real number), the two constraint couples are parallel or collinear.

2.2 Actuation singularity of 3T1R parallel mechanisms

Four constraint screws, $\$_i^a\,(i=1,2,3,4)$, are added to the 3T1R mechanisms when the actuators are locked. According to the input selection principles, $rank(\$_1^r\ \$_2^r\ \$_1^a\ \$_2^a\ \$_3^a\ \$_4^a)^T = 6$. The integrate Jacobian is composed by $\$_i^r\,(i=1,2)$ and $\$_i^a\,(i=1,2,3,4)$. When the mechanism is at an interior singularity configuration, $rank(J_A) < 6$. For the 3T1R mechanisms, there are two cases which cause the interior singularity:

(1) $\$_i^a\,(i=1,2,3,4)$ are linear dependent;

(2)When $\$_i^r\,(i=1,2)$ is linear independent, and $\$_i^a\,(i=1,2,3,4)$ is linear independent, but the rank of J_A is less than 6.

According to the linear dependence of screws in Ref. [28], all cases of the actuation singularity are shown in Table 1.

3 Synthesis of Branched Chains with Actuation Redundancy

3.1 Synthesis method for redundant actuation branched chain

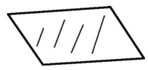
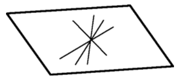
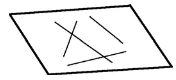
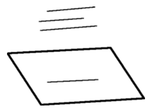
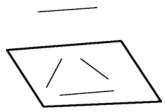
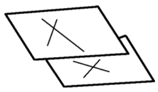
The method is as follows. The interior singularities of the mechanism can be analyzed with the Klein mapping theory based on the actuation screws and constraint screws of the mechanism. Then, the type of the redundant branched chains can be determined after confirming the type and the number of the constraint screws. According to the interior singularity, the layout of the redundant branch can be arranged reasonably. The actuation screws or constraint screws should keep at certain position (different surface and intersect). At last, the minimum number is determined for redundant branched chains.

3.2 Type synthesis of additional branched chains

When the 3T1R mechanisms are at the interior singularity configuration, $\$_i^r$ (or $\$_i^a$) can be added to eliminate the interior singularity($rank(J_A)=6$). The number of additional $\$_i^a$ is equivalent to the number of redundant branch, and the number of additional $\$_i^r$ is related to the structure of redundant branch. The number of $\$_i^r$ needs to be discussed only when the platform singularities occur.

When $\$_2^r = a\$_1^r$ (a is real number), $rank(\$_1^r\ \$_2^r\ ) = 1$. In order to eliminate the interior singularity, the redundant branched chain must provide at least one constraint couple. So the structures of the redundant chain can be 3T, 2R1T, 1R2T, 1R3T,2R2T,3T2R.

Table 1. Conditions of actuation singularity

Serial No.	Geometric characteristics of four forces	Geometric characteristics of two couples	Projection on Q_2^4	Graphic	Maximum No. of linear independent
1	Coaxial	Linear independent	A point		3
2	Coplanar and parallel		A line		4
3	Coplanar and concurrent		A straight line		4
4	Coplanar		A plane		5
5	Parallel in space		A plane		4
6	Three forces are coplanar and the fourth is parallel to the plane		A plane and a point outside the plane		5
7	Four forces are in two parallel planes		Two planes intersect at one point		5

The redundant branched chains are added to eliminate the singularity of the mechanism without changing the primary DOF. So, the rank of the constraint screws cannot be changed. The structures of 2R1T, 1R2T, 2R2T, 3T cannot meet the requirements, but 1R3T, 3T2R can. In addition, 3R3T can meet the requirements only when the actuation singularity concurs. All types of the redundant branched chains can be synthesized according to the constraint screw synthesis[29], some of them are shown in Table 2.

Table 2. Types of the branched chains with actuation redundancy

Type	Configuration	Structure
$3T1R^z$	3R1P	RRRP RRPR RRC RCR
	2R2P	RRPP RPRP PRRP RPPR RCP CRP PRC RPC
	1R3P	RPPP PRPP CPP PCP
$3T2R^{xz}$	5R	$R_xU_{xz}R_zR_x$ $R_xR_xU_{xz}R_z$ $R_xR_xR_zU_{xz}$ $U_{xz}R_xR_zR_x$ $R_xU_{xz}R_zR_x$ $R_xR_zU_{xz}R_x$ $R_xR_zR_xU_{xz}$ $R_xU_{xz}U_{xz}$ $U_{xz}R_xU_{xz}$ $U_{xz}U_{xz}R_x$
	4R1P	$R_xR_xR_xC_z$ $R_xC_xR_xR_z$ $R_xR_xC_xR_z$ $R_xC_xR_zR_x$ $R_xPR_xU_{xz}$ $R_xPU_{xz}R_x$ $C_xR_xR_zR_x$ $PR_xU_{xz}R_x$ $R_xR_{xz}R_zR$ $R_xR_xR_zC_z$ $R_xU_{xz}PR_z$ $R_xR_xC_zR_z$
	3R2P	$R_xR_xC_xP$ $C_xR_xPR_x$ $R_xPC_xR_x$ $R_xC_xPR_x$ $C_xPR_xR_x$ $PR_xR_xC_x$ $R_xC_xR_xP$ $R_xU_{xz}PP$ $C_xPR_xR_z$ $R_xPR_xC_z$ PR_xPU_{xz} $C_zR_xR_xP$ $PU_{xz}R_xP$ $PR_zR_xC_x$ $R_xC_xC_x$ $C_xR_xC_x$ $R_xC_xC_z$ $C_xC_xR_z$ C_xPU_{xz}
	2R3P	C_xPR_xP PC_xR_xP PR_xC_xP PR_xPC_x C_xR_xPP R_xPC_xP R_xPC_xP PC_xR_xP C_xPR_zP R_xPC_zP R_xPC_zP PR_xC_zP PC_xR_zP PR_xC_zP PR_xPC_z C_xPC_x C_xC_xP PC_xC_x C_xPC_z C_xC_zP PC_xC_z
3T3R	.6R.	$R_xR_yR_zS_{xyz}$ $R_xU_{xy}U_{yz}R_y$ $R_xU_{yz}S_{xyz}$ $R_xS_{xyz}U_{yz}$ $U_{yz}S_{xyz}R_x$
	5R1P	$S_{xyz}PR_xR_y$ $U_{xy}PS_{xyz}$ $PU_{xy}S_{xyz}$ $C_xU_{xy}U_{yz}$ $R_yC_xS_{xyz}$ $R_yS_{xyz}C_x$
	4R2P	$R_xR_yC_zC_x$ $S_{xyz}C_xP$ $S_{xyz}PC_x$ $C_xS_{xyz}P$ $U_{xy}C_zC_x$ $C_zU_{xy}C_x$
	3R3P	$PPPS_{xyz}$ $R_xC_yC_zP$ $PPU_{xy}C_z$ $C_yC_zR_xP$ $U_{xy}PPC_z$ $R_xC_yC_zP$

In Table 2, $3T1R^z$ means that the chain has three translational and one rotational DOF around the z axis direction. R shows that the rotation axis of the joint is z axis. R indicates that the rotation axis of the R joint is x axis. The relative pose of the joint is represented in the branch coordination system, as shown in Fig. 1.

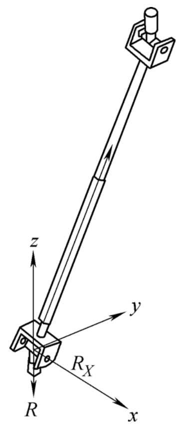


Fig. 1. R and Rx joint

3.3 Determination of the number and the layout of the redundant branched chains

3.3.1 Number and the layout of the redundant branched chains corresponding to the platform singularity

The 3T1R mechanisms are at the platform singularity configuration when $\boldsymbol{\$}_i^r$ $(i=1,2)$ are parallel. One branched chain, whose constraint screw $\boldsymbol{\$}^r$ is not in a plane or intersectant with $\boldsymbol{\$}_i^r$, can be added to eliminate the platform singularity[30].

3.3.2 Number and the layout of the redundant branched chain corresponding to the actuation singularities

Assuming that the 3T1R mechanisms can translate along x,y,z axis directions and rotate around z axis direction. The mechanisms are at the actuation singularity configuration, when all actuation screws are coaxial. Generally speaking, this situation will not occur in parallel mechanism. The mechanisms are also at the actuation singularity configuration, when four actuation screws are spatial crossing, but not in one plane. In general, the size of the mobile platform and fixed platform is not same, so this case is rare to occur.

The geometry relationship of actuation branches is equivalent to the relationship between the redundant branched chain and primary branched chain of parallel mechanisms. On the whole, the redundant chain is the minimum number when the mechanism has the specific geometry constraint relationship. The analysis can be summarized and they are shown in Table 3.

(1) All actuation screws are parallel and in one plane. Two redundant branches, whose $\boldsymbol{\$}^a$ intersect with $\boldsymbol{\$}_i^a$ of primary branch, are needed to eliminate the actuation singularity.

(2) All actuation screws are coplanar and intersect with each other. If the added $\boldsymbol{\$}^a$ is not in one plane with $\boldsymbol{\$}_i^a$, only one redundant chain is needed. If the added $\boldsymbol{\$}_i^a$ intersect with $\boldsymbol{\$}_i^a$, two redundant chains are needed.

(3) All actuation screws are coplanar. At least one redundant branched chain is needed, whose actuation screw is not in one plane with the actuation screws of primary branched chains.

(4) All actuation screws are parallel in spatial space. One redundant chain is needed, whose actuation is not in a plane or intersectant.

(5) Three actuation screws are coplanar, and the other one is parallel to this plane. At least one redundant chain is needed. The actuation screws can intersect with the actuation screw of the rest fourth branch. If there is a constraint couple in the redundant branched chain, the couple should be coplanar and concurrent with the constraint couples of primary branched chains.

(6) Four actuation screws are distributed on two different planes. At least one redundant branched chain is needed, whose actuation screws are not in a plane with actuation screws of the two primary chains.

(7) Two actuation screws are coplanar, and the other two are parallel to this plane. At least one redundant branched chain is needed. The relationship between the actuation screws of redundant chain and the actuation screws of primary chain is determined by the specific situation.

(8) Four actuation screws are spatially concurrent. At least one redundant branched chain is needed, whose actuation screw is not in one plane with the actuation screw of a primary chain. In addition, two redundant branched chains can be added, whose actuation screws intersect with the ones of primary chains and keep dependent with the constraint screws.

(9) Four actuation screws are distributed on two intersectant planes, and the intersection point is in the intersecting line. At least one redundant branched chain is needed. The actuation screws are not in one plane with the actuation screws, but they are dependent with constraint screws.

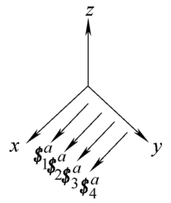
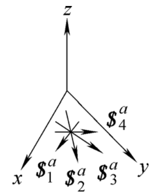
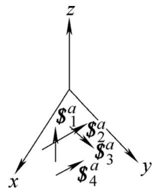
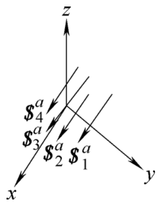
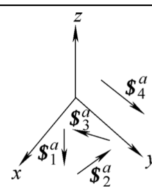
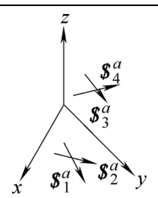
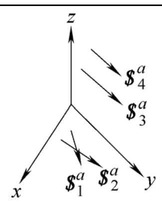
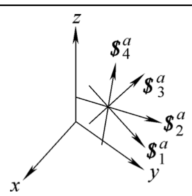
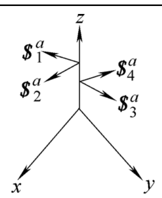
4 Case Analysis

A 4-UPU mechanism is taken as an example to illustrate the synthesis of the redundant branched chain, and to show how to get the redundant actuation parallel mechanisms without interior singularity.

4.1 Mechanism description

The 4-UPU mechanism is shown in Fig. 2, it consists of four identical UPU branched chains. The mobile platform is a square and the fixed platform is a rectangle. In each branched chain, the second and the third R joints are parallel to the fixed platform, and they are parallel to each other. The first and the fourth R joints are perpendicular to the mobile platform, but they are parallel to each other. Assuming that the P joint is the actuate joint, and the constraint screws and the actuation screws are shown in Fig. 3.

Table 3. Locations and the minimum number of the addition branched chains

Type	Geometric characteristics of the $\$_i^a$	Graphic	Geometry relationship of the additional branched chains	No. of the additional branched chains
1	Coplanar and parallel		Different surface	1
			Intersection	2
2	Coplanar and concurrent		Different surface	1
			Intersection	2
3	Coplanar		Different surface	1
4	Parallel in space		Different surface	1
			Intersection	2
5	Three $\$_i^a$ are coplanar, $\$_4^a$ parallel to this plane		Intersection	1
6	Four $\$_i^a$ are in two planes which are parallel to each other		Different surface	1
7	Two $\$_i^a$ are coplanar,the other two $\$_i^a$ parallel to this plane		Different surface or intersection	1
8	Four $\$_i^a$ are concurrent in space		Intersection	2
			Different surface	1
9	Four $\$_i^a$ are in two intersectant planes		Different surface	1

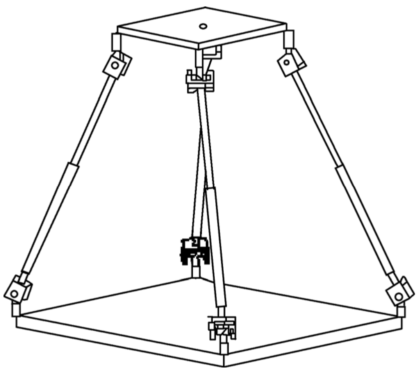


Fig. 2. 4-UPU PM

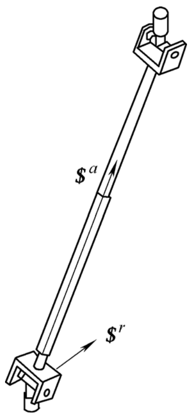


Fig. 3. UPU branched chain

4.2 Type synthesis of redundant branched chains

4.2.1 Interior singularities of 4-UPU mechanism

The 4-UPU mechanism has four constraint couples, which are coplanar and parallel to the fixed platform. The rank of the constraint screw system is 2, and it does not change with the configuration. So, there is no platform singularity, but there are actuation singularities as follows[31].

(1) When the mobile platform coincides with the fixed platform. The 4 actuation screws are coplanar, and the $rank(\$_1^a\ \$_2^a\ \$_3^a\ \$_4^a)^T = 3$. It is unavoidable for the occurrence of the singularity at this configuration.

(2) The singularity occurs when the mobile platform turns $\pi/2$ around the z axis from the initial pose as shown in Fig. 2. $Rank(\$_1^a\ \$_2^a\ \$_3^a\ \$_4^a)^{\mathrm{T}} = 3$, only one redundant branched chain is needed to eliminate the singularity.

4.2.2 Elimination of the singular configurations

When the 4 actuators are locked, 4 actuation screws are added to the mechanism. If the rank of the actuation screws is less than 4, the mechanism is at a singularity configuration, and the platform has a remanent freedom. In order to eliminate the singularity, one redundant branched chain is needed, whose actuation screw is not at one plane with the 3 of the actuation screws, $rank(\$_1^a\ \$_2^a\ \$_3^a\ \$_4^a)^{\mathrm{T}} = 4$.

Through the above analysis, one redundant actuation UPU or UPS can be added to eliminate the singularity. The hinge points of the redundant branched chain with mobile and fixed platform is B_5 and C_5 , respectively. Two redundant mechanism 4UPU/UPU and 4UPU/UPS are shown in Fig. 4.

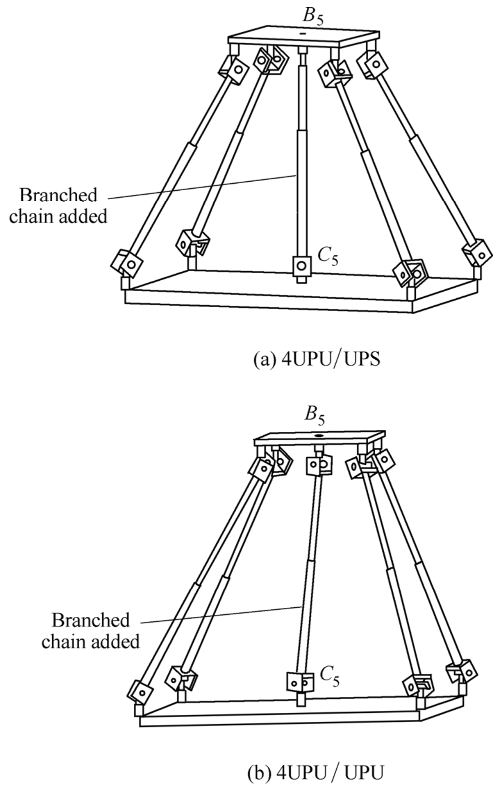


Fig. 4. Parallel mechanism with actuation redundancy

4.3 Performance comparison of 4-UPU PM with different redundant actuation branched chains

According to the previous method of type synthesis, many different redundant branched chains for 4 UPU PM are synthesized, such as UPU and UPS. All of them can eliminate the interior singularities of the mechanism. The indexes of stiffness, payload capability and dexterity are different with the different branched chain types. The performances of PMs with different redundant branched chains are studied in the following paragraphs.

4.3.1 Comparison of condition numbers

The parameters of this mechanism are: the length of mobile platform is $D = 0.4$ m, the length of fixed platform is $R_1 = 1000$ mm and the width is $R_2 = 800$ mm. The initial position of the center of the mobile platform is $(20\ 0\ 300)^{\mathrm{T}}$ mm . When the p_x and p_z is fixed, the $p_y = y$ and $\gamma = z$ is variable, the condition number of the 4-UPU is shown in Fig. 5. The condition numbers of redundant mechanism 4UPU/UPU and 4UPU/UPS are shown in Fig. 6 and Fig. 7, respectively.

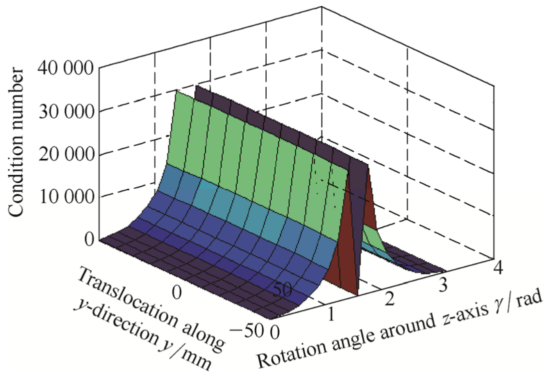


Fig. 5. 4UPU

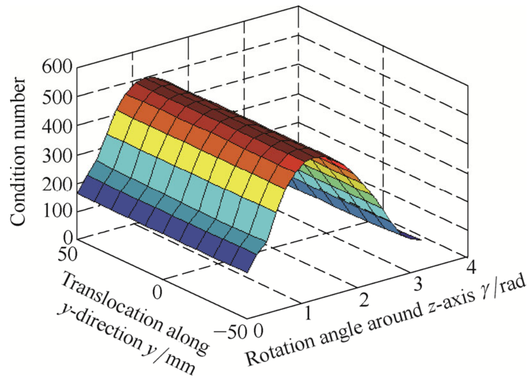


Fig. 6. 4UPU/UPU

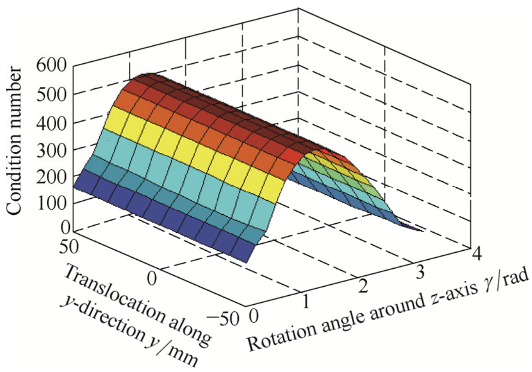


Fig. 7. 4UPU/UPS

From Fig. 5, it can be seen that the condition number of Jacobian changes acutely when the mechanism is at the singularity configuration ($z=\pi/2$) or its adjacent areas. But for the redundant actuation mechanisms, shown in Fig. 6 and Fig. 7, the condition number curve is smooth, without acute changes. The singularity is eliminated by the added redundant branched chain.

4.3.2 Stiffness analysis

The parameters of this mechanism are: the length of mobile platform is 0.1 m, the thickness is 5 mm, The length of fixed platform is 250 mm, the width is 200 mm and the thickness is 10 mm; the length of interior pole is 120 mm, the diameter is 5 mm; the length of outside pole is 90 mm, the diameter is 7 mm. All materials are alloy steel. It is analyzed via ANSYS. The fixed platform is fixed, and 100 N • m torque are applied on the center of mobile platform. The total deformation of the center and its orthogonal components are calculated. Then the stiffness simulation results are retreated. The last results of PMs with different redundant actuation branched chains are shown in Table 4, Table 5 and Table 6.

From the simulation results, we can conclude the following:

(1) In terms of the overall stiffness, the stiffness of normal mechanism in general position is smaller than in initial position. But for the parallel mechanism with actuation redundancy, the stiffness in general position is much bigger than the initial position. It is almost one order of magnitude. By comparing deformation of the mechanism, it can be seen that the stiffness can be improved by the added redundant branched chain. To a certain extent, it can also optimize the stiffness distribution.

(2) The stiffness of PM with actuation redundancy is bigger than normal mechanism. In general position, it is almost one order of magnitude. For the stiffness of branches and kinematic pairs is increased, the overall stiffness of the PM is improved.

Table 4. Max deformations of 4UPU in different positions m

Position	Coordinate direction	Max deformation	Max total deformation
	x	0.008 343 9	
Initial position	*y*	0.008 060 1	0.010 793 0
	z	0.000 993 9	
	x	0.009 766 4	
General position	*y*	0.011 145 0	0.012 395 0
	z	0.000 505 1	

Table 5. Max deformations of 4UPU/UPU in different positions m

Position	Coordinate direction	Max deformation	Max deformation
	x	0.007 312 8	
Initial position	*y*	0.006 800 7	0.009 384 8
	z	0.000 765 4	
	x	0.004 730 0	
General position	*y*	0.007 213 7	0.009 153 8
	z	0.002 148 2	

Table 6. Max deformations of 4UPU/UPS in different positions m

Position	Coordinate direction	Max deformation	Max total deformation
Initial position	x	0.008 258 8	0.010 534 0
	y	0.007 909 7	
	z	0.000 802 9	
General position	x	0.006 085 8	0.009 754 1
	y	0.007 710 4	
	z	0.001 709 4	

(3) The stiffness of the PMs with actuation redundancy are different from those that have different redundant actuation branched chains. The stiffness of UPU is bigger than UPS because the kinematic pairs of UPU have large stiffness.

4.3.3 Performance analysis

The calculation results and the performance analysis are listed as follows.

4.3.3.1 Calculation results of the 4UPU/UPU

(1) When the length of mobile platform is 400 mm, the length of fixed platform r is 300 mm–1000 mm, the width r_2 is 300 mm–1000 mm. The results are shown in Fig. 8 and Fig. 9.

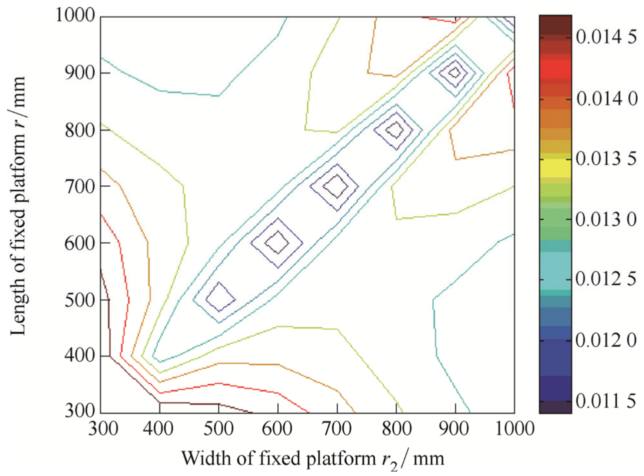


Fig. 8. Condition number in all domains

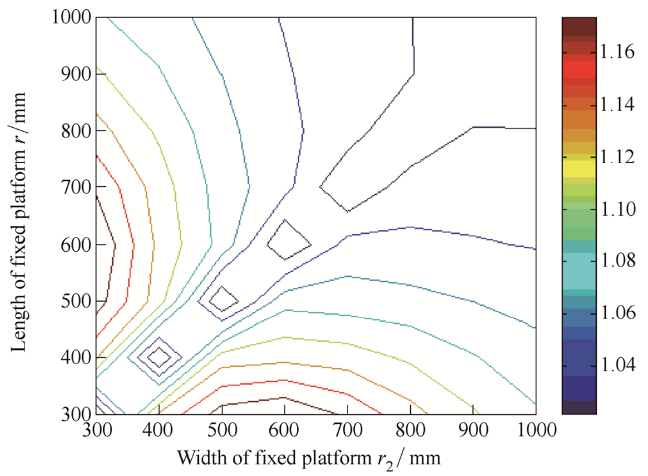


Fig. 9. Min payload force in all domains

(2) When the length of mobile platform d is 300 mm–800 mm, the length of fixed platform r is 400 mm–900 mm, the width r_2 is 400 mm. The outcome is shown in Fig. 10 and Fig. 11.

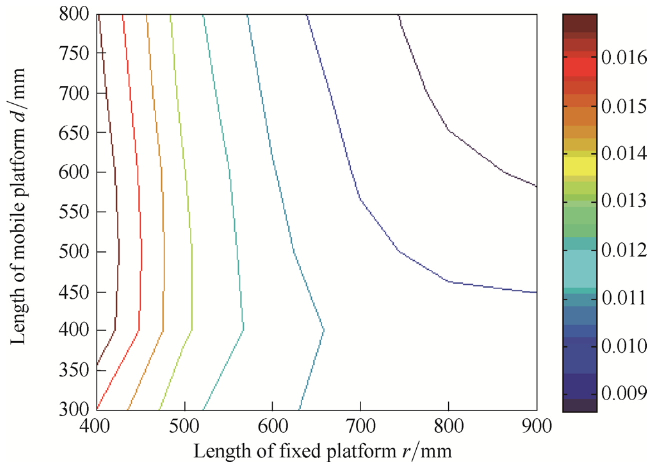


Fig. 10. Condition number in all domains

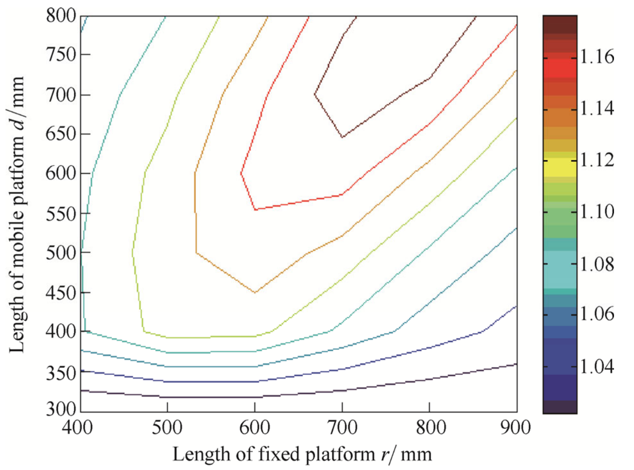


Fig. 11. Min payload force in all domains

4.3.3.2 Results of the 4UPU/UPS

(1) When the length of mobile platform d is 400 mm, the length of fixed platform r is 300 mm–900 mm, the width r_2 is 300 mm–900 mm. The outcome is shown in Fig. 12 and Fig. 13.

(2) When the length of mobile platform d is 400 mm–900 mm, the length of fixed platform r is 300 mm–800 mm, the width r_2 is 400 mm. The outcome is shown in Fig. 14 and Fig. 15.

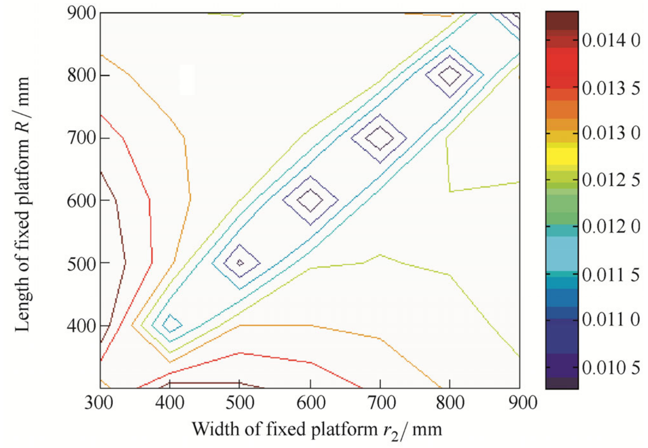


Fig. 12. Condition number in all domains

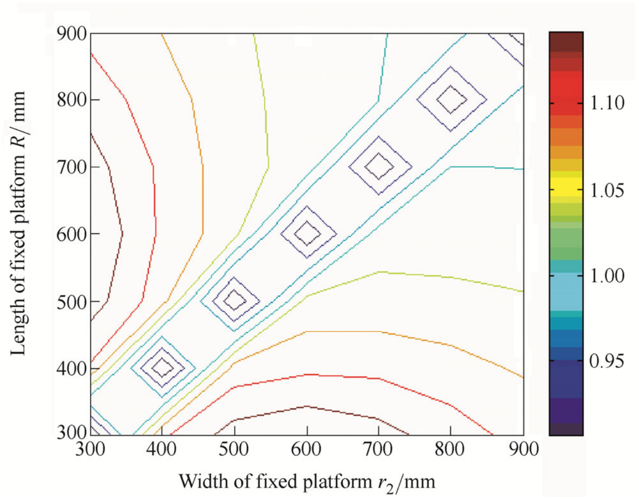


Fig. 13. Min payload force in all domains

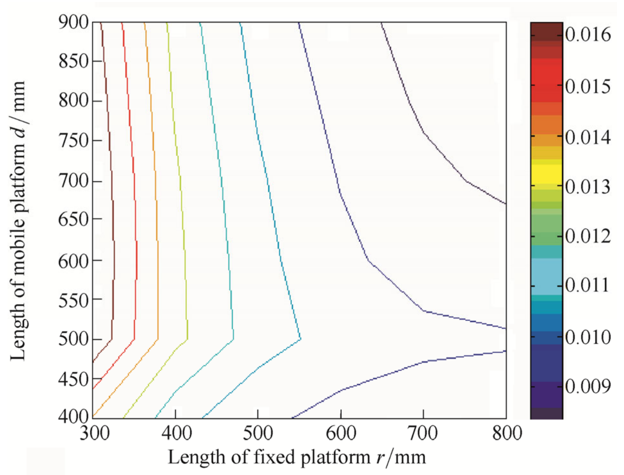


Fig. 14. Condition number in all domains

From the results, the following can be seen:

For the two redundant actuation parallel mechanisms, there is no redundant actuation branched chain which has good performance parameters at any domain. In the same size (d=400 mm, r=400 mm–700 mm, or r_2=400 mm–700 mm), the maximum condition number of 4UPU/UPU is closer to 1, so it's dexterity is comparatively better; The minimum payload force is comparatively larger. When r_2=400 mm, d=300 mm–800 mm, r=300 mm–800 mm, the difference of condition number is very small, the minimum payload force of the 4UPU/UPU is comparatively larger.

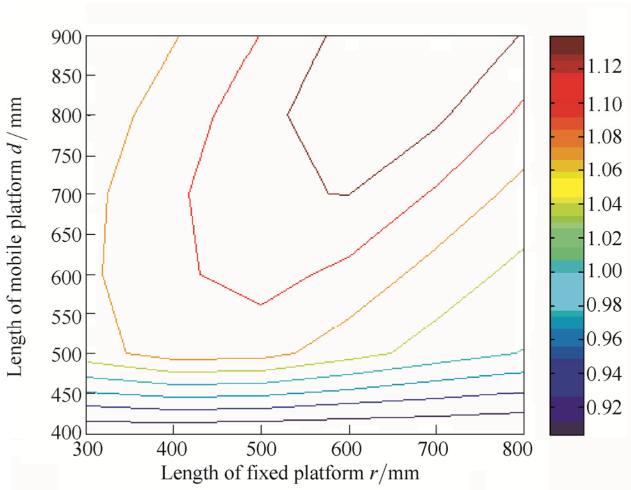


Fig. 15. Min payload force in all domains

5 Conclusions

(1) A new synthesis method is proposed for redundant actuation branched chain to eliminate the interior singularities of 3T1R parallel mechanisms. The synthesis method is also suitable for other parallel mechanisms with different degree of freedom.

(2) The possible configurations, installed position and quantity are given out for the redundant actuation branched chains of 3T1R parallel mechanism.

(3) The configuration synthesis of redundant actuation branched chains is presented to eliminate the interior singularity of 4-UPU parallel mechanism.

(4) This method can avoid the singular configurations and has the characteristics of improving the rigidity and the bearing capacity, and it is easy to be implemented.

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