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Least Squares Evaluations for Form and Profile Errors of Ellipse Using Coordinate Data

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Abstract: To improve the measurement and evaluation of form error of an elliptic section, an evaluation method based on least squares fitting is investigated to analyze the form and profile errors of an ellipse using coordinate data. Two error indicators for defining ellipticity are discussed, namely the form error and the profile error, and the difference between both is considered as the main parameter for evaluating machining quality of surface and profile. Because the form error and the profile error rely on different evaluation benchmarks, the major axis and the foci rather than the centre of an ellipse are used as the evaluation benchmarks and can accurately evaluate a tolerance range with the separated form error and profile error of workpiece. Additionally, an evaluation program based on the LS model is developed to extract the form error and the profile error of the elliptic section, which is well suited for separating the two errors by a standard program. Finally, the evaluation method about the form and profile errors of the ellipse is applied to the measurement of skirt line of the piston, and results indicate the effectiveness of the evaluation. This approach provides the new evaluation indicators for the measurement of form and profile errors of ellipse, which is found to have better accuracy and can thus be used to solve the difficult of the measurement and evaluation of the piston in industrial production.

Keywords: ellipse, form Error, profile error, least squares method, piston

1 Introduction

In the evaluation of geometrical tolerance, there is a special geometry-ellipse. According to mathematical definition, the ellipse is a curve such that the sum of the distances of every point on the curve to two foci is a constant. Obviously, circle is the degenerate form of a typical ellipse that has the same lengths of major axis and minor axis. As a special geometry, ellipse is generally applied to engineering, physics, and astronomy. For example, ellipse projection in pattern recognition, elliptical polarization in optics, and ellipse is also used to describe the orbit of planet or satellite in astronomy. Here, we focus on the applications of the ellipse measurement in geometric tolerance, such as measuring the skirt line of a piston of internal-combustion engine and measuring the form deviation of an elliptic bearing in an overloaded device.

Form error of ellipse reflects a deviation of the actual profile from the specified ideal ellipse. The quantitative description of this deviation usually depends on the evaluation methods and the relevant standards. However, there are no corresponding international standards and technical publications on the ellipse measurement at present. The international standard ISO 1101[1] does not introduce the form error of ellipse explicitly. From the view of geometric tolerance, the form error of ellipse can be regarded as a morphology tolerance in the geometrical product specifications. Thus, the concepts of geometric tolerances also can be extended to the evaluation of the form error of ellipse. Coordinate measuring machines (CMM), a measuring instrument determining physical dimension, is used as a main aided inspection tool being gradually applied to online measuring. Nowadays most CMMs can depend on least squares(LS) method to achieve the roundness measurement. For the elliptic section, obtaining physical dimensions by CMM is also easy to be performed. However, CMM has not the evaluation function for the form error of an ellipse. Therefore, developing more studies will be helpful for the practical applications of the ellipse measurement.

Until now, the definitions of form and profile errors of ellipse have not been included in the international standards, such as ISO and ASME. From the authors' point of view, the form and profile errors of ellipse are considered as the deviation between the physical profile and the ideal profile. But there are some differences between the form error and the profile error. Due to ellipse having unequal radii, the

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error models of form and profile of ellipse are complex geometric problems requiring lots of mathematical calculation. Least squares method, a solution based on regression analysis, is widely used in geometrical error evaluation due to the uniqueness of its solution and the convenience of implementation. Thus, the research study about the form and profile errors of ellipse combined with least squares method is a significant work. How to obtain an accurate ellipse profile, as an important research, has been referred by many researchers in image processing. Firstly, in the research studies of fitting method, WATSON[2] introduced Gauss-Newton method to fit circles and ellipses which takes account of the measurement design. KANATANI, et al[3], declared a new fitting ellipses method - Hyper LS, which relies on algebraic distance minimization with careful choosing scale normalization. FITZGIBBON, et al[4], presented a least squares fitting method which is specific to ellipses and direct at the same time. AHN, et al[5], proposed a simple and robust nonparametric algorithm based on the coordinate description for geometric fittings of ellipse. The above research studies have proposed different fitting methods for the ellipse profile. Secondly, for the form feature of ellipse, WALTER, et al[6], reported several algorithms to compute that the sum of the squares of the ellipse to the given points achieves minimal. CUI, et al[7], proposed an unbiased minimum variance estimator to estimate the parameters of an ellipse and presented a space decomposition scheme to direct search optimal parameters. CHAUDHURI[8] extended the 2D fitting method of circle or ellipse depending on the border points of the object to fit sphere or ellipsoid in 3D. Additionally, PAUL[9] proposed a variety of error of fit (EOF) functions used in the least-square fitting of ellipses. DILIP[10] investigated the various sources that can affect the accuracy of the geometric methods for detection of ellipses in images. ZOU, et al[11], studied the fragmental ellipse fitting algorithm based on least square. ZHANG, et al[12], presented an improved ellipse detector that may be used in real-time face detection. Moreover, as the research backgrounds for pattern recognition and the image processing, some research studies about the ellipse fitting still underway[13–15]. In the industry metrology and measurement field, a true and accurate evaluation for the form error of ellipse is also important. KURT, et al[16], presented a new approach for precision estimation for algebraic ellipse fitting based on combined least squares method. MURTHY[17] proposed three different methods for the evaluation of the elliptical profiles, based on normal least squares fit, bivariate Gaussian distribution and general second degree equation. LIU, et al[18], described an algorithm of evaluating the form error of elliptic profile in a plane, and the algorithm requires only one time of calculation instead of iterations as same as circle and sphere fitting algorithm. HOU, et al[19], gave the unified description of parameter vector function for the designed curve of complicated plane profiles, and derives the distance function from measured points to the design curve by the theory of differential geometry. Further, LEI, et al[20], presented an algorithm for evaluating elliptical profile error based on the minimum zone geometry optimization approach method. These methods have constructed a good foundation for the researches of the measurement and evaluation of ellipse error.

The above research studies introduced the LS fitting method of an ellipse profile and propose different solutions. However, the mathematical characteristics of ellipse having unequal radii determine a certain complexity for the evaluations of the form and profile errors, hence constructing the LS model of error evaluation is a challenging task. This work focus on two main problems in the error evaluation of the ellipse: form error and profile error. Additionally, LS method is adopted to establish a non-linear objective function based on the structure parameters of ellipse. Using structure parameters calculate foci of an ellipse and obtain the form error of ellipse, which does not require more calculations. The LS model based on the geometrical characteristics is not limited by the measurement space, and the ellipse can be in any position in the coordinate system. The advantage of the evaluation method is simple and easy to be implemented to obtain the form and profile errors of the ellipse without the complex calculations in 2D space, which reduces the difficulty of mathematical modelling. This paper argues that the form error and the profile error are two independent levels, and both of errors can be used to distinguish and quantify the machining error associated with different elliptic sections.

The remainder of this paper is organized as follows: Section 2 describes the definitions of the error evaluation of ellipse, including the form error and the profile error. Mathematical descriptions of the form and profile errors based on LS are presented in section 3. Next, experimental verification and applications by real datasets are presented in section 4. Finally, the discussions and the conclusions are drawn in sections 5 and 6, respectively.

2 Form and Profile Errors of Ellipse

The form and profile errors of ellipse reflect the deviation from an especial geometry. As a geometric measurement problem, the form and profile errors of ellipse have a direct relationship with mathematical properties and geometric characteristics of an ellipse. Thus, the research studies about the error evaluation of elliptic section are not much compared with those of others form error, such as roundness error and cylindricity error. Before performing an error analysis of ellipse, it is necessary to introduce the components of the elliptic section. The composing factors of an ellipse profile include roughness, waviness, and form profile(as shown in Fig. 1), as well as those are used to describe the general characteristics of the ellipse section. In three factors, the form profile as the main effect factor affects the form error of the measured section, the waviness is usually considered to be the source generating the profile error, and the roughness is generally ignored due to the fact that it is smaller than the instrumental error. In the design of the processing parameters, obtaining the accurate form profile is a basis for the reconstruction of the measurement parameters. Since the circle has isometric radii, the form error is equal to the profile error. On the contrary, the form error is not equal to the profile error in the error evaluation of the ellipse section. The form error of ellipse reveals the consistency and change of form, which belongs to a macroscopic error describing the measured shape. The profile error of ellipse is the presentation of morphology characteristics on the machined surface, which tends to a microcosmic error. In the early research studies, the profile error was considered as the main indicator of quality control of the ellipse section due to the limitations from the machining method. Nowadays, some new machining methods are applied to the ellipse machining. In those methods, only controlling and evaluating the profile error could cause some defects, such as machining eccentricity and deformation. Therefore, it is necessary to control the form error while controlling the profile error. And both errors must be separated in the error measurement of ellipse.

Fig. 2 shows the form error and profile error of ellipse. There are some differences between two kinds of errors in the definitions. In the authors' opinion, the form error is used to describe the deviation of the section form, while the profile error is considered as the maximum deviation from the ideal profile to the physical profile.

The form error E_{fe} of ellipse is stated as

$$E_{\mathrm{fe}} = \max(\delta_{\mathrm{oe}}) - \min(\delta_{\mathrm{ie}}), \tag{1}$$

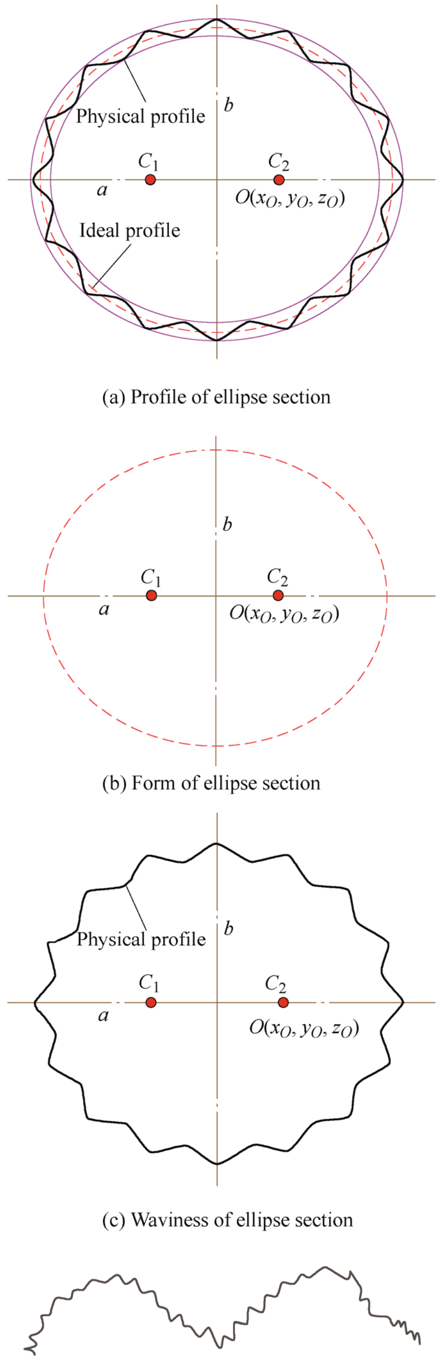
where δ_{oe} is the half of difference between the major axis and the sum of the distances from the point on the outer ellipse to the foci of least squares ellipse, and δ_{ie} is the half of difference between the major axis and the sum of the distances from the point on the inner ellipse to the foci of least squares ellipse.

The profile error E_{pe} of ellipse is given as

$$E_{\mathrm{pe}} = \max(d_{+}) - \min(d_{-}), \tag{2}$$

where d_{+} is the maximal deviation from the least squares ellipse to the outer ellipse, and d_{-} is the minimal deviation from the least squares ellipse to the inner ellipse.

Generally, the information of the whole profile of the measured section should be accurately described according to the error evaluation model. With the definitions of discrete sampling, the models of form error and profile error of ellipse are represented by sampling points that can be regarded as an accurate description of the entire elliptic section. At present, obtaining profile coordinates by CMMs is a main method for the measurement of ellipse form. The profile coordinates usually contain the information of the form error and the profile error of the measured section.



(a) Profile of ellipse section

(b) Form of ellipse section

(c) Waviness of ellipse section

(d) Roughness of ellipse profile

Fig. 1. Components of an elliptic section profile

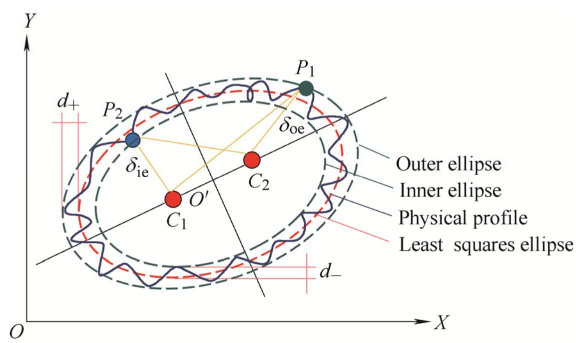


Fig. 2. LS evaluation model of the elliptic section

3 LS Models of Form and Profile Errors of Ellipse

3.1 LS model of the elliptic section

The form of elliptic section is a typical quadratic curve and its details are unknown before the measurement. The quadratic curve equation is adopted to fit an ellipse, which not only can describe the form error but also can obtain the profile error. To evaluate the two errors, the least squares method is used to establish a non-linear objective function based on the structure parameters of the ellipse.

In the measurement of an elliptic section, n sampling points are selected on the measured profile, and $n\geqslant 5$. The ellipse was defined as[17–18]

$$F(x,y)=x^2+C_1xy+C_2y^2+C_3x+C_4y+C_5, \tag{3}$$

where (x, y) are coordinates of the sampling point P, and C_1, C_2, $\cdots$, C_5 are the constant coefficients of the elliptic equation. Here, the monomial coefficient of x^2 is set to 1.

Let $F(x, y)$ be equal to an infinitesimal. As a minor deviation for approximation result, the infinitesimal can be ignored after linearizing the objective equation. Then, rewriting Eq. (3) as

$$F(x,\ y)=0,$$

provided $C_1^2-4C_2<0$. Let $\varDelta$ be the determinant

$$\varDelta=\begin{vmatrix}1 & C_1/2 & C_3/2\\ C_1/2 & C_2 & C_4/2\\ C_3/2 & C_4/2 & C_5\end{vmatrix}. \tag{4}$$

If $\varDelta<0$, the ellipse is a non-degenerate real ellipse; If $\varDelta>0$, the ellipse is an imaginary ellipse; if $\varDelta=0$, it is a point ellipse.

Each sampling point corresponds to an equation F_i, and a dataset of concentric ellipses f_k (k=1, 2,$\cdots$, n) composes a linear equation group E_k,

$$\begin{aligned}&E_k=f(C_{1k},C_{2k},C_{3k},C_{4k},C_{5k})=\\&\sum_{k=1}^{n}(x_k^2+C_1x_ky_k+C_2y_k^2+C_3x_k+C_4y_k+C_5\ )^2,\end{aligned} \tag{5}$$

where k is the number of sampling point, and k=1, 2,$\cdots$, n.

Our objective is to obtain the coefficients C_1, C_2, C_3, C_4, and C_5. Here, V, W, and U are defined by

$$V=(C_5,C_1,C_2,C_3,C_4),$$

$$W=\left(n,\sum x_ky_k,\sum y_k^2,\sum x_k,\sum y_k\right),$$

$$U=\left(\sum x_k^2,\sum x_k^3y_k,\sum x_k^2y_k^2,\sum x_k^3,\sum x_k^2y_k\right).$$

According to least squares theorem[21–22], the least squares solutions C_1, C_2, C_3, C_4 and C_5 of the real matrix V can be obtained by the generalized singular-value decomposition of the orthogonal matrix

$$V=-(W^{\mathrm{T}}W)^{-1}U^{\mathrm{T}}. \tag{6}$$

If the coefficients of ellipse are obtained, then the least squares centre of elliptic section can be calculated by

$$\begin{cases}x_o=\dfrac{2C_2C_3-C_1C_4}{C_1^2-4C_2},\\ y_o=\dfrac{2C_4-C_1C_3}{C_1^2-4C_2},\end{cases} \tag{7}$$

where $(x_O,\ y_O)$ are the coordinates of the least squares centre O of an ellipse.

In this method, the equal interval sampling and the strict rule of sampling distribution are not necessary. The advantage of the least squares method is to reduce the influences of form and sampling errors for the evaluation results, meanwhile, the calculation model is also not restricted by the measurement space.

The semi-major axis a and the semi-minor axis b are one half of the major and minor axes of the ellipse, respectively. a and b can be expressed as

$$\begin{cases}a=\sqrt{\dfrac{2(x_o^2+C_1x_oy_o+C_2y_o^2-C_5)}{1+C_3-\sqrt{(1-C_2)^2+C_3^2}}},\\ b=\sqrt{\dfrac{2(x_o^2+C_1x_oy_o+C_2y_o^2-C_5)}{1+C_3+\sqrt{(1-C_2)^2+C_3^2}}}.\end{cases} \tag{8}$$

When C_2=1, the ellipse is a standard circle. The relationship of a, b, and C_2 is as follows:

$$\begin{cases}a<b, & C_2<1,\\ a=b, & C_2=1,\\ a>b, & C_2>1.\end{cases}$$

The rotation angle of the ellipse in the Cartesian coordinate system is defined as θ, and it is expressed by

$$\theta=\frac{1}{2}\arctan\left(\frac{C_1}{1-C_2}\right). \tag{9}$$

The orthogonal projection point P' of the sampling point P mapped on the ellipse F can be calculated by

$$\begin{cases}x'^2+C_1x'y'+C_2y'^2+C_3x'+C_4y'+C_5=0,\\ (x-x')(C_1x'+2C_2y'+C_4)-(y-y')(2x'+C_1y'+C_3)=0,\end{cases} \tag{10}$$

where (x, y) are coordinates of the sampling point P, and (x', y') are coordinates of the orthogonal projection point P'.

The coordinates of two foci f_{C_1} (x_{fC_1}, y_{fC_1}) and f_{C_2} (x_{fC_2}, y_{fC_2}) of the ellipse are obtained by

$$\begin{cases} x_{fC_1} = x_o - \left(\dfrac{a^2 - b^2}{1+K^2}\right)^{0.5}, \\ y_{fC_1} = y_o - K\left(\dfrac{a^2 - b^2}{1+K^2}\right)^{0.5}, \end{cases}$$

$$\begin{cases} x_{fC_2} = x_o + \left(\dfrac{a^2 - b^2}{1+K^2}\right)^{0.5}, \\ y_{fC_2} = y_o + K\left(\dfrac{a^2 - b^2}{1+K^2}\right)^{0.5}, \end{cases}$$

$$K = \tan\theta . \tag{11}$$

Therefore, the sum of distances from any sampling point P_i (x_i, y_i) on the ellipse to those two foci can be represented by

$$D_i = \left\|P_i f_{C_1}\right\| + \left\|P_i f_{C_2}\right\| = \left[(x_i - x_{fC_1})^2 + (y_i - y_{fC_1})^2\right]^{\frac{1}{2}} + \left[(x_i - x_{fC_2})^2 + (y_i - y_{fC_2})^2\right]^{\frac{1}{2}}, \quad i=1, 2, \cdots, n. \tag{12}$$

3.2 Form error of ellipse based on LS evaluation

After obtaining the structure parameters of the least square ellipse, we hope not only to acquire the accurate form error of the elliptic section, but also to know the form error of entire profile. The form and profile errors of ellipse with LS evaluation can be worked out as follows.

We defined δ by

$$\delta = \frac{1}{2}(D - 2a), \tag{13}$$

let δ_i be the value of δ for P_i (x_i, y_i). Then the LS form errors of elliptic section can be written as

$$E_{\text{ef}} = \max\{\delta_i\} - \min\{\delta_i\}, \ i=1, 2, \cdots, n. \tag{14}$$

In this paper, we used δ as closely to true values as possible to denote the LS form errors of elliptic rather than use the radius deviation. In fact, δ is the deviation of every semi-major axis of the ellipse deviates from that of the LS ellipse. We defined the major axis as the reference axis in the form error of an ellipse, which is the basis for the evaluation. Meanwhile, the semi-major axis and the focus can be used as the evaluation parameters to judge the deviation degree for the form profile of ellipse. It is also to firstly propose an evaluation parameter for the form error of ellipse.

In addition, all sampling points corresponding to the ellipses must be consistent with two basic conditions in realizing the form error evaluation of ellipse: (1) All of the ellipses have the same centre; (2) All of the ellipses have the same foci.

3.3 Profile error of ellipse based on LS evaluation

We defined μ by

$$\mu_i = \left\|P_i P_i'\right\| = \left[(x_i - x_i')^2 + (y_i - y_i')^2\right]^{\frac{1}{2}}, \ i=1, 2, \cdots, n, \tag{15}$$

where $\begin{cases} \mu_i(+) = \mu_i, & D_i \geqslant 2a; \\ \mu_i(-) = -\mu_i, & D_i < 2a. \end{cases}$

Then the profile error of elliptic section can be expressed as

$$E_{\text{ep}} = \max\{\mu_i(+)\} - \min\{\mu_i(-)\}, \ i=1, 2, \cdots, n. \tag{16}$$

According to the definitions of form and profile errors of elliptic, the relationship is found:

$$E_{\text{ef}} \leqslant E_{\text{ep}}. \tag{17}$$

The difference between the form error E_{ef} and the profile error E_{ep} is defined as

$$D_{\text{s}} = |E_{\text{ef}} - E_{\text{ep}}|. \tag{18}$$

D_{s} can be used as a reference value of the profile error deviating from the form error and can also reflect the quality of the machined surface.

4 Data Estimation and Prediction

Obtaining a reasonable sample size is the foundation of accurate measuring. It was found that the Boltzmann curve can provide a better estimation for obtaining the sufficient prediction in Ref. [23]. Boltzmann curve fitting is a kind of improved mode of exponential curve fitting, which has not only advantages of exponential fitting but also a smaller fitting error. So a series of the different estimation results can also be obtained with the prediction method. According to Boltzmann curve, a nonlinear equation revealing the relationship between sample size and evaluation result is expressed by

$$y_{c_0,c_i}(x_i) = C_i + \frac{C_0 - C_i}{1 + \exp\left(\dfrac{x_i - x_0}{\mathrm{d}x}\right)}, \tag{19}$$

where x_i $(i = 0, \cdots, n.)$ is the sequence of sample size, x_0 is the first value of the sequence x_i, and dx is the interval of the neighboring sample sizes. C_0 and C_i are the corresponding least square results of sample sizes x_0 and x_i, respectively.

In Eq. (19), dx controls the speed which the evaluation result reaches a steady level, which indicates the stability of error evaluation system. As the known variables, C_i is relatively stable without any effects from other parameters. After a set of data fitting, x_0 can be obtained as the reference of sample size in error system.

5 Experimental Results

We were interested in the measurement of the heteromorphic form and focused on the geometric error analysis for a fixed number N of sampling points on the elliptic section. So an error evaluation model based on LS method is adopted to deal with the error measurement of ellipse using coordinate data from the CMM, and the proposed method has been applied to two examples.

5.1 Test and comparison

For the evaluations of form and profile errors of ellipse, it is difficult to define and evaluate the calculation accuracy of the modelling method with mathematical definitions. In this work, a specimen was used to test and verify the proposed method. In calculation process, the unit of the effective value was set to 0.000 1 mm, and the setup met the requirements of conventional computing. The calculation programs were developed based on Matlab and ran on a personal computer.

The used dataset in the specimen was published in Refs. [19–20] and is presented in Table 1, which was considered as a comparative sample to verify the evaluation method. Therein the profile error of the ellipse is 64.5 μm. The dataset includes 20 sampling points that are regularly distributed in 2D space, as shown in Fig. 3, and the results are listed in Table 2.

Table 1. Coordinates of sampling points from Ref. [19] mm

No.	x	y
1	18.642 2	9.739 5
2	15.676 9	17.992 1
3	10.970 3	24.751 9
4	5.564 0	28.726 6
5	−0.565 9	30.142 3
6	−6.791 7	28.692 3
7	−12.293 1	24.343 3
8	−16.880 5	17.138 5
9	−19.393 6	9.131 9
10	−20.227 8	−0.030 9
11	−19.033 6	−9.377 3
12	−16.030 3	−17.839 5
13	−11.635 3	−24.142 3
14	−5.840 1	−28.442 6
15	0.220 1	−29.849 9
16	6.458 6	−28.319 2
17	11.935 5	−24.003 6
18	16.345 3	−17.101 6
19	18.983 2	−8.871 7
20	19.745 6	0.986 3

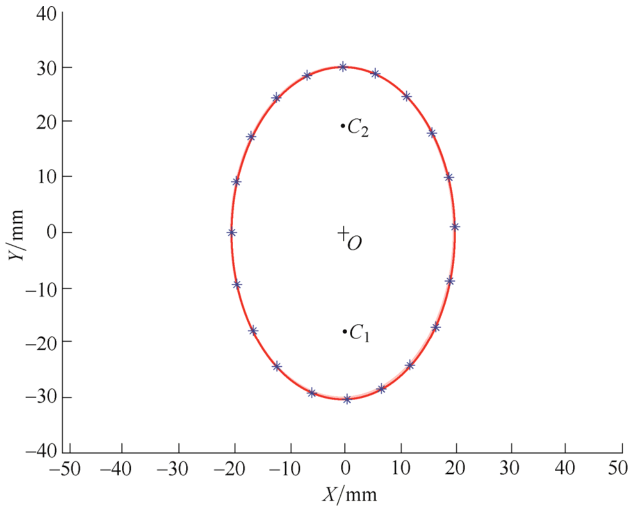


Fig. 3. Reconstruction model of the dataset from Ref. [19]

Table 2. Evaluation results of LS applied to the dataset from Ref. [19]

Parameter	Value
Coordinates of centre/mm	x=−0.206 6, y=0.158 4
Semi-major axis a/mm	30.004 9
Semi-minor axis b/mm	19.996 5
Angle of rotation/(°)	91.197 6
Form error/mm	0.061 0
Profile error/mm	0.064 4
Computation time/s	1.406 3

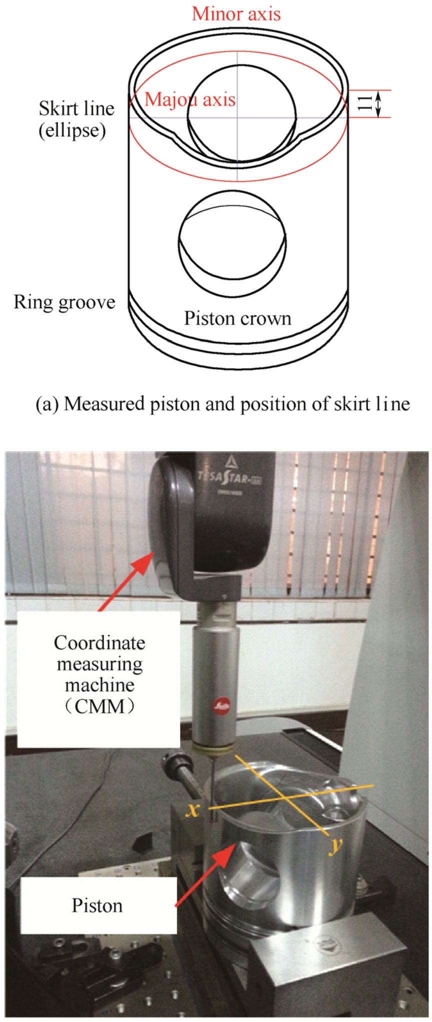
The form error of the ellipse evaluated by LS model is 61 μm, with the point 6 and the point 14 on the profile of the ellipse. As a comparison, the profile error of the ellipse obtained by LS model is 64.4 μm. The value of D_s is equal to 3.4 μm. It is clearly that the form error of the ellipse is smaller than the profile error of the ellipse. In addition, all of the structure parameters of an ellipse evaluation are given in Table 2, such as the coordinates of centre (x, y), the semi-major axis a, the semi-minor axis b, and the angle of rotation.

5.2 Experiment with real data

The experimental workpiece is a piston from a large power diesel engine. The form of piston skirt is an ellipse, which is a sealing surface in the cylinder of internal combustion engine. As an important component in powertrain is given, we hoped to obtain the form and profile errors of the piston skirt line from the measured data. Currently, there are two main measuring methods for the form error of the piston in industry. One method is to obtain the roundness error or the profile error of the piston using roundness or profile measuring instrument, and then, the maximum diameter and the roundness error are used to determine the profile error of the piston. The other method is the matching method, namely using the feeler gauge to measure the backlash between the piston and the cylinder. However, for industrial metrology and measurement, the form errors obtained by above methods are inadequate. Because, we want to obtain the accurate tolerance information from a batch of products, the applications of two methods would be restricted by measurement accuracy. Therefore, we depend on CMM to measure and evaluate the form error of the piston. Due to having a large measurement space, CMM can also measure the large size piston. Accurately obtaining the form parameters of piston skirt has certain requirement in product design and parts repair.

In this experiment, the diameter of the measured piston is 125.300 mm and the surface roughness R_a is 1.6. The profile error of the qualified piston is required in between 0.185 mm and 0.200 mm. The material of the piston is aluminum alloy AlSiCuMg. The measurement process was carried in a HEXAGON GLOBAL CLASS SR 575 (CMM), as shown in Fig. 4. The maximum permissible probing error (MPEP) and the positioning repeatability of the CMM are 2.3 μm and 0.5 μm, respectively. The positioning repeatability of the CMM and the surface roughness are reasonable values according to the referencing standard ISO 10360. The room temperature is 68 °F (20 ℃) and the humidity is 60%. The form and profile errors were calculated using LS method on the Lenovo-ThinkCentre M6000t.

For a piston, the hole of gudgeon pin is located at the direction of minor axis. The skirt line of the measured piston is located at the 11 mm below the edge and it is measured by the CMM with a tactile probe of 2 mm. The measurement consists of 32, 64, 128, 256, 512, 1024, and 2048 points. All of the points surround the entire elliptical surface. Different sample sizes correspond to independent measurement for the piston skirt line. The evaluation results of the form and the profile errors with LS method are presented in Table 3.



(a) Measured piston and position of skirt line

(b) Measuring equipment and piston

Fig. 4. Form and profile errors measurement of the piston skirt line

Table 3. The results of the LS applied to the evaluation of the form and profile errors of the ellipse

Evaluation model/No. of sampling points	Coordinates of centre/mm	Semi-major axis a/mm	Semi-minor axis b/mm	Angle of rotation θ/ (°)	Form error E_{ef}/mm	Profile error E_{ep}/mm
LS/32 points	x=–0.003 1 y=0.001 1	63.000 9	62.448 0	45.667 8	0.182 1	0.183 5
LS/64 points	x=–0.004 4 y=0.001 2	62.998 9	62.446 8	45.810 4	0.190 9	0.192 2
LS/128 points	x=–0.005 2 y=0.001 1	62.998 7	62.446 2	45.961 4	0.193 6	0.194 9
LS/256 points	x=–0.000 5 y=0.001 7	62.997 8	62.444 9	45.994 3	0.193 7	0.195 1
LS/512 points	x=–0.006 0 y=0.001 5	62.997 9	62.445 8	46.022 6	0.196 3	0.197 7
LS/1024 points	x=–0.005 6 y=0.001 8	62.997 9	62.445 3	46.035 6	0.196 3	0.197 7
LS/2048 points	x=–0.003 6 y=0.001 8	62.998 3	62.444 2	46.062 1	0.195 3	0.196 7

For each sample size and evaluation result, Boltzmann curve was adopted to fit all results with the minimum fitting error. In Fig. 5, the form error and the profile error of the piston skirt line have the same variation trend basically. The lower and upper deviations of the difference D_s between the form error and the profile error are 0.001 3 mm to 0.001 4 mm, respectively. Here, the form error is smaller than the profile error, which is consistent with our expectations. The value of D_s indirectly reflects the quality and the roughness of the machined surface. The final evaluation results are E_{ef}=0.195 23 and E_{ep}=0.196 63. These results are in line with the requirements of machining error, which also shows that the experiment is an effective measurement. Fig. 5 shows the relationships between the sample sizes and the evaluation results. In Table 3, the LS evaluation with 32 points makes a wrong result, which is often encountered in industrial measurement. A very large sample size could improve the accuracy of evaluation results, but measurement costs and time will be greatly increased. Boltzmann fitting can be used to obtain a predicted value, which is an initial value of sample size. In this measurement, the sample size was considered to achieve the acceptable accuracy at 418 points obtained by Eq. (19), the relative deviation of which is 0.5%. Of course, this is just a reference for this measurement.

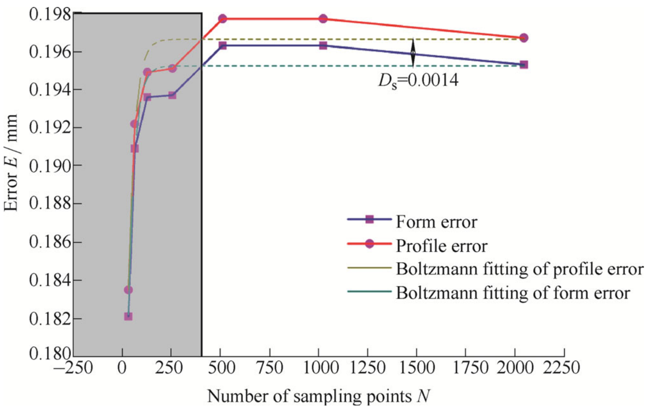


Fig. 5. Evaluation results of form and profile errors of the piston skirt line

In addition, it is found that the semi-major axis a and the semi-minor axis b have some changes by changing sample size. In Fig. 6, the values of a and b are decreased with the increase of the sample size, and their variation trend is the opposite of that of the form and profile errors. A detailed analysis of the results showed that the average error of the quadratic sum of the distances from foci to every sampling point is reduced with increasing the sample size. The maximum or minimum error in the least sum of squares fitting is considered as a relative error. Referencing Fig. 5 and Fig. 6, it is presented as a notable result that the values of a and b have the same variation trend. This implies that the area of the whole ellipse is shrinking when the sample size upgrades the evaluation result. The changes of rotation angle are shown in Fig. 7, the variation trend of which similar to that of the form error.

It can be seen from Tables 2 and 3 that the discrepancies among results are very small, which illustrates that the evaluations of the form and profile errors of ellipse based on LS model has better accuracy. Due to depending on the structure parameters of the measured profile rather than the overall surface, the established LS evaluation model is an accurate description for the elliptic section. In mass production, the form error and the profile error of ellipse must be controlled so as to achieve the purpose of precision production. Additionally, we analyzed the LS model with different sample sizes in measuring ellipse. If the relationship between the evaluation result and the sample size in the limitation of $N\to\infty$ is needed, a sampling strategy that specifies how the data positions increase on the ellipse would be necessary. This is the content of our research in future.

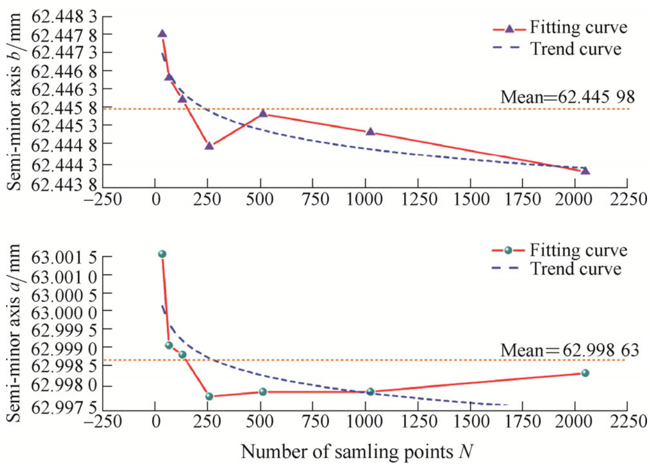


Fig. 6. Change of the semi-major axis a and the semi-minor axis b with increasing sample size in LS model

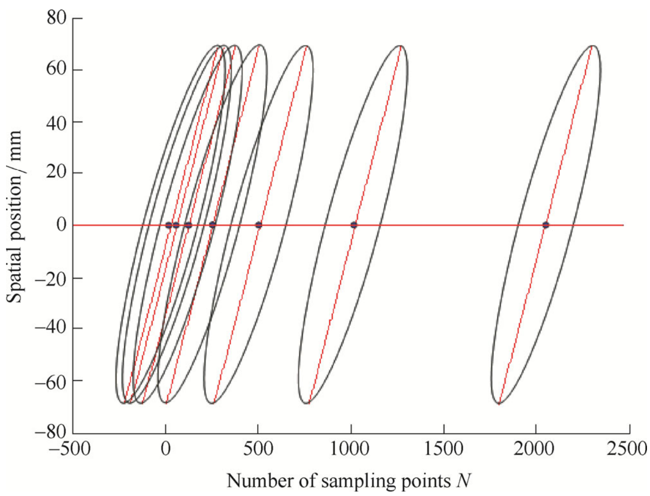


Fig. 7. Changes of rotation angle with increasing sample size

6 Conclusions

(1) With easy implementation and acceptable accuracy, a LS evaluation method is introduced to evaluate the form and profile errors of ellipse section.

(2) The definitions of form error and profile error in ellipticity evaluation are discussed, which are considered to be the basis of quantitative evaluation and are helpful to achieve that the evaluation results are consistent with the standards of form error inspection.

(3) The main feature of the evaluation method is able to differentiate error level, which can be used to calculate the uncertainty associated with the sample size.

(4) To reduce the measurement cost and improve the processing efficiency, Boltzmann fitting is used to provide

the flexibility to set a reasonable sample size.

(5) It is no doubt that this work provides a favorable solution for the measurement and evaluation of ellipse, the proposed method can also be applied to the measurement of renovating the worn piston.

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