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High Accurate Interpolation of NURBS Tool Path for CNC Machine Tools

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Abstract: Feedrate fluctuation caused by approximation errors of interpolation methods has great effects on machining quality in NURBS interpolation, but few methods can efficiently eliminate or reduce it to a satisfying level without sacrificing the computing efficiency at present. In order to solve this problem, a high accurate interpolation method for NURBS tool path is proposed. The proposed method can efficiently reduce the feedrate fluctuation by forming a quartic equation with respect to the curve parameter increment, which can be efficiently solved by analytic methods in real-time. Theoretically, the proposed method can totally eliminate the feedrate fluctuation for any 2nd degree NURBS curves and can interpolate 3rd degree NURBS curves with minimal feedrate fluctuation. Moreover, a smooth feedrate planning algorithm is also proposed to generate smooth tool motion with considering multiple constraints and scheduling errors by an efficient planning strategy. Experiments are conducted to verify the feasibility and applicability of the proposed method. This research presents a novel NURBS interpolation method with not only high accuracy but also satisfying computing efficiency.

Keywords: NURBS, interpolation, feedrate, machine tool, CNC

1 Introduction

Parametric curve interpolation is a very efficient way to realize high-speed and high-accuracy machining, as the tangency and curvature of parametric curves are both continuous for interpolators to generate smooth tool trajectories and feedrate profiles. The parametric curve interpolation involves several key fields such as parametric interpolation methods, feedrate planning algorithms and servo-loop control techniques, which are the research hotspots investigated by many scholars nowadays. Among all parametric curve models such as Bezier, B-spline and NURBS, NURBS has got the most attention as it offers a common mathematical form for the precise presentation of standard analytical shapes as well as free-from curves and surfaces[1]. Recently, STEP compliant NC programming, STEP-NC, has been specified as a new NC data model[2–3]. NURBS has been adopted by STEP-NC and becomes the standard interface for data exchange between CAD/CAM and CNC systems. In order to develop the next generation intelligent CNC machine tools and intelligent machining, it is really critical and urgent to design and realize a feasible, applicable and efficient NURBS interpolator.

The relationship between the curve parameter and arc length in NURBS is non-linear, and the calculation and derivation of NURBS are time-consuming, thus realizing a real-time and accurate NURBS interpolator is very challenging. Since approximation methods have been widely used for NURBS interpolation, feedrate fluctuation has been a crucial challenge due to approximation errors. HUANG, et al[4] and KOREN, et al[5], utilized the first- and second-order Taylor method to determine the approximate parameter value with constant feedrate. However, feedrate fluctuation related to truncation errors was inevitable, and the geometrical errors were not considered. LO[6], CHENG, et al[7], TSAI, et al[8], and ZHAO, et al[9], proposed several feedback interpolation methods, which can confine the feedrate fluctuation. However, these methods involved many iterations and the real-time performance deteriorated. YEH, et al[10], proposed a speed-controlled interpolator by incorporating a compensatory parameter into the Taylor method to minimize the effect of truncation errors. Nevertheless, longer CPU computational time was required owing to the algorithm's complexity. ERKORKMAZ, et al[11–12], adopted quintic polynomial for interpolation with minimal feedrate fluctuation. However, very time-consuming processes were needed. LEI, et al[13] and WU, et al[14], utilized fitting techniques to approximate the relationship between the arc length along the tool path and the curve parameter by polynomials and biarc splines, which can efficiently reduce the feedrate fluctuation. However, the time-consuming fitting processes limited the practical application of these techniques. LIN, et al[15] and SHEN, et al[16], introduced the axes dynamics into NURBS

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interpolation. However, feedrate fluctuation still existed. The aforementioned methods offered different schemes for NURBS interpolation with some practical effects, but the fast NURBS interpolation with minimal feedrate fluctuation in real-time context has not been realized yet.

The smooth feedrate planning is one of the most important factors for achieving the efficiency and quality of CNC machining. As the curvature along a NURBS curve varies frequently with many high curvature zones and smooth zones on the curve, the feedrate planning should decide where the feedrate accelerates or decelerates to ensure the interpolation accuracy and dynamic constraints. Most of existing feedrate planning methods can be classified into two groups: the look-ahead ones and the custom feedrate profile ones. The look-ahead methods, such as S-curve feedrate profile and sine-curve feedrate profile, plan the feedrate profile on single curve segment with look-ahead after the curve has been divided into several sub-curves, which have been investigated by many scholars for linear interpolation. LUO, et al[17], proposed a universal velocity profile generation approach for high-speed machining with look-ahead. YE, et al[18–19], proposed a look-ahead algorithm based on trapezoidal feedrate profile. CHENG, et al[20], presented a smooth S-curve feedrate profiling generation algorithm with continuous feedrate, acceleration, and jerk profiles. DONG, et al[21], introduced the contour error constraint into feedrate planning and provided a detailed planning algorithm based on S-curve profile. However, the aforementioned methods do not consider the rounding of the interpolation cycle count, which is very important to machining accuracy. LEE, et al[22], proposed the feedrate scheduling of NURBS interpolator for CNC machine tools with using sine-curve feedrate profile. The rounding errors of interpolation cycle count and the estimation errors of curve length were both considered by using a scheduling error compensation algorithm. However, the algorithm needs extra interpolation cycles to compensate the errors, which may reduce the machining efficiency. The custom feedrate profile methods, such as the feedrate interpolation method and the curve evolution method, plan the feedrate profile on the whole curve with using iteration methods or spline fitting techniques in non-real time. BEUDAERT, et al[23], proposed a feedrate interpolation method with axis jerk constraints on 5-axis NURBS tool path. SUN, et al[24–25], proposed a novel adaptive-feedrate interpolation method for NURBS tool path with drive constraints by using B-spline fitting and evolution techniques. However, these methods need time-consuming computing processes, thus the feedrate profile can only be customized offline and cannot be changed in real-time interpolation.

In order to solve the aforementioned problems and realize a feasible, applicable and efficient NURBS interpolator, this paper proposes a novel high accurate interpolation method of NURBS based on the authors' previous work presented by LIU, et al[26]. The proposed method improved the previous method by further reducing the feedrate fluctuation. Moreover, a smooth feedrate planning algorithm is presented with considering the rounding of the interpolation cycle count and other multiple constraints. The remainder of this paper is organized as follows: The NURBS curve and conventional interpolation method are presented in section 2. Section 3 proposes the improved quartic equation based interpolation method of NURBS with lower feedrate fluctuation. The smooth feedrate planning method for NURBS is presented in section 4. Experiments are conducted in section 5. Section 6 concludes this paper.

2 NURBS Curve and Conventional Interpolation Method

A NURBS curve $C(u)$ can be expressed as follows[1]:

$$C(u)=\frac{\sum_{i=0}^{n}N_{i,p}(u)P_iw_i}{\sum_{i=0}^{n}N_{i,p}(u)w_i},u\in[0,1], \tag{1}$$

where P_i is the control point, w_i is the corresponding weight of P_i, $(n+1)$ is the number of the control points, and p is the degree of the NURBS curve. $N_{i,\,p}(u)$ is the pth degree B-spline basis function defined on the non-uniform knot vector $U=\{u_0,\ u_1,\cdots,\ u_{n+p+1}\}$, recursively defined as follows:

$$N_{i,0}(u)=\begin{cases}0, \text{ if } u_i\leqslant u\leqslant u_{i+1},\\ 1, \text{ otherwise;}\end{cases}$$

$$N_{i,p}(u)=\frac{u-u_i}{u_{i+p}-u_i}N_{i,p-1}(u)+\frac{u_{i+p+1}-u}{u_{i+p+1}-u_{i+1}}N_{i+1,p-1}(u). \tag{2}$$

For generating a motion trajectory of the NURBS curve $C(u)$, the first step is to determine the curve parameter u. Taylor series expansion method is adopted in most NURBS interpolation algorithms. By employing Taylor's expansions of $u(t)$ at $t=t_i$ and neglecting the high-order terms, the second-order Taylor interpolation method is given as follows[26]:

$$u_{i+1}=u_i+\frac{L_i}{\left\|C'(u_i)\right\|}-\frac{\left\langle C'(u_i),C''(u_i)\right\rangle L_i^2}{2\left\|C'(u_i)\right\|^4}, \tag{3}$$

where L_i, $C'(u_i)$ and $C''(u_i)$ are the sampling step size, the first and second derivatives of the NURBS curve, respectively. For given feedrate V_i, acceleration A_i, jerk J_i and sampling time T_s, the sampling step size can be expressed as follows:

$$L_i=V_iT_s+\frac{1}{2}A_iT_s^2+\frac{1}{6}J_iT_s^3. \tag{4}$$

3 Improved Quartic Equation based Interpolation Method

Approximation errors inevitably exist when Eq. (3) is applied for NURBS interpolation, thus feedrate fluctuation ε arises, which can be defined by

$$\varepsilon = \frac{L_i - \left\| C(u_{i+1}) - C(u_i) \right\|}{L_i} \times 100\%. \tag{5}$$

In the previous work of the authors presented in Ref. [26], a quartic equation based interpolation(QEI) method was proposed to achieve minimal feedrate fluctuation. In this section, an improved quartic equation based interpolation (IQEI) method is presented to realize lower feedrate fluctuation than the previous one. Eq. (1) can be rewritten as follows:

$$C(u) = \frac{\boldsymbol{A}(u)}{B(u)}, u \in [0, 1], \tag{6}$$

where $\boldsymbol{A}(u)$ is a 3-dimension pth degree polynomial vector and $B(u)$ is a pth degree polynomial scalar.

Let u_i be a known quantity of curve parameter in the ith sampling time, and u_{i+1} be the unknown quantity of curve parameter in the $(i+1)$th sampling time, which should be calculated by the interpolation method in real-time. $\mathbf{A}(u_{i+1})$ and $B(u_{i+1})$ can be rewritten with using Taylor series expansions as follows:

$$\begin{cases} \boldsymbol{A}(u_{i+1}) = \sum\limits_{j=0}^{p} \dfrac{\boldsymbol{A}^{(j)}(u_i)(u_{i+1} - u_i)^j}{j!}, \\ B(u_{i+1}) = \sum\limits_{j=0}^{p} \dfrac{B^{(j)}(u_i)(u_{i+1} - u_i)^j}{j!}. \end{cases} \tag{7}$$

Ideally, the distance between two adjacent sampling points should be equal to the sampling step size in every sampling period, i.e. the following equation should be strictly satisfied:

$$\left\| C(u_{i+1}) - C(u_i) \right\| = L_i. \tag{8}$$

By integrating Eqs. (6)–(8), we can obtain

$$\left\| \frac{\sum\limits_{j=0}^{p} \dfrac{\boldsymbol{A}^{(j)}(u_i)(u_{i+1} - u_i)^j}{j!}}{\sum\limits_{j=0}^{p} \dfrac{B^{(j)}(u_i)(u_{i+1} - u_i)^j}{j!}} - \frac{\boldsymbol{A}(u_i)}{B(u_i)} \right\| = L_i. \tag{9}$$

Eq. (9) can be rewritten as follows:

$$\left\| \sum_{j=0}^{p} \frac{1}{j!} \left(\boldsymbol{A}^{(j)}(u_i) B(u_i) - \boldsymbol{A}(u_i) B^{(j)}(u_i) \right) (u_{i+1} - u_i)^j \right\| = \sum_{j=0}^{p} \frac{1}{j!} L_i B(u_i) B^{(j)}(u_i) (u_{i+1} - u_i)^j. \tag{10}$$

In calculating the next sampling parameter u_{i+1}, the value of u_i and the 0th to pth derivatives of $\boldsymbol{A}(u_i)$ and $B(u_i)$ are all known quantities. Define several variables as follows:

$$\begin{cases} \boldsymbol{a}_j = \dfrac{1}{j!} \left(\boldsymbol{A}^{(j)}(u_i) B(u_i) - \boldsymbol{A}(u_i) B^{(j)}(u_i) \right), j = 1, 2, \cdots, \\ b_j = \dfrac{1}{j!} L_i B(u_i) B^{(j)}(u_i), j = 0, 1, \cdots, \\ x = u_{i+1} - u_i. \end{cases} \tag{11}$$

By substituting Eq. (11) into Eq. (10), we can obtain a $(2p)$th polynomial equation with respect to variable x as follows:

$$a_{2p} x^{2p} + \cdots + a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 = 0. \tag{12}$$

By neglecting the high-order terms (orders $>$ 4), we can obtain a quartic equation with respect to x as follows:

$$a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 = 0, \tag{13}$$

where

$$\begin{aligned} a_4 &= \boldsymbol{a}_2^{\mathrm{T}} \boldsymbol{a}_2 + 2\boldsymbol{a}_3^{\mathrm{T}} \boldsymbol{a}_1 - b_2^2 - 2b_4 b_0 - 2b_3 b_1, \\ a_3 &= 2\boldsymbol{a}_2^{\mathrm{T}} \boldsymbol{a}_1 - 2b_3 b_0 - 2b_2 b_1, \\ a_2 &= \boldsymbol{a}_1^{\mathrm{T}} \boldsymbol{a}_1 - b_1^2 - 2b_2 b_0, \\ a_1 &= -2b_1 b_0, \\ a_0 &= -b_0^2. \end{aligned} \tag{14}$$

Eq. (13) can be solved by Ferrari's Formulas and Shengjin's Formulas very efficiently in real-time. The selection of parameter increment and the details of Ferrari's Formulas and Shengjin's Formulas are presented in Ref. [26]. After getting the proper value of variable x, the next sapling parameter can be obtained by

$$u_{i+1} = u_i + x. \tag{15}$$

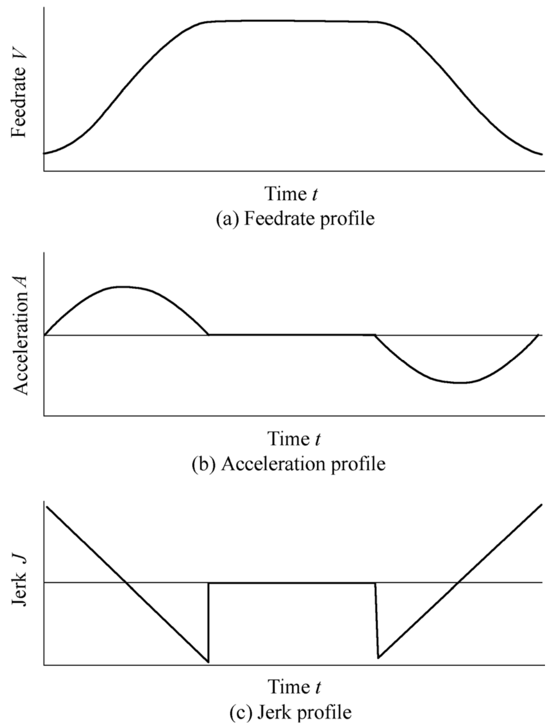
The above proposed method can achieve lower feedrate fluctuation than the previous QEI method, because the coefficients of Eq. (13) is more accurate than that of the QEI method, thus we call the proposed new method as improved quartic equation based interpolation method.

4 Smooth Feedrate Planning

The look-ahead feedrate planning method is adopted in this paper. First of all, the breakpoints with G^0 continuity and the critical points with large curvatures are detected. The suitable feedrate V_i at each of the points are adjusted to the limits of multiple constraints, such as chord error, centripetal acceleration and jerk, and axis dynamics[22]. Consequently, the NURBS curve is divided into small NURBS blocks after determining the breakpoints and critical points, which are called crucial points. The curve parameters u_i of crucial points are recorded simultaneously. The length S_i of each NURBS block between two adjacent crucial points are estimated by the adaptive quadrature method[13]. For a NURBS block, the suitable feedrate V_i of the two adjacent crucial points at the two ends of the NURBS block are set as the initial value of the start feedrate V_{is} and the end feedrate V_{ie}. The maximum feedrate V_{im} of each NURBS block are initialized by the command feedrate F. Finally, the data (u_i, V_{is}, V_{im}, V_{ie}, S_i) for each NURBS block are obtained and ready for the next stage of smooth feedrate planning.

4.1 Cubic polynomial feedrate profile

In industrial applications, the jerk-limited feedrate profile is essential for high-speed machining with high quality. The S-shape acceleration/deceleration profile is the most widely used method in current CNC systems with continuous acceleration and limited jerk. However, the complexity of the method leads to difficulties in realizing a real-time look-ahead module. Since the cubic polynomial feedrate profile shown in Fig. 1 is also jerk-limited and has continuous acceleration, meanwhile the computing process isn't complex and is easy to be implemented in a real-time look-ahead module, it is chosen to generate the feedrate profile for each NURBS block in this paper and its velocity equation is given as follows:



(a) Feedrate profile
(b) Acceleration profile
(c) Jerk profile

Fig. 1. Cubic polynomial feedrate profile

$$V_j=\begin{cases}(V_\mathrm{m}-V_\mathrm{s})\left[-2\left(\dfrac{j}{N_\mathrm{a}}\right)^3+3\left(\dfrac{j}{N_\mathrm{a}}\right)^2\right]+V_\mathrm{s}, \\ 0\leqslant j<N_\mathrm{a}; \\ V_\mathrm{m}, N_\mathrm{a}\leqslant j<N_\mathrm{a}+N_\mathrm{c}; \\ (V_\mathrm{m}-V_\mathrm{e})\left[2\left(\dfrac{j-N_\mathrm{a}-N_\mathrm{c}}{N_\mathrm{d}}\right)^3-3\left(\dfrac{j-N_\mathrm{a}-N_\mathrm{c}}{N_\mathrm{d}}\right)^2+1\right]+V_\mathrm{e}, \\ N_\mathrm{a}+N_\mathrm{c}\leqslant j<N_\mathrm{a}+N_\mathrm{c}+N_\mathrm{d},\end{cases} \tag{16}$$

where V_s, V_m and V_e denote the start velocity, maximum velocity and end velocity of a NURBS block, respectively. N_a, N_c and N_d denote the numbers of sampling time in the acceleration(ACC), constant feedrate(CF) and deceleration (DEC) sections, respectively. j denote the jth interpolation period. According to Eq. (16), the acceleration and jerk equations can be obtained as follows:

$$A_j=\begin{cases}\dfrac{V_\mathrm{m}-V_\mathrm{s}}{N_\mathrm{a}T_\mathrm{c}}\left[-6\left(\dfrac{j}{N_\mathrm{a}}\right)^2+6\left(\dfrac{j}{N_\mathrm{a}}\right)\right], 0\leqslant j<N_\mathrm{a}; \\ 0, N_\mathrm{a}\leqslant j<N_\mathrm{a}+N_\mathrm{c}; \\ \dfrac{V_\mathrm{m}-V_\mathrm{e}}{N_\mathrm{d}T_\mathrm{c}}\left[6\left(\dfrac{j-N_\mathrm{a}-N_\mathrm{c}}{N_\mathrm{d}}\right)^2-6\left(\dfrac{j-N_\mathrm{a}-N_\mathrm{c}}{N_\mathrm{d}}\right)\right], \\ N_\mathrm{a}+N_\mathrm{c}\leqslant j<N_\mathrm{a}+N_\mathrm{c}+N_\mathrm{d}.\end{cases} \tag{17}$$

$$J_j=\begin{cases}\dfrac{V_\mathrm{m}-V_\mathrm{s}}{\left(N_\mathrm{a}T_\mathrm{c}\right)^2}\left[-12\left(\dfrac{j}{N_\mathrm{a}}\right)+6\right], \\ 0\leqslant j<N_\mathrm{a}; \\ 0, N_\mathrm{a}\leqslant j<N_\mathrm{a}+N_\mathrm{c}; \\ \dfrac{V_\mathrm{m}-V_\mathrm{e}}{\left(N_\mathrm{d}T_\mathrm{c}\right)^2}\left[12\left(\dfrac{j-N_\mathrm{a}-N_\mathrm{c}}{N_\mathrm{d}}\right)-6\right], \\ N_\mathrm{a}+N_\mathrm{c}\leqslant j<N_\mathrm{a}+N_\mathrm{c}+N_\mathrm{d}.\end{cases} \tag{18}$$

When V_s, V_m, V_e, N_a, N_c and N_d are determined, the maximum acceleration and jerk in the ACC, CF and DEC sections can be obtained by Eqs. (17) and (18) as follows:

$$\max|A|=\begin{cases}\dfrac{3(V_\mathrm{m}-V_\mathrm{s})}{2N_\mathrm{a}T_\mathrm{c}}, 0\leqslant j<N_\mathrm{a}; \\ 0, N_\mathrm{a}\leqslant j<N_\mathrm{a}+N_\mathrm{c}; \\ \dfrac{3(V_\mathrm{m}-V_\mathrm{e})}{2N_\mathrm{d}T_\mathrm{c}}, N_\mathrm{a}+N_\mathrm{c}\leqslant j<N_\mathrm{a}+N_\mathrm{c}+N_\mathrm{d}.\end{cases}$$
$$\max|J|=\begin{cases}\dfrac{6(V_\mathrm{m}-V_\mathrm{s})}{\left(N_\mathrm{a}T_\mathrm{c}\right)^2}, 0\leqslant j<N_\mathrm{a}; \\ 0, N_\mathrm{a}\leqslant j<N_\mathrm{a}+N_\mathrm{c}; \\ \dfrac{6(V_\mathrm{m}-V_\mathrm{s})}{\left(N_\mathrm{d}T_\mathrm{c}\right)^2}, N_\mathrm{a}+N_\mathrm{c}\leqslant j<N_\mathrm{a}+N_\mathrm{c}+N_\mathrm{d}.\end{cases} \tag{19}$$

4.2 Planning strategy

For the ith NURBS block, the numbers of sampling time of each section in the feedrate profile are first calculated by

$$\begin{aligned} N_{\mathrm{ia}} &= \left[\max\left(\frac{3(V_{\mathrm{im}}-V_{\mathrm{is}})}{2A_{\mathrm{m}}T_{\mathrm{s}}}, \sqrt{\frac{6(V_{\mathrm{im}}-V_{\mathrm{is}})}{J_{\mathrm{m}}T_{\mathrm{s}}^2}}\right)\right], \\ N_{\mathrm{id}} &= \left[\max\left(\frac{3(V_{\mathrm{im}}-V_{\mathrm{ie}})}{2A_{\mathrm{m}}T_{\mathrm{s}}}, \sqrt{\frac{6(V_{\mathrm{im}}-V_{\mathrm{ie}})}{J_{\mathrm{m}}T_{\mathrm{s}}^2}}\right)\right], \\ N_{\mathrm{ic}} &= \left[\frac{2S_i-(V_{\mathrm{is}}+V_{\mathrm{im}})N_{\mathrm{ia}}T_{\mathrm{s}}-(V_{\mathrm{ie}}+V_{\mathrm{im}})N_{\mathrm{id}}T_{\mathrm{s}}}{2V_{\mathrm{im}}T_{\mathrm{s}}}\right], \end{aligned} \tag{20}$$

where operator '[ ]' denotes rounding down. The detailed algorithm of smooth feedrate planning is as follows:

Step 1: If $N_{\mathrm{ic}} > 0$, go to step 6; Otherwise, go to step 2.

Step 2: Update V_{im} by

$$V_{\mathrm{im}} = \frac{2S_{\mathrm{i}} - N_{\mathrm{ia}}T_{\mathrm{s}}V_{\mathrm{is}} - N_{\mathrm{id}}T_{\mathrm{s}}V_{\mathrm{ie}}}{N_{\mathrm{ia}}T_{\mathrm{s}} + N_{\mathrm{id}}T_{\mathrm{s}}}. \tag{21}$$

If $V_{\mathrm{im}} > \max(V_{\mathrm{is}}, V_{\mathrm{ie}})$, use Eq. (20) to recalculate N_{ia}, N_{id} and N_{ic}, then go to step 6; Otherwise, if $V_{\mathrm{im}} > \min(V_{\mathrm{is}}, V_{\mathrm{ie}})$, go to step 3; Otherwise, go to step 5.

Step 3: Reassign $V_{\mathrm{im}}=\max(V_{\mathrm{is}}, V_{\mathrm{ie}})$, use Eq. (20) to recalculate N_{ia}, N_{id} and N_{ic}, if $N_{\mathrm{ic}} > 0$, go to step 6; Otherwise, go to step 4.

Step 4: If $V_{\mathrm{is}} > V_{\mathrm{ie}}$, reassign V_{im} and V_{is} by

$$V_{\mathrm{im}} = V_{\mathrm{is}} = \frac{2S_{\mathrm{i}} - N_{\mathrm{id}}T_{\mathrm{s}}V_{\mathrm{ie}}}{N_{\mathrm{id}}T_{\mathrm{s}}}. \tag{22}$$

And assign $N_{\mathrm{ia}}=N_{\mathrm{ic}}=0$. Otherwise, reassign V_{im} and V_{ie} by

$$V_{im} = V_{\mathrm{ie}} = \frac{2S_{\mathrm{i}} - N_{\mathrm{ia}}T_{\mathrm{s}}V_{\mathrm{is}}}{N_{\mathrm{ia}}T_{\mathrm{s}}}. \tag{23}$$

And assign $N_{\mathrm{id}}=N_{\mathrm{ic}}=0$. Then go to step 6.

Step 5: If $\min(V_{\mathrm{is}}, V_{\mathrm{ie}}) > 0$, reassign $V_{\mathrm{is}}=V_{\mathrm{ie}}=V_{\mathrm{im}}=\min(V_{\mathrm{is}}, V_{\mathrm{ie}})$, and use Eq. (20) to recalculate N_{ia}, N_{id} and N_{ic}, then go to step 6.

Step 6: In order to ensure the continuity of global feedrate profile, the feedrate at the junction point between two adjacent NURBS blocks should be equal, thus assign $V_{(\mathrm{i}-1)\mathrm{e}}= V_{\mathrm{is}}$ and $V_{(\mathrm{i}+1)\mathrm{s}}=V_{\mathrm{ie}}$. If V_{is} has not been changed from steps 1 to 6, then update i by $i=i+1$, and go forward to plan the rest of NURBS blocks; Otherwise update i by $i=i-1$ and go back to re-plan the last NURBS block.

In order to compensate the rounding errors of sampling time numbers, a compensation strategy is proposed. Firstly, the rounding error of each NURBS block is calculated by

$$\Delta S_{\mathrm{i}} = S_{\mathrm{i}} - \sum_{j=0}^{N_{\mathrm{ia}}+N_{\mathrm{ic}}+N_{\mathrm{id}}-1} \left\| P_{j+1} - P_j \right\|, \tag{24}$$

where P_j are the sampling points on the ith NURBS block. Then the compensated feedrate ΔV_j in every sampling time can be obtained by

$$\Delta V_j = \begin{cases} V_{\mathrm{com}}\left[-2\left(\dfrac{2j}{N_{\mathrm{i}}}\right)^3 + 3\left(\dfrac{2j}{N_{\mathrm{i}}}\right)^2\right], j \leqslant \dfrac{N_{\mathrm{i}}}{2}; \\ V_{\mathrm{com}}\left[2\left(\dfrac{2j}{N_{\mathrm{i}}}\right)^3 - 3\left(\dfrac{2j}{N_{\mathrm{i}}}\right)^2 + 1\right], j > \dfrac{N_{\mathrm{i}}}{2}, \end{cases} \tag{25}$$

where j denotes the jth sampling period of the ith NURBS block, N_{i} and V_{com} are the total numbers of sampling time and the maximum compensated feedrate respectively, which can be calculated by

$$\begin{cases} N_{\mathrm{i}} = N_{\mathrm{ia}} + N_{\mathrm{ic}} + N_{\mathrm{id}}, \\ V_{\mathrm{com}} = \dfrac{2\Delta S_{\mathrm{i}}}{N_{\mathrm{i}}T_{\mathrm{s}}}. \end{cases} \tag{26}$$

Finally, the new feedrate after compensation can be expressed as

$$V_j^{\mathrm{new}} = V_j + \Delta V_j. \tag{27}$$

After the above processes, the rounding errors have been compensated. According to the value of N_{ia}, N_{id} and N_{ic}, there are several types of feedrate profile as shown in Figs. 1 and 2. Fig. 1 shows the feedrate profile type with ACC, CF and DEC sections. Fig. 2(a) shows the feedrate profile type with ACC and DEC sections. Fig. 2(b) shows the feedrate profile type with only CF section. Fig. 2(c) shows the feedrate profile type with ACC and CF sections. Fig. 2(d) shows the feedrate profile type with CF and DEC sections. Fig. 2(e) shows the feedrate profile type with only ACC section. Fig. 2(f) shows the feedrate profile type with only DEC section.

5 Experiments

The experiments are conducted on a 3-axis vertical machine tool developed by the authors. The CNC system is also implemented in TwinCAT by the authors with the ability of machining NURBS curves. The industrial PC, drivers and servo motors are both chosen from Beckhoff. Each axis includes a ball screw with 5 mm pitch. The servo motors both have 20 bit digital encoder, and are set in position control mode during the experiments. The interpolation sampling time is set to 1 ms. The following experiments will compare several interpolation methods, i.e. the first- and second-order Taylor method, the QEI method and the proposed IQEI method, on feedrate fluctuation by machining two NURBS curves. As the 2nd and 3rd degree NURBS curves are widely used in CNC machining, and high-order NURBS curves are rarely utilized, we will chose a 2nd degree NURBS curve and a 3rd degree NURBS curve as examples. The system parameters involved in the experiments are listed in Table 1.

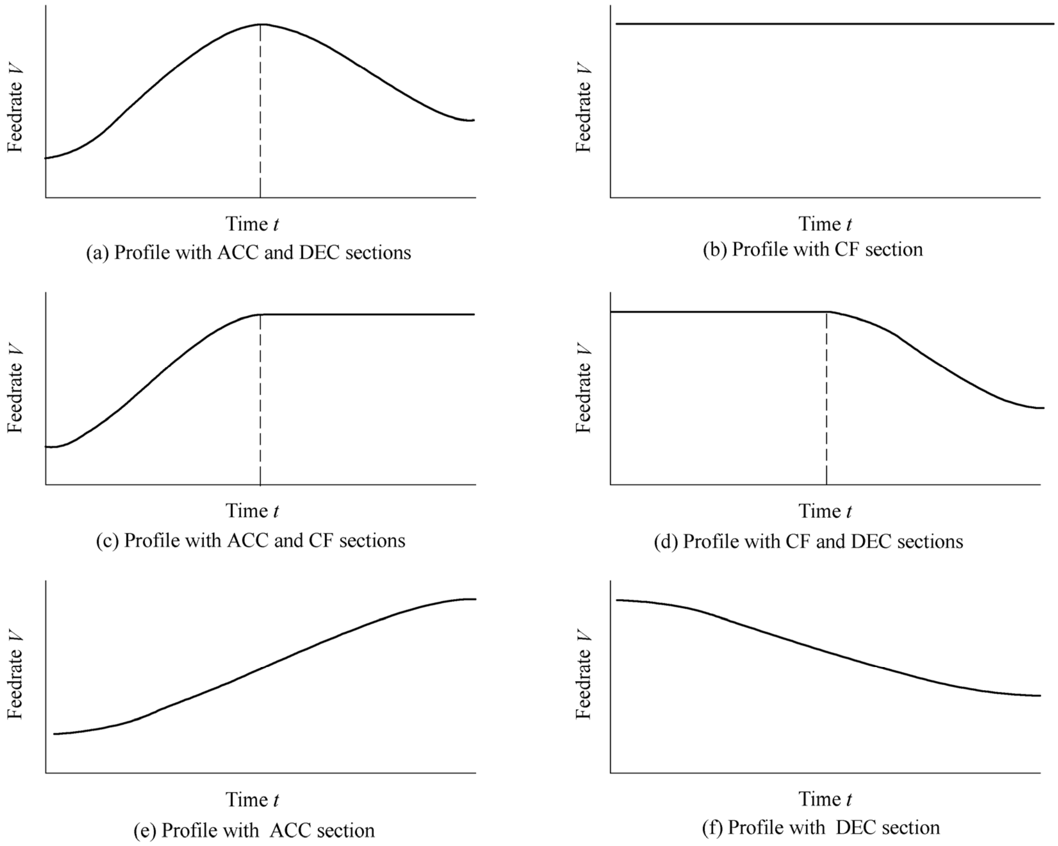


Fig. 2. Feedrate profile types

Table 1. System parameters

Parameter	Value
Sampling Time T_s / ms	1
Maximum chord error δ / μm	1
Maximum feedrate F / (mm • s^{-1})	100
Maximum acceleration A_m / (mm • s^{-2})	800
Maximum jerk J_m / (m • s^{-3})	25

5.1 Experiment 1

The first experiment is conducted on a 2nd degree NURBS curve shown in Fig. 3. Firstly, the proposed smooth feedrate planning algorithm is applied to generate smooth feedrate profile with the system parameters shown in Table 1. Fig. 4 shows the feedrate planning results of the 2nd degree NURBS curve. As we can see, the feedrate, acceleration and jerk are all within prescribed values with the proposed smooth feedrate planning algorithm. Then, the four interpolation methods are both applied to interpolate the NURBS curve based on the feedrate planning results. Fig. 5 shows the feedrate fluctuation profiles of the four interpolation methods during real-time interpolation. And Table 2 summarizes the results.

As we can see in Fig. 5(a), the first-order Taylor method gets the worst feedrate precision, and the maximum feedrate fluctuation is about 0.65%. The second-order Taylor method gets a better result, and the maximum feedrate fluctuation is about 0.084%. The QEI method and the proposed IQEI method get almost zero-fluctuation in feedrate as shown in Figs. 5(c) and (d). Thus, we can see that the QEI method and IQEI method are better than the first- and second-order Taylor method, and they have same accuracy for interpolating the 2nd degree NURBS curve. In addition, the average feedrate fluctuations of the four interpolation methods presented in Table 2 also demonstrate the same conclusion. Fig. 6 shows the machining result of the 2nd degree NURBS curve.

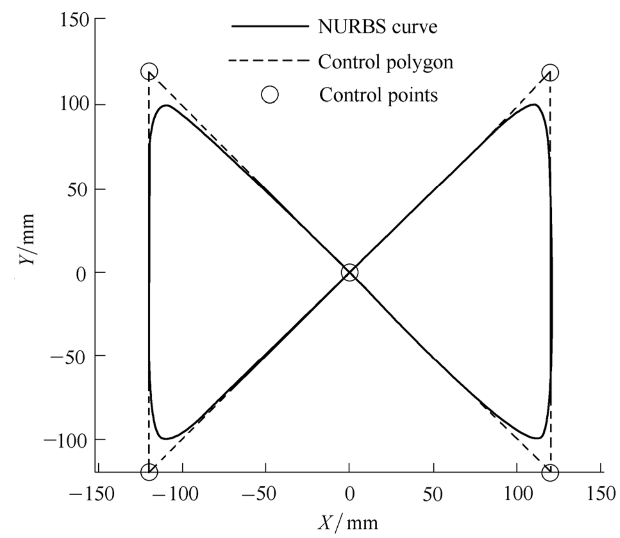
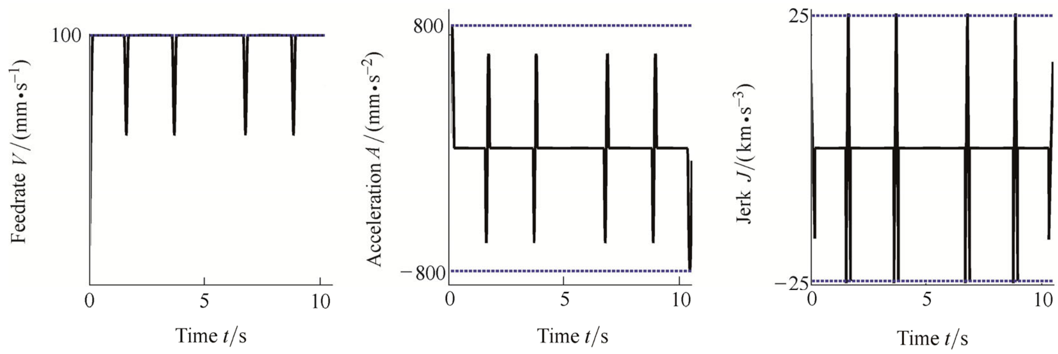
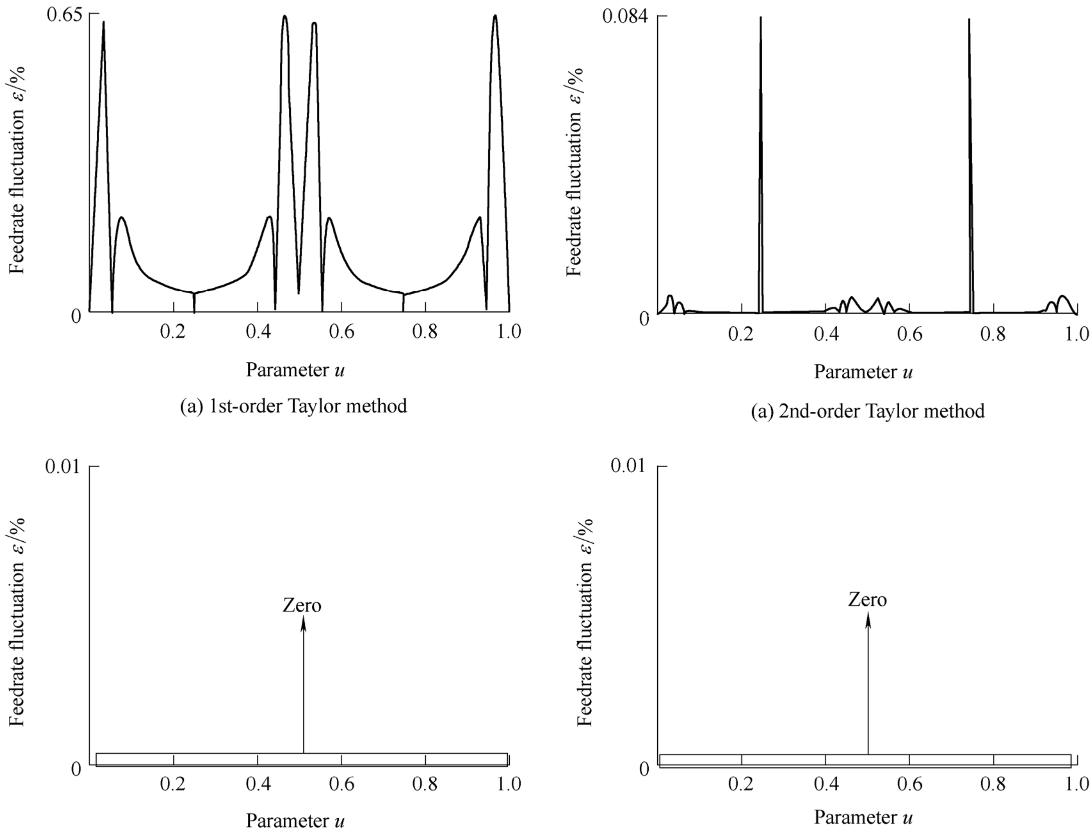


Fig. 3. The 2nd degree NURBS curve



(a) Feedrate profile (b) Acceleration profile (c) Jerk profile

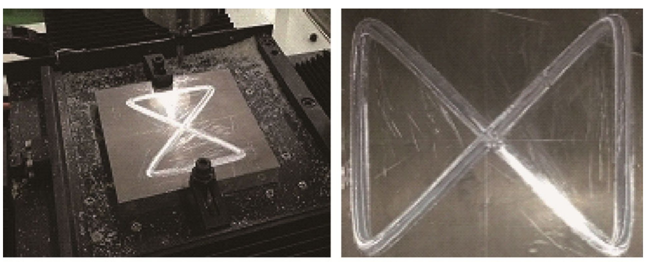
Fig. 4. Feedrate planning results of the 2nd degree NURBS curve



(a) 1st-order Taylor method (a) 2nd-order Taylor method

(c) QEI method (d) IQEI method

Fig. 5. Experimental results of the 2nd degree NURBS curve



(a) Workpiece machining (b) Machining result

Fig. 6. Machining result of the 2nd degree NURBS curve

5.2 Experiment 2

The butterfly shaped NURBS curve of 3rd degree shown in Fig. 7 is used as the second example to test the proposed method. The smooth feedrate planning algorithm is also first applied to generate the smooth feedrate profile, as shown in Fig. 8. As well, the feedrate, acceleration and jerk are all within prescribed values. Fig. 9 shows the experimental results of the four interpolation methods. As we can see, the maximum feedrate fluctuations of the first- and second-order Taylor methods are about 2.8% and 0.136%, respectively, which are not good results for high-quality machining. The QEI method gets a satisfied value about 0.013%, which is usually enough for industrial applications. The proposed IQEI method is the improved version of the QEI method, and the maximum feedrate fluctuation is 0.01%, which is smaller than that of the QEI method. It is not very clear to compare the merits between the QEI method and IQEI method on the value of maximum feedrate fluctuation as 0.013% and 0.01% are very close. As we can see in Table 3, the average feedrate fluctuation of the QEI method is 0.001 9%, while the average feedrate fluctuation of the IQEI method is only 0.000 029%, about 1/65 of that of the QEI method. Thus, the proposed IQEI method makes a great improvement in further reducing the feedrate fluctuation over the QEI method. Fig. 10 shows the machining result of the 3rd degree NURBS curve.

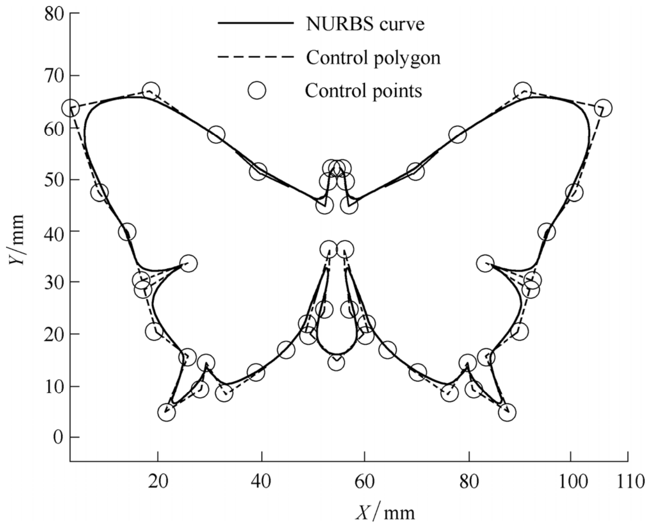


Fig. 7. The 3rd degree NURBS curve

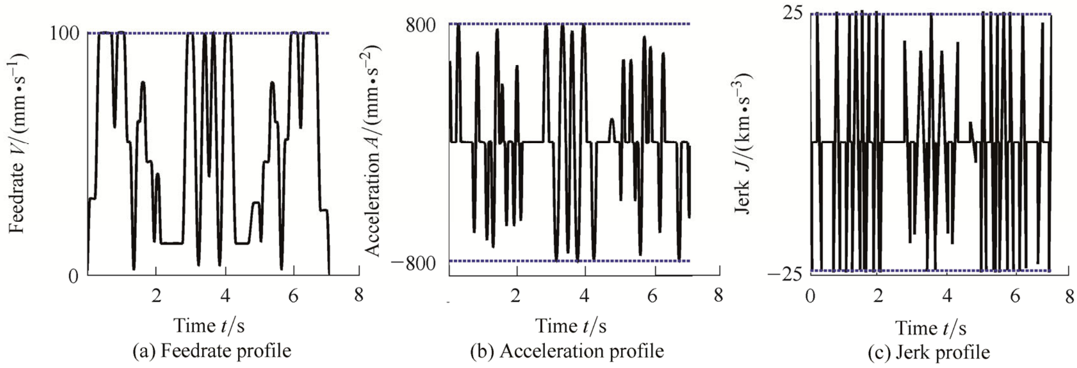


Fig. 8. Feedrate planning results of the 3rd degree NURBS curve

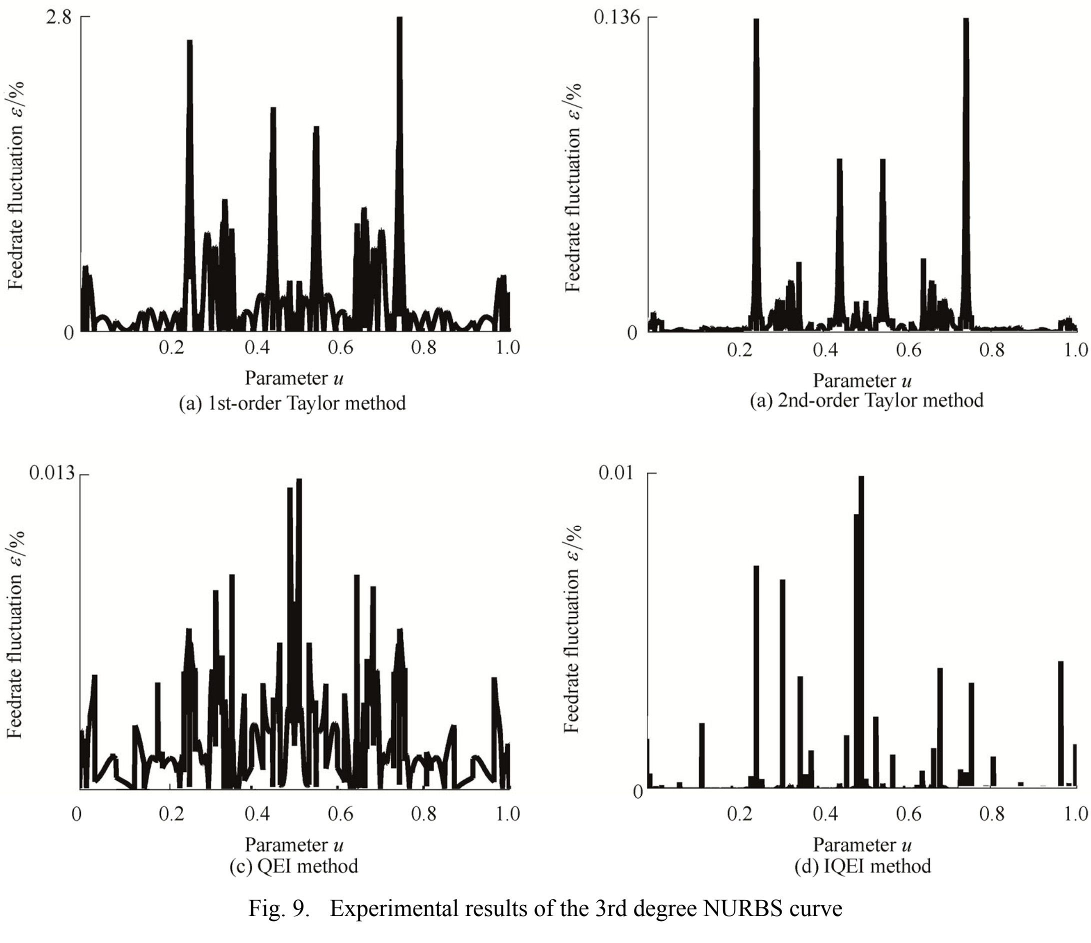
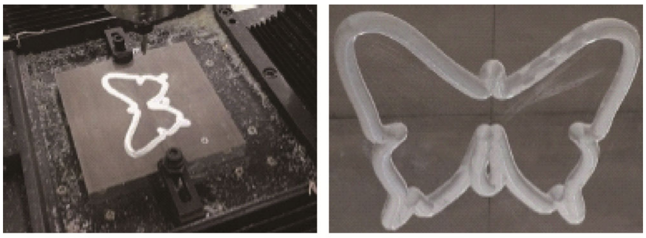


Fig. 9. Experimental results of the 3rd degree NURBS curve



(a) Workpiece machining (b) Machining result

Fig. 10. Machining result of the 3rd degree NURBS curve

5.3 Computing efficiency analysis

Accuracy and computing efficiency always contradict with each other, and high accuracy means low computing efficiency. In the proposed IQEI method, the quartic equation is solved by analytic methods, i.e. the Ferrari's formulas and Shengjin's formulas. In order to verify the computing efficiency of the proposed method, comparisons between the four interpolation methods on average interpolation time are conducted on a same PC with an Intel(R) Core™2 Duo CPU P9300@2.26GHz. The test program is written by C# in Visual Studio 2013, and the "Stopwatch" class is used to measure the running time of each interpolation. The test results of the 2nd degree NURBS curve and the 3rd degree NURBS curve are shown in Table 2 and Table 3, respectively.

Table 2. Experimental results of the 2nd degree NURBS curve

Interpolation method	Max • federate fluctuation / %	Ave • federate fluctuation / %	Ave • interpolation time / μs
1st-order Taylor	0.65	0.1252	6.27
2nd-order Taylor	0.084	0.000 37	7.76
QEI	0	0	7.23
IQEI	0	0	7.24

Table 3. Experimental results of the 3rd degree NURBS curve

Interpolation method	Max. feedrate fluctuation / %	Ave. federate fluctuation / %	Ave. interpolation time / μs
1st-order Taylor	2.8	0.3146	6.53
2nd-order Taylor	0.136	0.006 3	9.38
QEI	0.013	0.001 9	9.62
IQEI	0.01	0.000 029	10.50

From Table 2, even if the 1st-order Taylor method consumes the least computing time, it gets the worst feedrate fluctuation both in maximum feedrate fluctuation and average feedrate fluctuation. The 2nd-order Taylor method gets a relative low feedrate fluctuation, meanwhile the computing time increases. The computing time of QEI method and IQEI method is almost same and smaller than that of the 2nd-order Taylor method. Meanwhile the maximum feedrate fluctuation and average feedrate fluctuation of the QEI method and the IQEI method are both zero. Thus, for interpolating the simple 2nd degree NURBS curve, the QEI method and the IQEI method have great advantages over the 1st- and 2nd-order Taylor methods when taking both the computing efficiency and feedrate fluctuation into account. It is hard to decide whether the QEI method is the best method or the IQEI method is the best method among the four interpolation methods when only consider interpolating the simple 2nd degree NURBS curve, thus further analysis on interpolating the 3rd degree NURBS curve should be carried out, which will be shown in the following paragraph.

In Table 3, it is no doubt that the 1st-order Taylor method consumes the least computing time as only the first order derivative of NURBS curve is needed in real time interpolation, but the worst feedrate fluctuation determines that it cannot be used for industrial application. Though the 2nd-order Taylor method gets a much lower feedrate fluctuation than the 1st-order Taylor method, it is not precise enough for high speed and high precision CNC machining. The QEI method and the IQEI method get satisfying feedrate fluctuation levels, which are both under 1/10 of that of the 2nd-order Taylor method. Although the proposed IQEI method consumes a little more time (less than 1 μs) than the QEI method, it can further reduce the feedrate fluctuation, i.e. the maximum feedrate fluctuation has been further reduced by 23.08% and the average feedrate fluctuation has been further reduced by 98.47% when compared with the QEI method, which are significant improvements on feedrate precision. Moreover, the extra computing time can be almost neglected when compared with the prescribed sampling time (1 ms). Compared with the significant improvement on feedrate precision presented by the aforementioned experiments and analysis, the negligible reduction on efficiency is worthwhile. From the above, the proposed IQEI method is the best method among the four interpolation methods when taking both the computing efficiency and feedrate fluctuation into account.

6 Conclusions

(1) A novel improved quartic equation based interpolation method of NURBS tool path is proposed, which can totally eliminate the feedrate fluctuation for any 2nd degree NURBS curves and can interpolate any 3rd degree NURBS curves with minimal feedrate fluctuation.

(2) A smooth feedrate planning algorithm is presented to generate smooth motion, which considers both the rounding of interpolation cycle count and other multiple constraints.

(3) Experiments are conducted to verify the feasibility and applicability of the proposed interpolation method. Results show that, the proposed method can achieve not only high accuracy but also satisfying computing efficiency when compared with other methods.

(4) In the future, the proposed method needs to be further improved as it is good for any 2nd and 3rd degree NURBS curves, but not good enough for high degree NURBS curves in the present. The later work of the authors is to

develop the IQEI method to interpolate high degree NURBS curves with satisfying feedrate fluctuation level.

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