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Theoretical, Numerical and Experimental Study on Synchronization of Three Identical Exciters in a Vibrating System

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Abstract: The theory on synchronization of two exciters is more widely used in engineering, while that of more than two exciters is less considered. So it is of great significant to investigate synchronization of three exciters. Firstly by introducing the average method of modified small parameters, the dimensionless coupling equations(DCE) of three exciters are derived, which convert the problem of synchronization into that of existence and stability of zero solutions for the DCE and lead to the construction on criterions of synchronization and stability in the simplified form for three exciters. Then the synchronization criterion is discussed numerically, as well as the abilities of synchronization and stability, some results thereof indicate that the synchronization ability increases with the increase of the coupling moment among three exciters, but decreases with that of their phase differences. Finally, an experiment on synchronization with three exciters is carried out. Through the comparison and analysis of experimental data on phase differences among three exciters, responses of system, and phases of three exciters recorded by high-speed camera, the parameters of system satisfying the above two criterions can ensure the synchronous and stable operation of three exciters. As a result, the average method of modified small parameters can be used as a theoretical apparatus studying reasonably the synchronization mechanism of three exciters, it is also proved to be useful and feasible by numeric and experiment. The present research lays the foundation and guidance for the establishment of synchronization theory system with multi-exciter and engineering design.

Key words: synchronization, vibrating system, stability, coupling dynamic, double-equilibrium

1 Introduction

Synchronization is a distinctive phenomenon in nonlinear system, it was first described more than three hundred years ago by HUYGENS. Synchronization is defined as the process of accommodation of the responses of two or more coupled nonlinear oscillators until a periodic steady state is achieved in which all oscillators lock in to a common period[1]. So far synchronization theory is well developed, such as synchronization of clocks, phase oscillators or oscillatory networks[2–5]. In mechanical engineering, especially, with the invention of a variety of vibrating machinery, synchronization theory of the vibrating machines with two or multiple exciters solved a number of practical problems, e.g., self-synchronous vibrating feeders, conveyors, vibrating coolers, and so on.

For the origin of self-synchronization of two unbalanced rotors (exciters), about sixty years ago, in Leningrad, it was accidently discovered that two unbalanced rotors driven by two motors on a single base trended to operate synchronously, later Dr. BLEKHMAN, et al[1, 6–10], gave the first theoretical explanations of this synchronization phenomenon. These remarkable facts stimulated greatly the interests of many researchers, one of the representative personages was Chinese scholar professor WEN, et al[11–14], who extended such synchronization theory and applied it to engineering successfully, so as to establish vibration utilization engineering, by considering adequately the damping effect of the vibrating system. Due to the complexity of the system structure, there are many factors to influence the index of synchronization, so controlled or hybrid synchronization[8, 9, 11, 15–16] is needed to meet the requirement in engineering.

The main theoretical methods used on synchronization of exciters at present, are the method of direct separation of motions[1, 6–10] and the averaging method of small parameters[11–23]. In the former method, the effect of damping and dynamic characteristics of induced motors are less considered, which results in the absence of the effect of electric-mechanic coupling analysis. Overcoming the abovementioned imperfection, synchronization and vibratory synchronization transmission of two exciters on a single base were discussed in detail by authors[17–20], as well as the general dynamic symmetry for a vibrating system with two exciters[21–22].

Besides the cited publications, extensive literature is devoted to the investigation on synchronization of more

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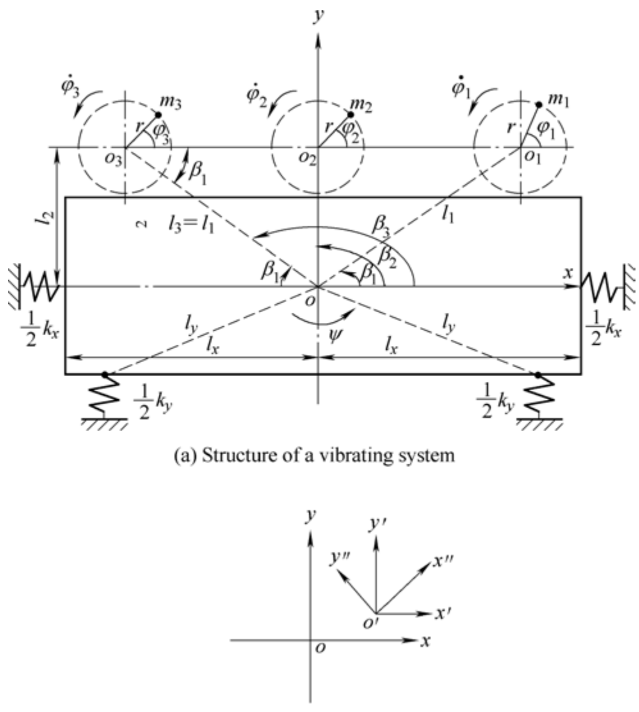
than two exciters, the corresponding results lies in the fact that synchronization of four identical exciters on double-base and some characteristics of selecting motion were proposed[23], and some comments on the numerical simulation of self-synchronization of four non-ideal exciters were also given by BALTHAZAR, et al[24].

The theoretical investigation on synchronization of multi-exciter aims at utilizing vibration or eliminating one, based on the principle of superposition of system. In light of the former, the properly positive superposition of the exciting forces is implemented to enhance the effective power of the vibrating system; while for the latter, the case is reverse. Generally in engineering, most researchers focus on synchronization for vibration utilization, i.e., the operation with zero phase difference for two or more exciters is implemented to improve the power of system. Although the significant theory achievements are emerged, the problem is far from being exhausted, such as that the coupling dynamics of more than two exciters should be understood perfectly, and the corresponding theoretical results are needed further verification by experiment or engineering application in practice. In order to both realize this purpose and keep the compact structure of system, we use three exciters instead of two on a single base, to analyse its coupling characteristics and verify whether the effective power of system is enhanced or not, thus lay a foundation for the establishment of synchronization theory system with multi-exciter and its engineering design supervision, by theoretical, numerical and experimental method.

In this paper, our attentions are restricted to the far-resonant vibrating system of plane motion, the average method of modified small parameters is employed to investigate synchronization of three exciters. In the next section, equations of motion of the vibrating system are described. Section 3 is devoted to deriving the criterions of synchronization and stability of synchronous states. Numeric results and discussions are in section 4 and experimental results are in section 5. Finally, conclusions are provided.

2 Equations of Motion of System

Fig. 1 shows the dynamic model of a considered vibrating system, which consists of a rigid frame and three exciters driven separately by three induction motors rotating in the same directions. The rigid frame is supported on an elastic foundation consisting of four springs installed symmetrically. The spin axis center of the middle exciter 2 is in the vertical central line of the rigid frame, the other two exciters are installed symmetrically on both sides of the vertical center line of the rigid frame, and the three exciters' pivots are all in a level line paralleling to x-axis, as shown in Fig. 1(a). The frame oxy is a fixed frame, and its origin o is the equilibrium point of centroid of the rigid frame; $o'x'y'$ is the non-rotating moving frame, which undergoes the translation motion and parallels to oxy; the moving frame $o'x''y''$, is fixed to the rigid frame, as shown in Fig. 1(b). Three reference frames coincide with each other when the vibrating system does not operate.



(a) Structure of a vibrating system

(b) Coordinate system

Fig. 1. Dynamic model of a vibrating system with three exciters

Because the rigid frame is supported by an elastic foundation, it exhibits three degrees of freedom. The mass center coordinates, x and y, and the angular rotation ψ are set as independent coordinates. Exciters 1, 2 and 3 rotate about their own spin axes, which are denoted by φ_1, φ_2 and φ_3, respectively.

The kinetic energy, potential energy and viscous dissipation function of system can be deduced, denoted by T, V, and D, respectively, which are substituted into the following Lagrange's equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial(T-V)}{\partial\dot{q}_i}-\frac{\partial(T-V)}{\partial q_i}+\frac{\partial D}{\partial\dot{q}_i}=Q_i. \qquad (1)$$

Here, we assume that three exciters are identical, i.e., $m_1=m_2=m_3=m_0$. If $\boldsymbol{q}=(x\ y\ \psi\ \varphi_1\ \varphi_2\ \varphi_3)^{\mathrm{T}}$ is chosen as the generalized coordinates, the generalized forces $Q_i=T_{ei}$ (i=1, 2, 3) and the others are zero. According to Refs. [11–12], m_0 is far smaller than m and ψ far smaller than 1. Hence, the inertia coupling stemming from asymmetry of three exciters can be neglected. The equations of motion of the vibrating system can be simplified as follows:

$$M\ddot{x}+f_x\dot{x}+k_x x=\sum_{i=1}^{3}m_0 r(\dot{\varphi}_i^2\cos\varphi_i+\ddot{\varphi}_i\sin\varphi_i),$$

$$M\ddot{y}+f_y\dot{y}+k_y y=\sum_{i=1}^{3}m_0 r(\dot{\varphi}_i^2\sin\varphi_i-\ddot{\varphi}_i\cos\varphi_i),$$

$$J\ddot{\psi}+f_\psi\dot{\psi}+k_\psi\psi=\sum_{i=1}^{3}m_0rl_i[\dot{\varphi}_i^2\sin(\varphi_i-\beta_i)-\ddot{\varphi}_i\cos(\varphi_i-\beta_i)],$$
$$m_0r^2\ddot{\varphi}_i+f_i\dot{\varphi}_i=T_{ei}-m_0r[\ddot{y}\cos\varphi_i-\ddot{x}\sin\varphi_i+l_i\ddot{\psi}\cos(\varphi_i-\beta_i)+l_i\dot{\psi}^2\sin(\varphi_i-\beta_i)],$$
$$i=1,2,3,\ l_3=l_1,\ l_2=l_1\sin\beta_1. \quad (2)$$

where m—Mass of the rigid frame,
m_0—Mass of each exciter,
$M=m+3m_0$,
k_x, k_y, k_ψ—Constants of springs in x-, y- and ψ-directions, respectively,
$k_\psi=(k_yl_x^2+k_xl_y^2)/2$,
f_x, f_y, f_ψ—Damping constants in x-, y-, and ψ-directions, respectively,
$f_\psi=(f_yl_x^2+f_xl_y^2)/2$,
f_i—Damping constant of rotor of the motor i,
l_i—Distance between the rotational centre o_i of exciter i and the mass centre o of the rigid frame,
r—Eccentric radius of each exciter,
l_e—Equivalent rotating radius of the vibrating system about the centroid of the rigid frame,
J_m—Moment of inertia of the rigid frame,
T_{ei}—Electromagnetic torque of the motor i,
$(\dot{\bullet})$—$d(\bullet)/dt$,
$(\ddot{\bullet})$—$d^2(\bullet)/dt^2$,
$$J=Ml_e^2=J_m+3m_0r^2+\sum_{i=1}^{3}m_0l_i^2,$$
$\beta_1=\beta_1$, $\beta_2=\pi/2$, $\beta_3=\pi-\beta_1$.

3 Synchronization of Three Exciters and Stability of Synchronous States

We assume that the average phase of three exciters is φ, the phase difference between exciters 1 and 2 is $2\alpha_1$, and that between exciters 2 and 3 is $2\alpha_2$, i.e.,

$$\varphi=\frac{1}{3}\sum_{i=1}^{3}\varphi_i,\ \ \varphi_1-\varphi_2=2\alpha_1,\ \ \varphi_2-\varphi_3=2\alpha_2. \quad (3)$$

Because the vibration of the vibrating system is periodic, the change of average angular velocity $\dot{\varphi}$ of three exciters is also periodic. If the least common multiple period of three exciters is assumed to be T_{LCMP}, the average value of the average angular velocity $\dot{\varphi}$ over the time T_{LCMP} must be a constant, i.e.,

$$\omega_{m0}=\frac{1}{T_{LCMP}}\int_{t'}^{t'+T_{LCMP}}\dot{\varphi}dt=\text{constant}. \quad (4)$$

According to the modified average method of small parameters[18–23], we assume

$$\dot{\varphi}=(1+\varsigma_0)\omega_{m0},\ \dot{\alpha}_i=\varsigma_i\omega_{m0},\ i=1,2, \quad (5)$$

where ς_0, ς_i (ς_0, ς_i are functions of time t, $i=1, 2$) are the coefficients of the instantaneous change of $\dot{\varphi}$ and $\dot{\alpha}_i$ around ω_{m0}, respectively.

Differentiating Eq. (3) with respect to time t and considering Eq. (5) yield

$$\begin{aligned}
&\varphi_1=\varphi+4\alpha_1/3+2\alpha_2/3=\varphi+\upsilon_1,\\
&\varphi_2=\varphi-2\alpha_1/3+2\alpha_2/3=\varphi+\upsilon_2,\\
&\varphi_3=\varphi-2\alpha_1/3-4\alpha_2/3=\varphi+\upsilon_3,\\
&\dot{\varphi}_1=(1+\varsigma_0+4\varsigma_1/3+2\varsigma_2/3)\omega_{m0}=(1+\varepsilon_1)\omega_{m0},\\
&\dot{\varphi}_2=(1+\varsigma_0-2\varsigma_1/3+2\varsigma_2/3)\omega_{m0}=(1+\varepsilon_2)\omega_{m0},\\
&\dot{\varphi}_3=(1+\varsigma_0-2\varsigma_1/3-4\varsigma_2/3)\omega_{m0}=(1+\varepsilon_3)\omega_{m0},\\
&\ddot{\varphi}_1=\dot{\varepsilon}_1\omega_{m0},\ \ddot{\varphi}_2=\dot{\varepsilon}_2\omega_{m0},\ \ddot{\varphi}_3=\dot{\varepsilon}_3\omega_{m0}.
\end{aligned} \quad (6)$$

If the average values of ε_1, ε_2 and ε_3 over the single period ($T_0=2\pi/\omega_{m0}$) are zero, i.e., $\bar{\varepsilon}_1=0$, $\bar{\varepsilon}_2=0$ and $\bar{\varepsilon}_3=0$, the three motors operate synchronously. When the vibrating system operates in the steady-state, the angular accelerations of the three motors change very little (close to zero), so $\ddot{\varphi}_1$, $\ddot{\varphi}_2$ and $\ddot{\varphi}_3$ can be neglected in the first three formulae of Eq. (2).

According to Refs. [18–23], in a far-resonant vibrating system with small damping, the responses of the steady-state in x-, y- and ψ-directions can be expressed in the form:

$$\begin{cases}
x=-\dfrac{r_mr}{\mu_x}\displaystyle\sum_{i=1}^{3}\cos(\varphi+\upsilon_i+\gamma_x),\\
y=-\dfrac{r_mr}{\mu_y}\displaystyle\sum_{i=1}^{3}\sin(\varphi+\upsilon_i+\gamma_y),\\
\psi=-\dfrac{r_mr}{\mu_\psi l_e}[r_{l1}\sin(\varphi+\upsilon_1-\beta_1+\gamma_\psi)+r_{l2}\sin(\varphi+\upsilon_2-\beta_2+\gamma_\psi)+r_{l1}\sin(\varphi+\upsilon_3-\beta_3+\gamma_\psi)],
\end{cases} \quad (7)$$

where ω_{ni}—Natural frequency of the vibrating system in i-direction, $i=x, y, \psi$,
$\omega_{nx}^2=k_x/M$, $\omega_{ny}^2=k_y/M$, $\omega_{n\psi}^2=k_\psi/J$,
ξ_{ni}—Corresponding damping ratio of spring,
$$\xi_{nx}=\frac{f_x}{2\sqrt{k_xM}},\ \xi_{ny}=\frac{f_y}{2\sqrt{k_yM}},\ \xi_{n\psi}=\frac{f_\psi}{2\sqrt{k_\psi J}},$$
$$\mu_i=1-\omega_{ni}^2/\omega_{m0}^2,\ \gamma_i=\arctan\frac{2\xi_{ni}(\omega_{ni}/\omega_{m0})}{1-(\omega_{ni}/\omega_{m0})^2},$$
$r_m=m_0/M$,
$\pi-\gamma_i$—Phase angle in i-direction,
$r_{lj}=l_j/l_e$, $j=1, 2, 3$.

Differentiating Eq. (7) to obtain $\ddot{x}$, $\ddot{y}$, $\dot{\psi}$, and $\ddot{\psi}$, inserting them into the differential equations of three exciters in Eq. (2), then integrating them over $\varphi=0$–2π, the average differential equations of three exciters are deduced as follows:

$$m_0r^2\omega_{m0}\dot{\bar{\varepsilon}}_i+f_i\omega_{m0}(1+\bar{\varepsilon}_i)=\bar{T}_{ei}-\bar{T}_{li},\ i=1,2,3, \quad (8)$$

with

$$\overline{T}_{li} = m_0 r^2 \omega_{m0} \left[\sum_{j=1}^{3} (\chi'_{ij} \dot{\overline{\varepsilon}}_j + \chi_{ij} \overline{\varepsilon}_j) + \chi_{fi} + \chi_{ai} \right], \quad i=1, 2, 3, \tag{9}$$

where

$$\chi_{f1} = \omega_{m0}[W_{s1} + W_{sc12}\cos(2\overline{\alpha}_1 + \theta_{s12}) + W_{sc13}\cos(2\overline{\alpha}_1 + 2\overline{\alpha}_2 + \theta_{s13})]/2,$$
$$\chi_{a1} = \omega_{m0}[W_{cc12}\sin(2\overline{\alpha}_1 + \theta_{c12}) + W_{cc13}\sin(2\overline{\alpha}_1 + 2\overline{\alpha}_2 + \theta_{c13})]/2,$$
$$\chi_{f2} = \omega_{m0}[W_{s2} + W_{sc12}\cos(2\overline{\alpha}_1 + \theta_{s12}) + W_{sc23}\cos(2\overline{\alpha}_2 + \theta_{s23})]/2,$$
$$\chi_{a2} = \omega_{m0}[-W_{cc12}\sin(2\overline{\alpha}_1 + \theta_{c12}) + W_{cc23}\sin(2\overline{\alpha}_2 + \theta_{c23})]/2,$$
$$\chi_{f3} = \omega_{m0}[W_{s3} + W_{sc23}\cos(2\overline{\alpha}_2 + \theta_{s23}) + W_{sc13}\cos(2\overline{\alpha}_1 + 2\overline{\alpha}_2 + \theta_{s13})]/2,$$
$$\chi_{a3} = \omega_{m0}[-W_{cc23}\sin(2\overline{\alpha}_2 + \theta_{c23}) - W_{cc13}\sin(2\overline{\alpha}_1 + 2\overline{\alpha}_2 + \theta_{c13})]/2,$$
$$\chi'_{ii} = -W_{ci}/2, \quad i = 1, 2, 3,$$
$$\chi'_{12} = \chi'_{21} = W_{cc12}\cos(2\overline{\alpha}_1 + \theta_{c12})/2,$$
$$\chi'_{13} = \chi'_{31} = W_{cc13}\cos(2\overline{\alpha}_1 + 2\overline{\alpha}_2 + \theta_{c13})/2,$$
$$\chi'_{23} = \chi'_{32} W_{cc23}\cos(2\overline{\alpha}_2 + \theta_{c23})/2,$$
$$\chi_{ii} = \omega_{m0} W_{si}, \quad i = 1, 2, 3,$$
$$\chi_{12} = -\chi_{21} = \omega_{m0} W_{cc12}\sin(2\overline{\alpha}_1 + \theta_{c12}),$$
$$\chi_{13} = -\chi_{31} = \omega_{m0} W_{cc13}\sin(2\overline{\alpha}_1 + 2\overline{\alpha}_2 + \theta_{c13}),$$
$$\chi_{23} = -\chi_{32} = \omega_{m0} W_{cc23}\sin(2\overline{\alpha}_2 + \theta_{c23}),$$
$$W_{sj} = r_m \left(\frac{\sin\gamma_x}{\mu_x} + \frac{\sin\gamma_y}{\mu_y} + \frac{r_{lj}^2 \sin\gamma_\psi}{\mu_\psi} \right), j=1, 2, 3,$$
$$W_{cj} = r_m \left(\frac{\cos\gamma_x}{\mu_x} + \frac{\cos\gamma_y}{\mu_y} + \frac{r_{lj}^2 \cos\gamma_\psi}{\mu_\psi} \right), j=1, 2, 3,$$
$$a_{scij} = \frac{\sin\gamma_x}{\mu_x} + \frac{\sin\gamma_y}{\mu_y} + \frac{r_{li} r_{lj} \sin\gamma_\psi}{\mu_\psi}\cos(\beta_i - \beta_j), \quad i = 1, 2;\ i < j \leqslant 3,$$
$$b_{scij} = \frac{r_{li} r_{lj} \sin\gamma_\psi}{\mu_\psi}\sin(\beta_i - \beta_j), \quad i = 1, 2,\ i < j \leqslant 3,$$
$$a_{ccij} = -\frac{\cos\gamma_x}{\mu_x} - \frac{\cos\gamma_y}{\mu_y} - \frac{r_{li} r_{lj} \cos\gamma_\psi}{\mu_\psi}\cos(\beta_i - \beta_j), \quad i = 1, 2;\ i < j \leqslant 3,$$
$$b_{ccij} = \frac{r_{li} r_{lj} \cos\gamma_\psi}{\mu_\psi}\sin(\beta_i - \beta_j), \quad i = 1, 2;\ i < j \leqslant 3,$$
$$W_{scij} = r_m\sqrt{a_{scij}^2 + b_{scij}^2}, \quad i = 1, 2;\ i < j \leqslant 3,$$
$$W_{ccij} = r_m\sqrt{a_{ccij}^2 + b_{ccij}^2}, \quad i = 1, 2;\ i < j \leqslant 3,$$
$$l_1 = l_3, \quad l_2 = l_1 \sin\beta_1, \quad r_{l1} = r_{l3}, r_{l2} = r_{l1}\sin\beta_1,$$
$$\theta_{sij} = \begin{cases} \arctan(-b_{scij}/a_{scij}), & a_{scij} \geqslant 0, \\ \pi + \arctan(-b_{scij}/a_{scij}), & a_{scij} < 0, \end{cases}$$
$$\theta_{cij} = \begin{cases} \arctan(b_{ccij}/a_{ccij}), & a_{ccij} \geqslant 0, \\ \pi + \arctan(b_{ccij}/a_{ccij}), & a_{ccij} < 0, \end{cases}$$
$$i = 1, 2,\ i < j \leqslant 3.$$

Compared with the change of φ ($\dot{\varphi} = \omega_{m0}$) with respect to time t, that of α_1, and α_2 are very small, so ε_i (i=1, 2, 3) are considered to be slow-changing parameters. According to the method of direct separation of motions[1, 6], α_j (j=1, 2), ε_i and $\dot{\varepsilon}_i$ are assumed to be the middle values of their integration $\overline{\alpha}_j$, $\overline{\varepsilon}_i$ and $\dot{\overline{\varepsilon}}_i$, respectively, during the aforementioned integration. On the other hand, the damping of the vibrating system is very small[11–12], so the terms in the expressions of χ'_{ij}, χ_{ij}, χ_{fi} and χ_{ai} related to $\sin\gamma_x$, $\sin\gamma_y$ and $\sin\gamma_\psi$ can be neglected.

When the vibrating system operates in a steady-state, the electromagnetic torque of an induction motor in the vicinity of ω_{m0} can be expressed as[19]

$$T_{ei} = T_{e0i} - \overline{k}_{e0i}\overline{\varepsilon}_i, \quad i = 1, 2, 3, \tag{10}$$

where T_{e0i} and $\overline{k}_{e0i}$ are respectively electromagnetic torque and stiffness coefficient of angular velocity when an induction motor operates steadily at the angular velocity ω_{m0}, i=1, 2, 3.

Inserting Eqs. (9) and (10) into Eq. (8) yields the dimensionless coupling equations of three exciters as

$$\boldsymbol{A}\dot{\overline{\boldsymbol{\varepsilon}}} = \boldsymbol{B}\overline{\boldsymbol{\varepsilon}} + \boldsymbol{u}, \tag{11}$$

with

$$\overline{\boldsymbol{\varepsilon}} = (\overline{\varepsilon}_1\ \overline{\varepsilon}_2\ \overline{\varepsilon}_3)^T, \quad \boldsymbol{u} = (u_1\ u_2\ u_3)^T,$$
$$\boldsymbol{A} = \begin{pmatrix} \rho_1 & \chi'_{12} & \chi'_{13} \\ \chi'_{21} & \rho_2 & \chi'_{23} \\ \chi'_{31} & \chi'_{32} & \rho_3 \end{pmatrix},$$
$$\boldsymbol{B} = -\omega_{m0}\begin{pmatrix} \kappa_{11} & \chi_{12}/\omega_{m0} & \chi_{13}/\omega_{m0} \\ \chi_{21}/\omega_{m0} & \kappa_{22} & \chi_{23}/\omega_{m0} \\ \chi_{31}/\omega_{m0} & \chi_{32}/\omega_{m0} & \kappa_{33} \end{pmatrix},$$
$$\rho_i = 1 - \frac{W_{ci}}{2}, \quad \kappa_{ii} = \frac{\overline{k}_{e0i}}{m_0 r^2 \omega^2_{m0}} + \frac{f_i}{m_0 r^2 \omega_{m0}} + W_{si},$$
$$u_i = \frac{T_{e0i}}{m_0 r^2 \omega_{m0}} - \frac{f_i}{m_0 r^2} - \chi_{ai} - \chi_{fi}, \quad i = 1, 2, 3,$$

where $\boldsymbol{A}$—Dimensionless inertia-coupling matrix,
$\boldsymbol{B}$—Dimensionless stiffness-coupling matrix of angular velocity,
$\boldsymbol{u}$—Dimensionless load torque coupling.

3.1 Criterion of synchronization

When three exciters operate synchronously, we have $\dot{\overline{\boldsymbol{\varepsilon}}} = 0$ and $\overline{\boldsymbol{\varepsilon}} = 0$ in Eq. (11). So $\boldsymbol{u}=\boldsymbol{0}$, i.e.,

$$T_{0i} = T_{e0i} - f_i\omega_{m0} = m_0 r^2 \omega_{m0}(\chi_{fi} + \chi_{ai}), \quad i = 1, 2, 3, \tag{12}$$

where T_{0i} is called as the output electromagnetic torque of the motor i, which is the difference between the electromagnetic torque of one motor and the damping torque of its rotor.

During the process of synchronous operation, the vibrating system transmits electromagnetic torque among three exciters to overcome the output torque difference between arbitrary two motors by virtue of adjusting their phase differences. Dimensionless rearrangement of Eq. (12) yields

$$\Delta T_{0ij}/T_{\rm u}-(W_{si}-W_{sj})=\tau_{cij}(\bar{\alpha}_1,\bar{\alpha}_2)\,,\quad ij=12,23,31\,, \tag{13}$$

with

$$\tau_{c12}(\bar{\alpha}_1,\bar{\alpha}_2)=2W_{cc12}\sin(2\bar{\alpha}_1+\theta_{c12})-W_{cc23}\sin(2\bar{\alpha}_2+\theta_{c23})+W_{cc13}\sin(2\bar{\alpha}_1+2\bar{\alpha}_2+\theta_{c13})-W_{sc23}\cos(2\bar{\alpha}_2+\theta_{s23})+W_{sc13}\cos(2\bar{\alpha}_1+2\bar{\alpha}_2+\theta_{s13})\,, \tag{14}$$

$$\tau_{c23}(\bar{\alpha}_1,\bar{\alpha}_2)=2W_{cc23}\sin(2\bar{\alpha}_2+\theta_{c23})-W_{cc12}\sin(2\bar{\alpha}_1+\theta_{c12})+W_{cc13}\sin(2\bar{\alpha}_1+2\bar{\alpha}_2+\theta_{c13})+W_{sc12}\cos(2\bar{\alpha}_1+\theta_{s12})-W_{sc13}\cos(2\bar{\alpha}_1+2\bar{\alpha}_2+\theta_{s13}), \tag{15}$$

$$\tau_{c31}(\bar{\alpha}_1,\bar{\alpha}_2)=-[\tau_{c12}(\bar{\alpha}_1,\bar{\alpha}_2)+\tau_{c23}(\bar{\alpha}_1,\bar{\alpha}_2)], \tag{16}$$

where $T_{\rm u}$—Kinetic energy of each exciter, $T_{\rm u}=m_0r^2\omega_{\rm m0}{}^2/2$,
ΔT_{0ij}—Difference of output torque between the motors i and j, $T_{0ij}=T_{0i}-T_{0j}$, ij=12, 23, 31.

It should be noted that the left-hand sides of Eq. (13) represent the difference of the dimensionless residual torque between the motors i and j; while $\tau_{cij}(\bar{\alpha}_1,\bar{\alpha}_2)$ (ij=12, 23, 31) describes the dimensionless coupling torque between exciters i and j. $\tau_{cij}(\bar{\alpha}_1,\bar{\alpha}_2)$ is limited function of $\bar{\alpha}_1$ and $\bar{\alpha}_2$, i.e.,

$$\left|\tau_{cij}(\bar{\alpha}_1,\bar{\alpha}_2)\right|\leqslant\tau_{cij\max}\,,\quad ij=12,23,31\,. \tag{17}$$

When the structural parameters of the vibrating system satisfy the following criterion

$$\left|\Delta T_{0ij}/T_{\rm u}-(W_{si}-W_{sj})\right|\leqslant\tau_{cij\max}\,,\quad ij=12,23,31\,, \tag{18}$$

Eq. (12) can be solved for $\omega_{\rm m0}$, $\bar{\alpha}_1$ and $\bar{\alpha}_2$, which are denoted by $\omega^*_{\rm m0}$, $\bar{\alpha}_{10}$ and $\bar{\alpha}_{20}$, respectively. The left-hand sides of Eq. (18) are referred to as the dimensionless residual torque difference between arbitrary two motors. Therefore, the synchronization criterion of three exciters is that the absolute value of dimensionless residual torque difference between arbitrary two motors is less than or equal to the maximum of their dimensionless coupling torque.

Adding T_{01}, T_{02} and T_{03} in Eq. (12) up and rearranging the result thereof, we have

$$\tau_{\rm a}(\bar{\alpha}_1,\bar{\alpha}_2)=\sum_{i=1}^{3}T_{0i}\Big/3T_{\rm u}=\frac{1}{3}\sum_{i=1}^{3}W_{si}+\frac{1}{3}[2W_{sc12}\cos(2\bar{\alpha}_1+\theta_{s12})+2W_{sc13}\cos(2\bar{\alpha}_1+2\bar{\alpha}_2+\theta_{s13})+2W_{sc23}\cos(2\bar{\alpha}_2+\theta_{s23})], \tag{19}$$

where $\tau_{\rm a}(\bar{\alpha}_1,\bar{\alpha}_2)$ describes the average dimensionless loading torque of the three motors and it is a limited function, i.e.,

$$\left|\tau_{\rm a}(\bar{\alpha}_1,\bar{\alpha}_2)\right|\leqslant\tau_{\rm a\max}\,, \tag{20}$$

$\tau_{\rm amax}$ is the maximum of average dimensionless loading torque of the three motors.

According to Refs. [21–22], the synchronization of the vibrating system stems from the coupling dynamic characteristics of exciters. The greater the coupling moment of two exciters, the stronger the synchronization ability of two exciters. Hence, from Eq. (12) and considering Eqs. (14)–(16), one can see that $\tau_{cij}(\bar{\alpha}_1,\bar{\alpha}_2)$ (ij=12, 23, 31) represents the relationship among three exciters, that is, $\tau_{cij}(\bar{\alpha}_1,\bar{\alpha}_2)$ describes the vibrating system's ability of adjusting the loading torque of each motor to reach synchronization. Here we define the coefficient of synchronization ability between arbitrary two exciters i and j as

$$\zeta_{ij}=\tau_{cij\max}/\tau_{\rm a\max}\,,\ ij=12,\ 23,\ 31\,. \tag{21}$$

The larger the coefficient of synchronization ability ζ_{ij}, the easier the vibrating system can achieve synchronization. Since $\tau_{cij}(\bar{\alpha}_1,\bar{\alpha}_2)$ and $\tau_{\rm a}(\bar{\alpha}_1,\bar{\alpha}_2)$ are transcendental functions of $\bar{\alpha}_1$ and $\bar{\alpha}_2$ and depend on the parameters of the vibrating system, their expressions of maximum are founded difficultly. In next section of the paper, we will numerically discuss them in detail.

3.2 Criterion of stability of synchronous states

When $\boldsymbol{u}=\boldsymbol{0}$, Eq. (11) is the generalized system[25], i.e.,

$$\boldsymbol{A}'\dot{\bar{\boldsymbol{\varepsilon}}}=\boldsymbol{B}'\bar{\boldsymbol{\varepsilon}}\,, \tag{22}$$

where $\boldsymbol{A}'_{ij}=(a'_{ij})_{3\times3}$ and $\boldsymbol{B}'_{ij}=(b'_{ij})_{3\times3}$ denote the values of $\boldsymbol{A}$ and $\boldsymbol{B}$ for $\bar{\alpha}_1=\bar{\alpha}_{10}$, $\bar{\alpha}_2=\bar{\alpha}_{20}$ and $\omega_{\rm m0}=\omega^*_{\rm m0}$. As shown in Eq. (11), the matrix $\boldsymbol{A}'$ is symmetrical and the matrix $\boldsymbol{B}'$ is antisymmetrical, so when the matrix $\boldsymbol{A}'$ is positive definite and all its elements are positive, i.e., the parameters of the vibrating system satisfy the following criterion:

$$a'_{ij}>0\,,\ \det(\boldsymbol{A}'_2)>0\,,\ \det(\boldsymbol{A}')>0\,,\quad i=1,2,3,\ j=1,2,3. \tag{23}$$

$\boldsymbol{A}'$ and $\boldsymbol{B}'$ satisfy the generalized Lyapunov equations[25]:

$$\boldsymbol{I}^{\rm T}\boldsymbol{B}'+\boldsymbol{B}'^{\rm T}\boldsymbol{I}=-\omega_{\rm m0}{\rm diag}\{\kappa_{11},\kappa_{22},\kappa_{33}\}, \tag{24}$$

$$\boldsymbol{A}'^{\rm T}\boldsymbol{I}=\boldsymbol{I}\boldsymbol{A}'>0\,, \tag{25}$$

where $\boldsymbol{I}$ is unit matrix.

Because the generalized system Eq. (22) is concessional and without pulse, Eq. (22) is stable if $\lim_{t\to\infty}\boldsymbol{A}'\bar{\boldsymbol{\varepsilon}}=0$ [22]. $\lim_{t\to\infty}\bar{\boldsymbol{\varepsilon}}=0$ means that the electromagnetic torques of the three motors are stably balanced with the load torques that the vibrating system acts on them, i.e.,

$$T_{e0i}-f_i\omega_{m0}=m_0r^2\omega_{m0}(\chi_{fi}+\chi_{ai}),\ \ i=1,\ 2,\ 3. \tag{26}$$

Linearizing Eq. (26) around $\bar{\alpha}_{10}$, $\bar{\alpha}_{20}$ and ω^*_{m0}, and neglecting W_{s0}, W_{scij} ($i=1,\ 2;\ i<j\leqslant 3$), f_1, f_2 and f_3 [19, 23], we have

$$\kappa_{11}\omega^*_{m0}\left(\varsigma_0+\frac{4}{3}\varsigma_1+\frac{2}{3}\varsigma_2\right)=-\sum_{i=1}^{2}\left(\frac{\partial\chi_{a1}}{\partial\bar{\alpha}_i}\right)_0\Delta\alpha_i\,, \tag{27}$$

$$\kappa_{22}\omega^*_{m0}\left(\varsigma_0-\frac{2}{3}\varsigma_1+\frac{2}{3}\varsigma_2\right)=-\sum_{i=1}^{2}\left(\frac{\partial\chi_{a2}}{\partial\bar{\alpha}_i}\right)_0\Delta\alpha_i\,, \tag{28}$$

$$\kappa_{33}\omega^*_{m0}\left(\varsigma_0-\frac{2}{3}\varsigma_1-\frac{4}{3}\varsigma_2\right)=-\sum_{i=1}^{2}\left(\frac{\partial\chi_{a3}}{\partial\bar{\alpha}_i}\right)_0\Delta\alpha_i\,, \tag{29}$$

where $(\bullet)_0$ —Values for $\bar{\alpha}_1=\bar{\alpha}_{10}$, $\bar{\alpha}_2=\bar{\alpha}_{20}$, $\Delta\alpha_i=\bar{\alpha}_i-\bar{\alpha}_{i0}$, $i=1, 2$.

Summing Eqs. (27)–(29), yields

$$\varsigma_0=\delta_1\varsigma_1+\delta_2\varsigma_2\,, \tag{30}$$

where $\delta_1=\dfrac{2\kappa_{33}/3+2\kappa_{22}/3-4\kappa_{11}/3}{\kappa_{11}+\kappa_{22}+\kappa_{33}}$,

$\delta_2=\dfrac{4\kappa_{33}/3-2\kappa_{22}/3-2\kappa_{11}/3}{\kappa_{11}+\kappa_{22}+\kappa_{33}}$.

It should be noticed that $\omega_{m0}\varsigma_1=\Delta\dot{\alpha}_1$ and $\omega_{m0}\varsigma_2=\Delta\dot{\alpha}_2$. In other words, Eqs. (27)–(29) are the differential equations of $\Delta\boldsymbol{\alpha}=(\Delta\alpha_1\ \Delta\alpha_2)^{\mathrm{T}}$. Subtracting Eq. (28) from Eq. (27), subtracting Eq. (29) from Eq. (28), and rewriting them in a matrix form:

$$\Delta\dot{\boldsymbol{\alpha}}=\boldsymbol{D}\Delta\boldsymbol{\alpha}\,, \tag{31}$$

where $\boldsymbol{D}=(d_{ij})_{2\times2}$,

$$d_{11}=-\frac{\omega^*_{m0}}{2}\left[\left(\frac{W_{cc12}}{\kappa_{11}}+\frac{W_{cc12}}{\kappa_{22}}\right)\cos(2\bar{\alpha}_{10}+\theta_{c12})+\frac{W_{cc13}}{\kappa_{11}}\cos(2\bar{\alpha}_{10}+2\bar{\alpha}_{20}+\theta_{c13})\right],$$

$$d_{12}=\frac{\omega^*_{m0}}{2}\left[\frac{W_{cc23}}{\kappa_{22}}\cos(2\bar{\alpha}_{20}+\theta_{c23})-\frac{W_{cc13}}{\kappa_{11}}\cos(2\bar{\alpha}_{10}+2\bar{\alpha}_{20}+\theta_{c13})\right],$$

$$d_{21}=\frac{\omega^*_{m0}}{2}\left[\frac{W_{cc12}}{\kappa_{22}}\cos(2\bar{\alpha}_{10}+\theta_{c12})-\frac{W_{cc13}}{\kappa_{33}}\cos(2\bar{\alpha}_{10}+2\bar{\alpha}_{20}+\theta_{c13})\right],$$

$$d_{22}=-\frac{\omega^*_{m0}}{2}\left[\left(\frac{W_{cc23}}{\kappa_{22}}+\frac{W_{cc23}}{\kappa_{33}}\right)\cos(2\bar{\alpha}_{20}+\theta_{c23})+\frac{W_{cc13}}{\kappa_{33}}\cos(2\bar{\alpha}_{10}+2\bar{\alpha}_{20}+\theta_{c13})\right].$$

Exponential time-dependence of the form $\Delta\boldsymbol{\alpha}=\boldsymbol{v}\exp(\lambda t)$ is now assumed, and inserted into Eq. (31), then solving the determinant equation $\det(\boldsymbol{D}-\lambda\boldsymbol{I})=0$, the characteristic equation for eigenvalue λ is obtained as

$$\lambda^2-(d_{11}+d_{22})\lambda+(d_{11}d_{22}-d_{12}d_{21})=0\,. \tag{32}$$

The zero solutions of Eq. (31) are stable if and only if all the roots of λ in Eq. (32) have the negative real parts. Using the Routh-Hurwitz criterion, Eq. (33) can satisfy the above requirements[26],

$$-d_{11}-d_{22}>0,\ \ d_{11}d_{22}-d_{12}d_{21}>0. \tag{33}$$

$\lim_{t\to+\infty}\Delta\boldsymbol{\alpha}=0$ means $\lim_{t\to+\infty}\varsigma_i=0$, $i=0,\ 1,\ 2$. Using Eq. (6), we have $\lim_{t\to+\infty}\boldsymbol{\varepsilon}=0$, i.e., $\lim_{t\to+\infty}\boldsymbol{A\varepsilon}=0$.

In engineering, the parameters of the three induction motors are usually chosen to be similar[11–14], i.e.,

$$\kappa_{ii}\approx\kappa_0,\ T_{e1}\approx T_{e2}\approx T_{e3},\ \ i=1, 2, 3. \tag{34}$$

We assume that

$$p_{12}=W_{cc12}\cos(2\bar{\alpha}_{10}+\theta_{c12}),\ \ p_{23}=W_{cc23}\cos(2\bar{\alpha}_{20}+\theta_{c23}),$$
$$p_{13}=W_{cc13}\cos(2\bar{\alpha}_{10}+2\bar{\alpha}_{20}+\theta_{c13})\,.$$

From Eq. (23), we have

$$p_{12}>0,\ p_{13}>0,\ p_{23}>0. \tag{35}$$

Substituting Eq. (34) into Eq. (33) and considering Eq. (35) yield

$$-d_{11}-d_{22}=\frac{\omega^*_{m0}}{\kappa_0}(p_{12}+p_{13}+p_{23})>0\,, \tag{36}$$

$$d_{11}d_{22}-d_{12}d_{21}=\frac{3\omega^{*2}_{m0}}{4\kappa_0^2}(p_{12}p_{13}+p_{12}p_{23}+p_{13}p_{23})>0. \tag{37}$$

Hence, Eq. (33) can also satisfy Routh-Hurwitz criterion of Eq. (31) when Eq. (23) satisfies the generalized Lyapunov criterion of Eq. (22), i.e., when the parameters of three motors are similar, the stability criterion of synchronous states is also Eq. (23), which describes that the dimensionless inertia-coupling matrix of the generalized system is positive definite and all its elements are positive.

According to the stability criterion, Eq. (23), the intervals of the stable phase differences among three exciters can be expressed as

$$\begin{cases}2\bar{\alpha}_{10}\in(-\pi/2-\theta_{c12},\pi/2-\theta_{c12}),\\ 2\bar{\alpha}_{20}\in(-\pi/2-\theta_{c23},\pi/2-\theta_{c23}),\\ 2\bar{\alpha}_{10}+2\bar{\alpha}_{20}\in(-\pi/2-\theta_{c13},\pi/2-\theta_{c13}).\end{cases} \tag{38}$$

4 Numeric Results and Discussions

In this section, the parameters of the three motors are assumed to be the same (three-phase squirrel-cage). The parameters of the vibrating system are: M=152 kg, m_0=4 kg, J_m=17 kg • m^2, k_x=k_y=79 kN/m (μ_x=μ_y=0.95), k_ψ=8.8 kN/rad (μ_ψ=0.95), f_x=f_y=0.485 kN • s/m, f_ψ=0.054 kN • s/rad (ξ_{nx}=ξ_{ny}=$\xi_{n\psi}$=0.07).

4.1 Criterion of synchronization and general dynamic symmetry characteristics

As aforementioned section 3.1, we can see that $\tau_{c12}(\bar{\alpha}_1,\bar{\alpha}_2)$ and $\tau_{c23}(\bar{\alpha}_1,\bar{\alpha}_2)$ are limited functions of $\bar{\alpha}_1$, $\bar{\alpha}_2$, μ_x, μ_y, μ_ψ, β_1, r_m and r_{l1}. In addition, μ_x, μ_y and μ_ψ less change (0.9–0.99) in a far-resonant vibrating system[20–22]. As a result, according to Eqs. (14)–(16), we can confine r_m=0.026 to find the values of $\tau_{cij\max}$ (ij=12, 23, 31) versus r_{l1} and β_1, as illustrated in Fig. 2.

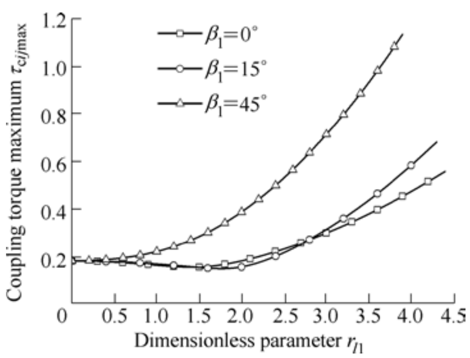


Fig. 2. Values of $\tau_{cij\max}$ versus r_{l1} and β_1 (ij=12, 23, r_m=0.026)

It should be noticed that r_{l1} has a maximum value, and based on its expression, we have

$$\begin{cases} r_{l1}^2 = \dfrac{1}{(l_{e0}/l_1)^2(1-3r_m)+r_m(2+\sin^2\beta_1)}, \\ r_{l1\max}^2 = \lim\limits_{l_1\to+\infty} r_{l1}^2 = 1/[r_m(2+\sin^2\beta_1)], \end{cases} \quad (39)$$

where $l_{e0}{}^2$=J_m/m.

For a given value r_m=0.026, according to the second formula of Eq. (39), we have that $r_{l1\max}$ value is 4.4, 4.3 and 3.9 when β_1 value is 0°, 15° and 45°, respectively.

In Fig. 2, $\tau_{c12\max}$=$\tau_{c23\max}$, r_{l1} arranges from 0 to its maximum (as well as the following discussions), and the greater the value of $\tau_{cij\max}$, the easier three exciters implement synchronization.

In order to clearly reflect the synchronization ability of the vibrating system, the coefficients of synchronization ability among three exciters should be presented. As shown in Eq. (21), the coefficient of synchronization ability between exciters i and j, is the ratio between the maximum of the dimensionless coupling torque of exciters i and j, $\tau_{cij\max}$, and the maximal average dimensionless loading torque of the three motors, $\tau_{a\max}$. It is independent on the parameters of the three motors, and is also called the coefficient of the general dynamic symmetry (CGDS)[21]. The better the CGDS is, the stronger the synchronization ability is.

Fig. 3 shows the CGDSs versus r_{l1} for different β_1 and r_m. Here, according to Eq. (39), l_1 is in the range from 0 m to 30 m.

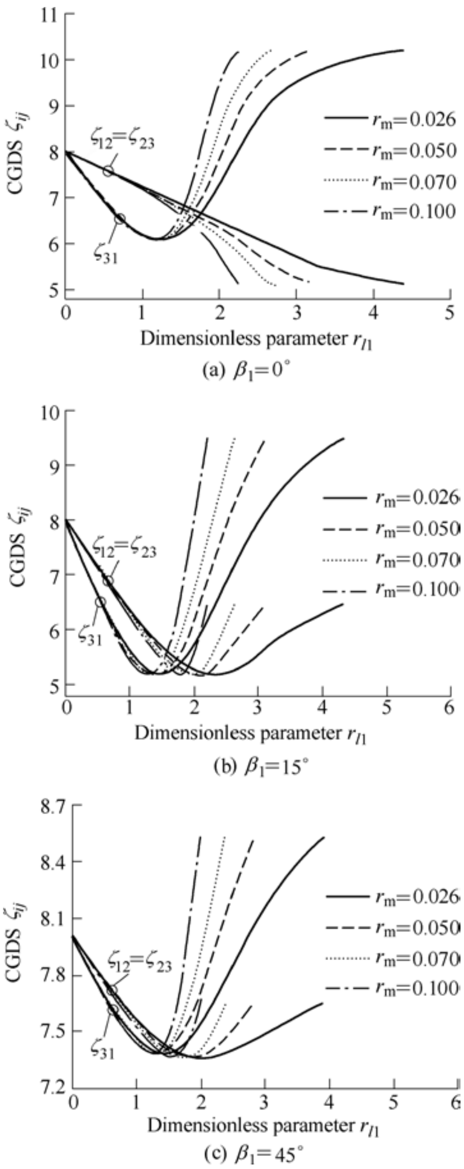


Fig. 3. CGDSs among three exciters

In Fig. 3(a), β_1=0°, so l_2=0 m, and three exciters are identical, which means that the structure of the vibrating system is complete symmetry. In this case, the coefficients of synchronization ability between adjacent two exciters are the same, i.e., ζ_{12}=ζ_{23}, and all decrease with the increasing r_{l1}; while that between separated two exciters, ζ_{31}, firstly decreases, and followed by increasing, with the increasing r_{l1}. Additionally, synchronization ability between separated two exciters is greater than that between adjacent two exciters for r_{l1} being more than a certain value (at the intersections of lines in Fig. 3(a)), under which the absolute value of phase difference between exciters 3 and 1 should be smaller than that between exciters 1 and 2 (or exciters 2 and 3). As for the reason, we can find it in Refs. [21–22], i.e., the greater the coupling moment between arbitrary two exciters, the smaller the absolute value of their phase difference, it can be verified in the next discussions of section 4.2.

In Figs. 3(b) and 3(c), $\beta_1 \neq 0°$ has $l_2 \neq 0$ m, i.e., the structure of the vibrating system is not complete symmetry. Herein, CGDSs among three exciters all present a down and up trend with the increase of r_{l1}. The others are similar to the above discussions in Fig. 3(a).

4.2 Synchronization stability

According to the balanced equation of three exciters, Eq. (13), and considering the stability criterion of the vibrating system, Eq. (23), the stable phase difference-value among three exciters (SPDATE) can be solved approximately by numeric method, as shown in Fig. 4 (the synchronous speed ω^*_{m0} changes in the interval of 102.3–104 rad/s versus r_{l1}).

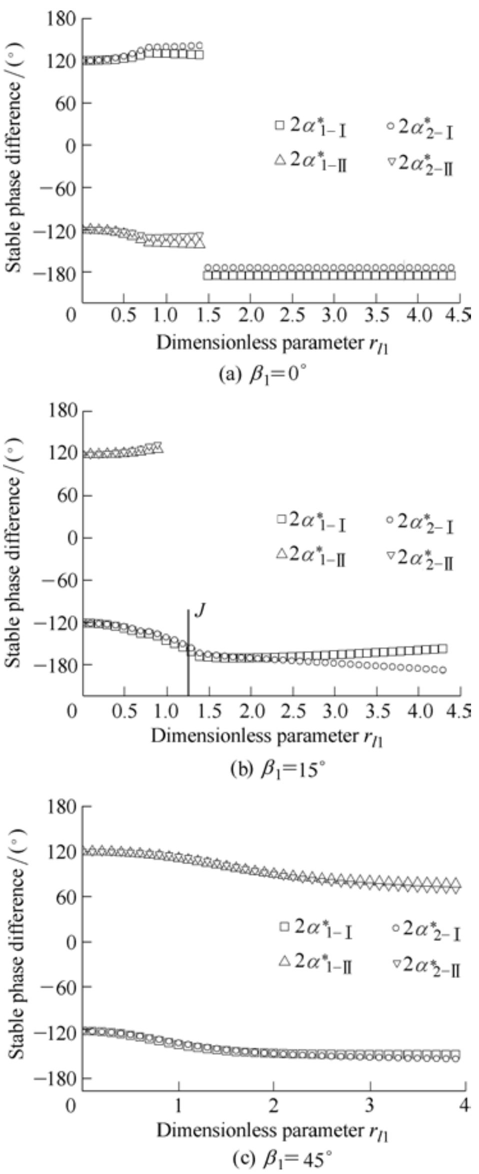


Fig. 4. Stable phase difference versus r_{l1} for different β_1

It should be noticed that, for convenient discussion, if the parameters of the three motors are identical, the first term on the left-hand sides of Eq. (13), $\Delta T_{0ij}/T_u$ (ij=12, 23, 31), might be assumed to be zero, i.e., $\Delta T_{0ij}/T_u \approx 0$ (in engineering, although the parameters of the three motors are completely identical, the output electromagnetic torque difference between arbitrary two motors, ΔT_{0ij}, is not completely equal to zero). Hence, the structural parameter, r_m, has no effect on the synchronous phase difference solutions because it can be eliminated in Eq. (13). But r_m plays an important role in adjusting the stability of the vibrating system, which can be seen in Refs. [16–20].

In Fig. 4(a), β_1=0°, SPDATEs have two groups values in the interval of $0 \leqslant r_{l1} < 1.414$ ($\sqrt{2}$). It indicates that the vibrating system has two equilibrium points in this interval, which is called the diversity of nonlinear system[27]. $2\alpha^*_{1\text{-I}}$ ($2\alpha^*_{1\text{-II}}$) denotes the I-group (II-group) stable phase difference between exciters 1 and 2; $2\alpha^*_{2\text{-I}}$ ($2\alpha^*_{2\text{-II}}$) that between exciters 2 and 3, as well as what are shown in Figs. 4(b) and 4(c). One can see that the interval of double-equilibrium is $0 \leqslant r_{l1} < 1$ in Fig. 4(b), while the vibrating system always has the state of double-equilibrium for β_1=45° in Fig. 4(c). Moreover, in Fig. 4(a), if $r_{l1} > 1.414(\sqrt{2})$, the SPDATE between exciters 3 and 1, $-(2\alpha^*_{1\text{-I}} + 2\alpha^*_{2\text{-I}})$, nears zero (or 360°), which is smaller than that between exciters 1 and 2 (or exciters 2 and 3), this fact coincides with the discussions in section. 4.1.

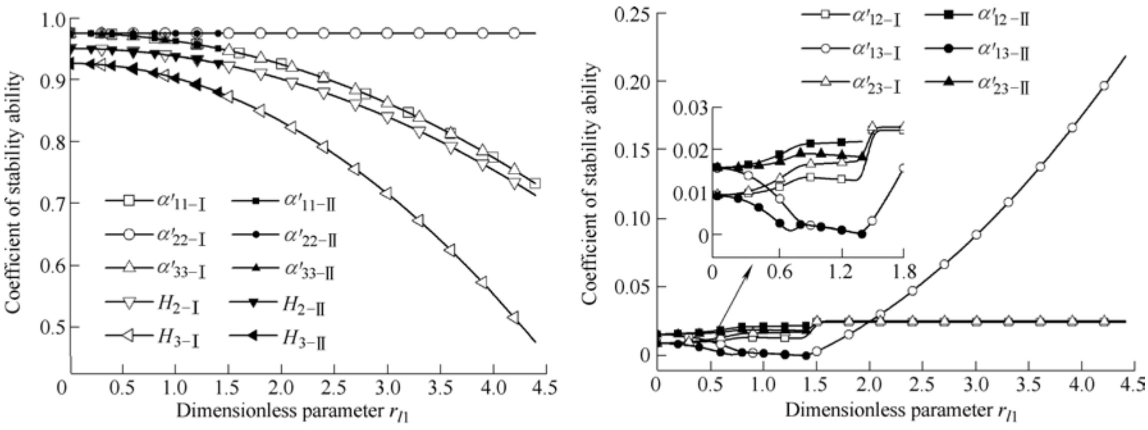
As we known, the motion type of the vibrating system is the result of mutual compensation of the three exciting forces, and such mutual compensation principle depends on SPDATEs. Taking Fig. 4(a) for example, SPDATEs among three exciters all near gradually 120° with the decrease of r_{l1}. For r_{l1}=0, the rigid frame implements no vibration for the sake of the mutual cancel of the three exciting forces; while for r_{l1}>1.414, SPDATE between exciters 1 and 3 nears 0, and that between exciters 1 and 2 (or 2 and 3) nears π, thus the exciting force of exciter 2 is always opposite to that of the other two exciters, that is, exciter 2 can't play a role of enhancing the effective power of system, but decrease it.

On the basis of the above analysis, we know that the double-equilibrium state of the considered vibrating system is related to the structural parameters, β_1 and r_{l1}. In engineering, to guarantee the reliability of the vibrating machine, it should be designed to have single-equilibrium state by adjusting its structural parameters. In addition, using three exciters instead of two, can not improve actually the effective total power of machinery.

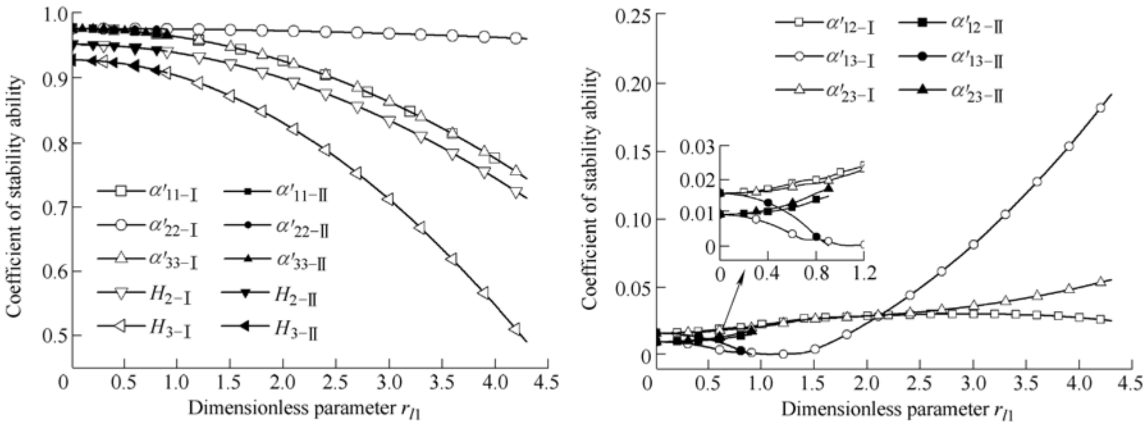
Considering the criterion of synchronization stability of the vibrating system, Eq. (23), we assume

$$\begin{cases} H_2 = \det(\boldsymbol{A}'_2), \\ H_3 = \det(\boldsymbol{A}'). \end{cases} \tag{40}$$

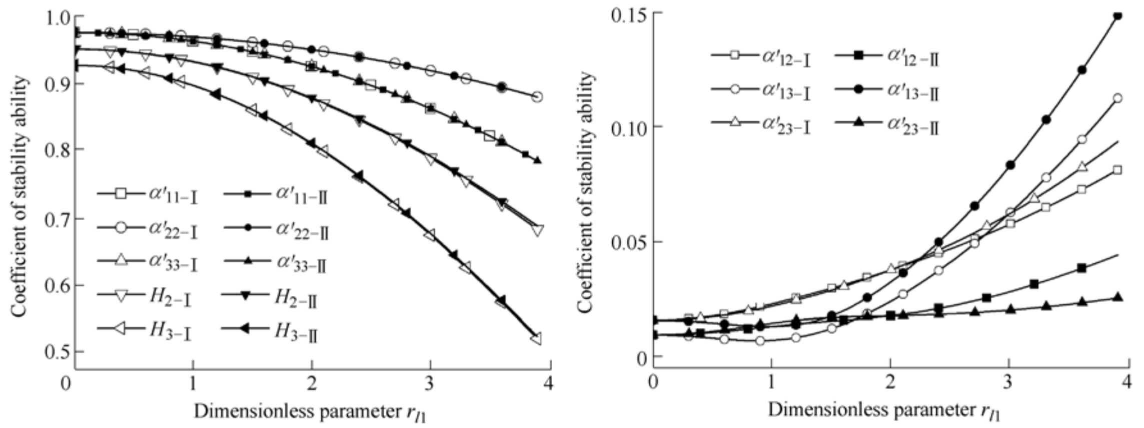
Inserting SPDATEs in Fig. 4 into Eq. (23), and confining r_m=0.026, we obtain the coefficients of ability of synchronization stability for β_1=0°, 15°, and 45°, respectively, as shown in Fig. 5.



(a) $\beta_1=0°$



(b) $\beta_1=15°$



(c) $\beta_1=45°$

Fig. 5. Coefficient of synchronization stability ability for different β_1 (r_m=0.026)

In Fig. 5, $a'_{ij\text{-I}}$, $H_{2\text{-I}}$ and $H_{3\text{-I}}$ denote the values for inserting $2\bar{\alpha}_{10}=2\alpha^*_{1\text{-I}}$ and $2\bar{\alpha}_{20}=2\alpha^*_{2\text{-I}}$ (in Fig. 4) into Eqs. (23) and (40), as well as $a'_{ij\text{-II}}$, $H_{2\text{-II}}$ and $H_{3\text{-II}}$ denote that for $2\alpha^*_{1\text{-II}}$ and $2\alpha^*_{2\text{-II}}$. As illustrated in Fig. 5, all elements are positive, in other words, the generalized system, Eq. (22), is stable. By the comparison among various coefficients of ability of synchronization stability in Fig. 5, the values of a'_{12}, a'_{13}, and a'_{23} are smaller than the others, that is, the critically decided factors of synchronization stability are as follows: $a'_{12}>0$, $a'_{13}>0$ and $a'_{23}>0$.

5 Experiment Results and Discussions

In this section we address the validity of the above theoretical and numerical results, by comparing to experimental results for a laboratory model.

5.1 Experiment description

Fig. 6 shows the setup schematically, the three exciting motors (exciters) rotating in the same directions are installed on the main rigid body. Three acceleration sensors and three Hall-sensors are used to measure the accelerations of experimental system in x-, y-, ψ-directions and the phases of three exciting motors, respectively, by Multifunctional Resistance to Mix Filter Amplifier (INV-6) and Intelligent Signal Acquisition and Processing Analyzer (INV306DF). The measured data are transferred to computer, after the software procedure, imaged by Originpro-8 lastly. In the meantime, the instantaneous phases of three exciting motors with synchronous operation are continuously recorded by high-speed camera.

Fig. 6. Vibrating synchronization bedstand

Each exciting motor has two pairs eccentric lumps distributing symmetrically on both ends of axis. The included angle θ between two eccentric lumps can be adjusted to accommodate a certain exciting force. Three exciting motors are identical, model VB-326-W (380 V, 50 Hz, 6-pole, Y-connected, rated speed 980 r/min, 0.2 kW), $\beta_1=15°$, $l_1=0.437$ m, $r_{l1}=1.25$, $r_m=0.026$. Equivalent mass of eccentric lumps for each exciting motor, $m_0=4$ kg; and its equivalent rotational radius, $r=0.05$ m. The other parameters of system are the same as that in section 4.

5.2 Experiment results

Fig. 7 shows some experimental results of three identical exciters rotating in the same directions. During the starting few seconds, when three exciters are supplied with electric source at the same time, their angular velocities pass through the resonant region of system, and excite the resonant responses in x-, y- and ψ-directions, in the meanwhile, the angular accelerations of the three motors are the same with each other because their moments of inertia are identical, as shown in Figs. 7(a)–7(f). In this case, the high frequency vibration of the system is not excited and the load torques of the motors that the vibrating system acts on them is very small.

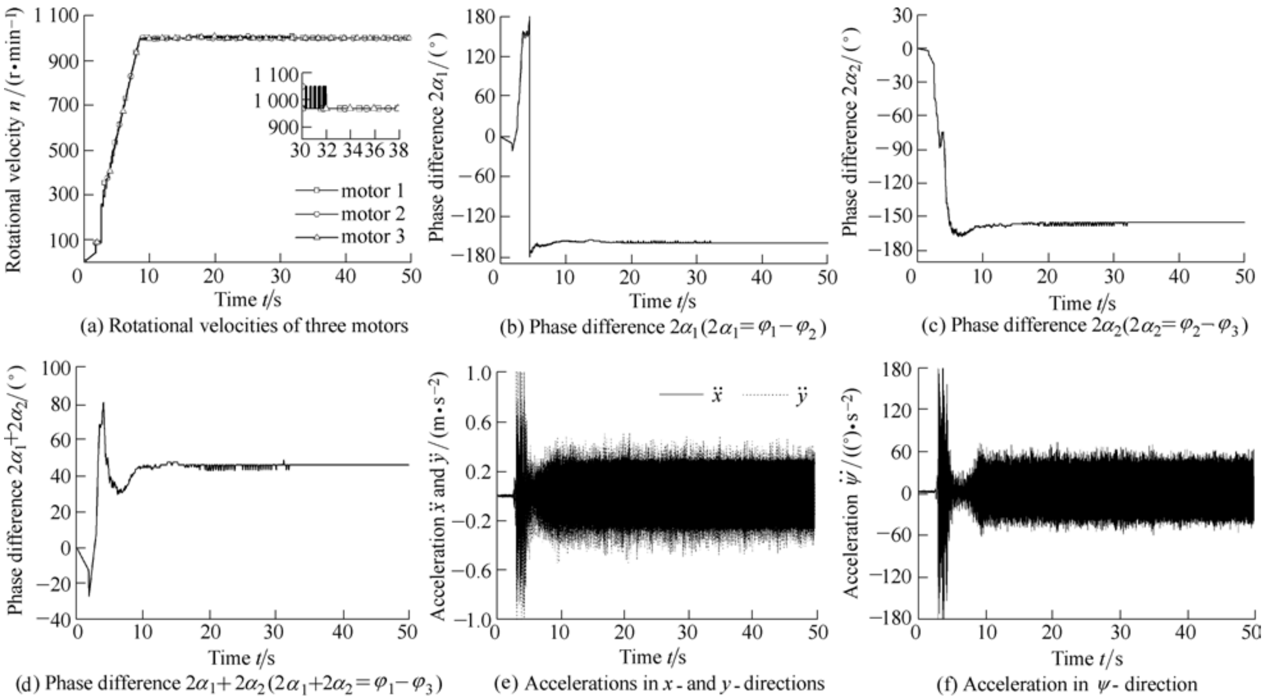


Fig. 7. Experimental results of three identical exciters rotating in the same directions

When the angular velocities of the three motors reach the operation value, the high frequency vibrations are excited and beat phenomenon occurs. This moment, the coupling torques occur and play a role of adjusting the loading torque of each motor to reach synchronization by regulating SPDATEs, which results that SPDATEs stabilize rapidly. The rotational velocity of synchronization is 996.6 r/min. SPDATEs are illustrated in Figs. 7(b), 7(c), 7(d): $2\alpha_1=-159.02°$, $2\alpha_2=-154.93°$ and $2\alpha_1+2\alpha_2=-46.05°$ (or $-313.95°$), which are approximately coincident with the point $\boldsymbol{J}$ ($r_{l1}=1.25$, $2\alpha^*_{1\text{-I}}=-157.9°$, $2\alpha^*_{2\text{-I}}=-152.8°$) in Fig. 4(b). It should be noted here that, since SPDATEs are not equal to zero, the three exciting forces are not be positively superimposed, which results in decreasing the effective power of the vibrating system. Above facts can be also shown in Figs. 7(e), 7(f), the accelerations in x- and y-directions all near roughly 0.3 m/s^2, which is smaller than the tested accelerations (0.5 m/s^2) in x- and y-directions excited by only two exciters operating with zero phase difference in practice. Meanwhile, the mutual superposition of the exciting forces gives rise to the occurrence of swing, and such swing is not the desire in engineering.

There are some errors by comparing the SPDATEs in experiment with that in Fig. 4(b), as for the causes, obviously, in section 4.2, although the parameters of the three motors are completely identical, their output electromagnetic torques are not absolutely equal to each other in fact. So such errors don't affect the investigation on synchronization of three exciters too much.

In addition, before 32 s, rotational velocity of synchronization and SPDATEs present some small fluctuations in Figs. 7(a)–7(d). As for the reasons, according to Ref. [11], in a vibrating system with multi-motor drives, due to the complexity of the mechanical structure, there are many factors influencing the synchronization: (1) from the mechanical system point of view, the disturbance in speed or phase difference is caused by the uneven loads among the motors that the vibrating system acts on them, such as the coupling effect of the mechanical system which makes the loads of the motors vary with a certain rule, the variation of mass and media characters and uneven distribution of media mass in a mechanical system cause the redistribution of the external loads of the motors, and so on; (2) from the motor aspect, the fluctuation in supply voltage, and the changes in motor parameters with humidity and temperature also cause certain disturbances. After 32 s, the vibrating system operate synchronously and stably with the best state. The other plots versus time t are shown in the relevant figures of Fig. 7, respectively.

To further give some visual results, during the above process of synchronous and stable operation in experiment, we recorded continuously the phases of three exciters within one cycle by high-speed camera, high-speed camera shooting frequency is 62/s, the horizontal x-axis positive direction is set as the reference line, as shown in Fig. 8: $2\alpha_1=\varphi_1-\varphi_2\approx-156.5°\sim-153°$, $2\alpha_2=\varphi_2-\varphi_3\approx-161.5°\sim-157°$, which are roughly the same as that of in Figs. 7(b), 7(c), 7(d) by comparing with each other. These facts verify that three exciters operate synchronously and stably, as well as the feasibility of the above test method. Additionally, from Fig. 8, we can also see that the phase of the middle exciter 2 always opposites approximately to that of the other two exciters 1 and 3, which shows visually that the total effective power of the vibrating system is decreased. The reasons related to the fluctuation for phase differences have been discussed previously, it need not to illustrate.

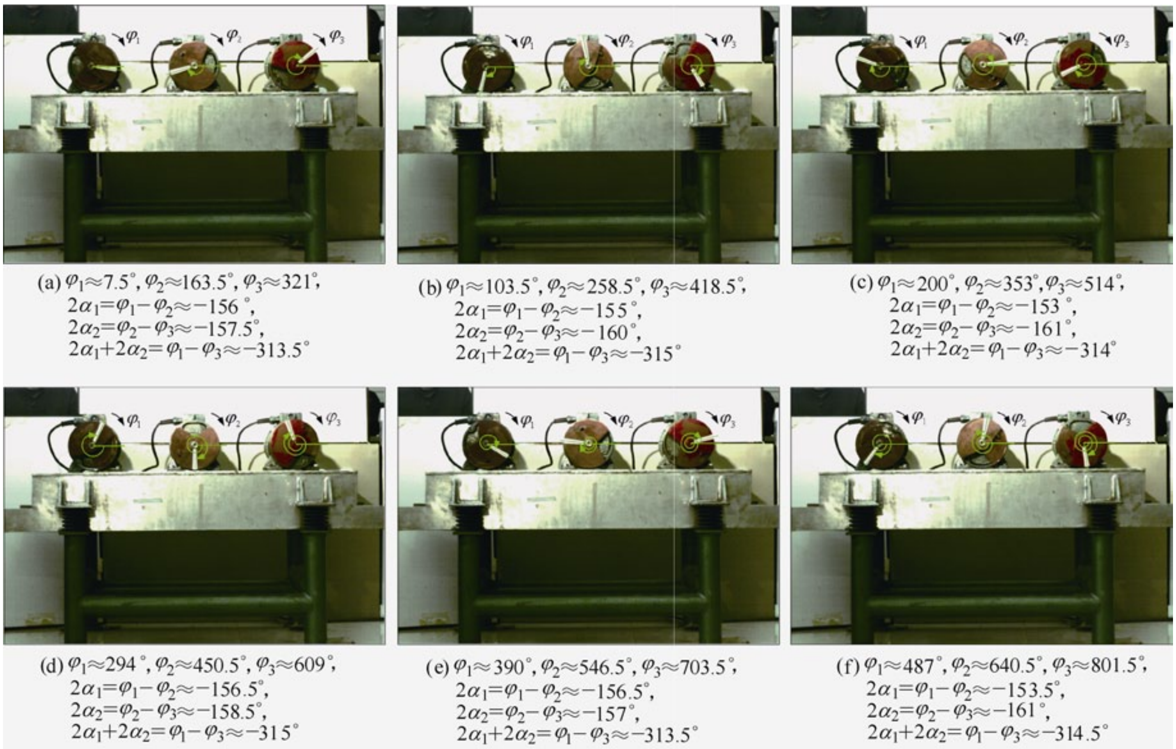


Fig. 8. Phases of three exciters recorded continually by high-speed camera in the steady-state

6 Conclusions

(1) To guarantee the synchronous and stable operation of three exciters, the parameters of system should satisfy both the criterion of synchronization and that of stability of synchronous states.

(2) Synchronization of three exciters stems from the coupling dynamic characteristics of system, the greater the coupling moment among three exciters, the smaller their phase differences, and the stronger the synchronization ability.

(3) In the far-resonant vibrating system with small damping, using three exciters instead of two on a single

base can not improve the effective power of system, duo to the mutual compensation of the exciting forces. To overcome the above defect, it is only by controlled synchronization that three exciters can operate synchronously with zero phase difference, this is the desire of our future work.

(4) Strong Background and design supervision are provided for the further theory investigation on synchronization of multi-exciter and its engineering application.

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